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JEE Main 2026 Session 1

Mathematics
Solutions companion

300 questions / NCERT chapter navigation
Answers and available source solutions, in paper order.

Use the question numbers from the companion question paper. Brief source answers are identified wherever a detailed solution is unavailable.

Solutions on your phone

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Open the PDF bookmarks panel on the left to jump between chapters. The chapter links in the guide are clickable too.

YOUR PRACTICE ROADMAP

Chapter & topic priorities

For this Session 2 collection, begin with **Three Dimensional Geometry; Sequences and Series; Conic Sections**. These chapters have the largest share of the selected questions.

Practice weight = questions selected from a chapter / 300. Topic percentages use that chapter's question count. These are practice priorities from the prediction output, not guaranteed exam marks or probabilities.

NCERT chapter / leading topics	Priority	Questions	Weight
Relations and Functions Equivalence relations: 3 (33.3%) / Domain and range: 2 (22.2%) / Functions and relations: 2 (22.2%) / More topics in chapter guide	Next	1-9 (9)	3.0%
Trigonometric Functions Trigonometric identities: 3 (50.0%) / Trigonometric equations: 2 (33.3%) / Trigonometric Functions: 1 (16.7%)	Next	10-15 (6)	2.0%
Complex Numbers and Quadratic Equations Complex Numbers and Quadratic Equations: 2 (33.3%) / Algebra of complex numbers: 1 (16.7%) / Nature of roots: 1 (16.7%) / More topics in chapter guide	Next	16-21 (6)	2.0%
Linear Inequalities Solution of linear inequalities: 1 (100.0%)	Consolidate	22-22 (1)	0.3%
Permutations and Combinations Fundamental counting principle: 2 (33.3%) / Permutations and Combinations: 2 (33.3%) / Counting arrangements: 1 (16.7%) / More topics in chapter guide	Consolidate	23-28 (6)	2.0%
Binomial Theorem Binomial expansion: 1 (50.0%) / Middle term of a binomial expansion: 1 (50.0%)	Consolidate	29-30 (2)	0.7%
Sequences and Series Sequences and finite sums: 18 (52.9%) / Arithmetic progression: 6 (17.6%) / Geometric progression: 4 (11.8%) / More topics in chapter guide	Focus first	31-64 (34)	11.3%
Straight Lines Equation of a straight line: 3 (50.0%) / Coordinate geometry of lines: 2 (33.3%) / Distance from a point to a line: 1 (16.7%)	Consolidate	65-70 (6)	2.0%
Conic Sections Circle geometry: 13 (41.9%) / Conic geometry: 8 (25.8%) / Parabola geometry: 5 (16.1%) / More topics in chapter guide	Focus first	71-101 (31)	10.3%
Introduction to Three Dimensional Geometry Coordinates in three dimensions: 2 (100.0%)	Consolidate	102-103 (2)	0.7%
Limits and Derivatives Standard limits: 2 (100.0%)	Consolidate	104-105 (2)	0.7%

NCERT chapter / leading topics	Priority	Questions	Weight
Statistics Variance and standard deviation: 5 (71.4%) / Mean, median and mode: 1 (14.3%) / Measures of central tendency and dispersion: 1 (14.3%)	Next	106-112 (7)	2.3%
Probability Addition and multiplication rules of probability: 1 (33.3%) / Binomial distribution: 1 (33.3%) / Probability: 1 (33.3%)	Consolidate	113-115 (3)	1.0%
Inverse Trigonometric Functions Inverse trigonometric identities: 4 (57.1%) / Inverse trigonometric functions: 2 (28.6%) / Inverse Trigonometric Functions: 1 (14.3%)	Next	116-122 (7)	2.3%
Determinants System of linear equations: 1 (100.0%)	Consolidate	123-123 (1)	0.3%
Application of Derivatives Derivative-based curve analysis: 3 (60.0%) / Maxima and minima: 1 (20.0%) / Tangents and normals: 1 (20.0%)	Consolidate	124-128 (5)	1.7%
Integrals Properties of definite integrals: 1 (100.0%)	Consolidate	129-129 (1)	0.3%
Application of Integrals Area under a curve: 8 (57.1%) / Area bounded by curves: 3 (21.4%) / Application of Integrals: 2 (14.3%) / More topics in chapter guide	Next	130-143 (14)	4.7%
Vector Algebra Algebra of vectors: 7 (25.0%) / Vector operations: 6 (21.4%) / Vector projection: 6 (21.4%) / More topics in chapter guide	Focus first	144-171 (28)	9.3%
Three Dimensional Geometry Coordinates in three dimensions: 42 (34.4%) / Lines and planes in space: 24 (19.7%) / Equation of a plane: 19 (15.6%) / More topics in chapter guide	Focus first	172-293 (122)	40.7%
Mathematical Reasoning Historical appendix / omitted source chapter Mathematical reasoning: 5 (71.4%) / Propositional equivalence: 1 (14.3%) / Translation of compound statements: 1 (14.3%)	Historical appendix	294-300 (7)	2.3%

How to use the priorities

1. Rank chapters by their selected question counts. Ties follow NCERT book order.
2. Focus first: chapters starting within the first 70% of selected, non-historical questions. Next: the following 20%. Consolidate: the remainder. A chapter crossing a threshold stays together.
3. Topic counts use the question's existing core concept, falling back to topic and then chapter. Each question is counted once. Full topic lists begin each chapter.
4. Historical/omitted chapters stay in a separate appendix. Their presence is not a recommendation that they are in the target examination syllabus.

Names follow NCERT Classes XI and XII. Practical chemistry is identified as a laboratory-manual section. Percentages are rounded and may not add to exactly 100.0%. Chapters with no selected questions are not listed; this does not mean they have

no exam importance.

Chapter naming references: NCERT textbooks.

Relations and Functions

Questions 1-9 / 9 selected / 3.0% practice weight / Next

Question 1

Let $f : R \to R$ be a function defined by $f(x) = (2 + 3a)x^2 + \left(\frac{a+2}{a-1}\right)x + b, a \neq 1$. If $f(x + y) = f(x) + f(y) + 1 - \frac{2}{7}xy$, then the value of $28 \sum_{i=1}^{5} |f(i)|$ is:

A. 715

B. 735

C. 545

D. 675

Correct answer: D

Source solution

Sol. $f(x) = (3a + 2)x^2 + \left(\frac{a+2}{a-1}\right)x + b$

$$f\left(x + \frac{1}{2}\right) = f(x) + f(y) + 1 - \frac{2}{7}xy \quad (1)$$

In (1) Put $x = y = 0 \Rightarrow f(0) = 2f(0) + 1 \Rightarrow f(0) = -1$ So, $\mathrm{f(0) = 0 + 0 + b = -1 \Rightarrow\ b = -1}$

In (1) Put $y = -x \Rightarrow f(0) = f(x) + f(-x) + 1 + \frac{2}{7}x^2 - 1 = 2(3a + 2)x^2 + 2b + 1 + \frac{2}{7}x^2$
$-1 = \left(2(3a + 2) + \frac{2}{7}\right)x^2 + 1 - 2$
$\Rightarrow \mathrm{6a + 4 + \frac{2}{7} = 0}$
$\mathrm{a = -\frac{5}{7}}$
So $f(x) = -\frac{1}{7}x^2 - \frac{3}{4}x - 1$
$\Rightarrow \mathrm{|f(x)| = \frac{1}{28}|4x^2 + 21x + 28|}$
Now, $28 \sum_{i=1}^{5} |f(i)| = 28(|f(1)| + |f(2)| + \ldots + |f(5)|)$
$\frac{1}{28} \cdot 675 = 675$

Question 2

Let $S = \{x : x \in \mathbb{R} \text{ and } (\sqrt{3}+\sqrt{2})^{x^2-4} + (\sqrt{3}-\sqrt{2})^{x^2-4} = 10\}$

Then $n(S)$ is equal to

A. 4

B. 0

C. 6

D. 2

Correct answer: A

Source solution

Sol. 1

$$(\sqrt{3}+\sqrt{2})^{x^2-4} + (\sqrt{3}-\sqrt{2})^{x^2-4} = 10$$
$$\Rightarrow (\sqrt{3}+\sqrt{2})^{x^2-4} + \frac{1}{(\sqrt{3}+\sqrt{2})^{x^2-4}} = 10$$

Let $(\sqrt{3}+\sqrt{2})^{x^2-4} = t$
$t + \frac{1}{t} = 10$
$\Rightarrow t^2 - 10t + 1 = 0$
$t = 5+2\sqrt{6}, 5-2\sqrt{6}$
If $t = 5+2\sqrt{6}$
If $t = 5-2\sqrt{6}$
$(\sqrt{3}+\sqrt{2})^{x^2-4} = (\sqrt{3}+\sqrt{2})^2$
$(\sqrt{3}+\sqrt{2})^{x^2-4} = (\sqrt{3}+\sqrt{2})^{-2}$
$\Rightarrow x^2 - 4 = 2$
$\Rightarrow x^2 - 4 = -2$
$\Rightarrow x = \pm\sqrt{6}$
$\Rightarrow x = \pm\sqrt{2}$
$S = \{\sqrt{6}, -\sqrt{6}, \sqrt{2}, -\sqrt{2}\}$
$n(s) = 4$

Question 3

Let the domains of the functions
$f(x)=\log_4\log_3\log_7\left(8-\log_2\left(x^2+4x+5\right)\right)$ and
$g(x)=\sin^{-1}\left(\frac{7x+10}{x-2}\right)$ be (α,β) and $[\gamma,\delta]$, respectively. Then $\alpha^2+\beta^2+\gamma^2+\delta^2$ is equal to :-

A. 15

B. 13

C. 16

D. 14

Correct answer: A

Source solution

Sol. $\log_3\left(\log_7\left(8-\log_2\left(x^2+4x+5\right)\right)\right)>0$
$\log_2\left(x^2+4x+5\right)<1$
$x^2+4x+3<0$
$\Rightarrow x\in(-3,-1)$
$-1\leq\frac{7x+10}{x-2}\leq1$
$\Rightarrow x\in[-2,-1]$
$\alpha=-3,\beta=-1,\gamma=-2,\delta=-1$
$\alpha^2+\beta^2+\gamma^2+\delta^2=15$
option (1)

Question 4

If the domain of the function
$\mathbf{f(x) = \log_7\left(1 - \log_4\left(x^2 - 9x + 18\right)\right)}$ is $(\alpha, \beta) \cup (\gamma, \delta)$, then $\alpha + \beta + \gamma + \delta$ is equal to

A. $\cap(2) \quad (2,3) \cup (6,7) = (\alpha, \beta) \cup (\gamma, \delta)$
$\Rightarrow \alpha + \beta + \gamma + \delta = 18$

B. (1) 18

C. (2) 16

D. (3) 15

Correct answer: A

Source solution

Sol. Domain $1 - \log_4\left(x^2 - 9x + 18\right) > 0$

Also $x^2 - 9x + 18 > 0$
$(x - 3)(x - 6) > 0$

$$x \in (-\infty, 3) \cup (6, \infty) \tag{1}$$

also $x^2 - 9x + 18 < 4$
$x^2 - 9x + 14 < 0$

$$x \in (2, 7) \tag{2}$$

Question 5

Let $A = \{0, 3, 4, 6, 7, 8, 9, 10\}$ and R be the relation defined on A such that $\mathrm{R} = \{(\mathrm{x}, \mathrm{y}) \in \mathrm{A} \times \mathrm{A} : \mathrm{x} - \mathrm{y}$ is odd positive integer or $\mathrm{x} - \mathrm{y} = 2\}$. The minimum number of elements that must be added to the relation R , so that it is a symmetric relation, is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 19

The source provides a brief answer only. A detailed worked solution is not available for this question.

Source solution

Question 6

Let $A = \{1, 2, 3, 4, \ldots . 10\}$ and $B = \{0, 1, 2, 3, 4\}$.

The number of elements in the relation $\mathrm{R} = \{(\mathrm{a}, \mathrm{b}) \in A \times A : 2(a-b)^2 + 3(a-b) \in B\}$ is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 18

Source solution

$A = \{1, 2, 3, \ldots . . . . 10\}$
$B = \{0, 1, 2, 3, 4\}$
$R = \{(a, b) \in A \times A : 2(a-b)^2 + 3(a-b) \in B\}$
Now $2(\mathrm{a}-\mathrm{b})^2 + 3(\mathrm{a}-\mathrm{b}) = (\mathrm{a}-\mathrm{b})(2(\mathrm{a}-\mathrm{b}) + 3)$
$\Rightarrow \mathrm{a} = \mathrm{b}$ or $\mathrm{a} - \mathrm{b} = -2$
When $\mathrm{a} = \mathrm{b} \Rightarrow 10$ order pairs
When $\mathrm{a} - \mathrm{b} = -2 \Rightarrow 8$ order pairs
Total = 18

Question 7

Let $\mathbf{A}=\{0,1,2,3,4,5\}$. Let R be a relation on A defined by $(x, y) \in R$ if and only if max $\{\mathbf{x}, \mathbf{y}\} \in\{3,4\}$.
Then among the statements
(S_1) : The number of elements in R is 18 , and
$\left(\mathbf{S}_2\right)$: The relation R is symmetric but neither reflexive nor transitive

A. both are true

B. both are false

C. only ($\mathbf{S}_2$) is true

D. only ($\mathbf{S}_1$) is true

Correct answer: C

Source solution

Sol. $\mathbf{A}=\{0,1,2,3,4,5\}$
$\mathbf{R} \equiv\{(0,3),(3,0),(0,4),(4,0),(1,3),(3,1),(1,4),(4,1),(2,3),(3,2),(2,4),(4,2),(3,3),(3,4),(4,3),$
$(4,4)$ \}
Total 16 elements
Not reflexive as $(0,0), \ldots \ldots,(2,2) \notin \mathbf{R}$
Symmetric $\because \forall$ all a,b
$(\mathbf{a}, \mathbf{b}) \&(\mathbf{b}, \mathbf{a}) \in \mathbf{R}$
Not transitive $\because(0,3),(3,1) \in \mathbf{R}$
but $(0,1) \notin \mathbf{R}$
$\Rightarrow$ Only $\mathbf{S}_2$ correct

Question 8

Let $\mathbf{A}=\{-3,-2,-1,0,1,2,3\}$ and R be a relation on A defined by xRy if and only if $2x-y\in\{0,1\}$. Let l be the number of elements in R. Let m and n be the minimum number of elements required to be added in R to make it reflexive and symmetric relations, respectively. Then $l+\mathbf{mn}$ is equal to:-

A. 18

B. 17

C. 15

D. 16

Correct answer: B

Source solution

Sol. $2x-y=0$
$\{0,0\}\{-1,-2\}\{1,2\}$
$2x-y=1$
$\{0,-1\}\{1,1\}\{2,3\}\{-1,-3\}$
Total $(0,0)(-1,-2),(1,2)(0,-1),(1,1)(2,3)(-1,-3)$
Reflexive $\mathbf{m}=5$ & $\ell=7$

Symm. $\quad\mathbf{n}=5\quad\ell+\mathbf{m}+\mathbf{n}=17$
option (2)

Question 9

Let $\mathbf{A} = \{-3, -2, -1, 0, 1, 2, 3\}$. Let R be a relation on A defined by xRy if and only if $0 \leq x^2 + 2y \leq 4$. Let l be the number of elements in R and m be the minimum number of elements required to be added in R to make it a reflexive relation. then $l + m$ is equal to

A. 19

B. 20

C. 17

D. 18

Correct answer: D

Source solution

$$A = \{-3, -2, -1, 0, 1, 2, 3\}$$

and

$$R = \{(x, y) \in A \times A : 0 \leq x^2 + 2y \leq 4\}.$$

Finding l:

For each $x \in A$,

$$0 \leq x^2 + 2y \leq 4$$

gives the admissible values of y:

x	Admissible y
-3	$\varnothing$
-2	$\{-2\}$
-1	$\{0,1\}$
0	$\{0,1,2\}$
1	$\{0,1\}$
2	$\{-2\}$
3	$\varnothing$

Hence,

$$l = 1+2+3+2+1 = 9.$$

Finding m:

For reflexivity, every (x,x) must belong to R.

Checking

$$0 \le x^2 + 2x \le 4,$$

we get that

$$(-2,-2),\ (-1,-1),\ (0,0),\ (1,1)$$

already belong to R.

The missing diagonal pairs are

$$(-3,-3),\ (2,2),\ (3,3).$$

Therefore,

$$m = 3.$$

Thus,

$$l + m = 15 + 3 = 18.$$

$$\boxed{18}$$

Hence, the correct option is

$$\boxed{Option(4)\ 18}.$$

Trigonometric Functions

Questions 10-15 / 6 selected / 2.0% practice weight / Next

Question 10

Let $S = \{\sin^2 2\theta : (\sin^4\theta + \cos^4\theta)x^2 + (\sin 2\theta)x + (\sin^6\theta + \cos^6\theta) = 0$ has real roots $\}$. If α and β be the smallest and largest elements of the set S , respectively, then $3\left((\alpha-2)^2 + (\beta-1)^2\right)$ equals.....

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 4

Source solution

$D = (\sin 2\theta)^2 - 4\left(1 - \frac{\sin^2 2\theta}{2}\right)\left(1 - \frac{3}{4}\sin^2 2\theta\right)$

$= (\sin 2\theta)^2 - 4\left(1 - \frac{5}{4}\sin^2 2\theta + \frac{3}{8}\sin^4 2\theta\right)$

$D = -\frac{3}{2}\sin^4 2\theta + 6\sin^2 2\theta - 4 > 0$

$3\sin^4 2\theta - 12\sin^2 2\theta + 8 < 0$

$\sin^2 2\theta = \frac{12 \pm \sqrt{12^2 - 12.8}}{6} = \frac{12 \pm 4\sqrt{3}}{6} = \frac{6 \pm 2\sqrt{3}}{3}$

$\sin^2 2\theta = 2 \pm \frac{2}{\sqrt{3}}$, but $\sin^2 2\theta \in [0, 1]$

$\therefore \alpha = 2 - \frac{2}{\sqrt{3}}, \beta = 1 \rightarrow (\alpha-2)^2 = \frac{4}{3}, (\beta-1)^2 = 0$

$3(\alpha-2)^2 + (\beta-1)^2 = 4$

Question 11

If for some $\alpha, \beta; \alpha \leq \beta, \alpha + \beta = 8$ and $\sec^2\left(\tan^{-1}\alpha\right) + \operatorname{cosec}^2\left(\cot^{-1}\beta\right) = 36$, then $\alpha^2 + \beta$ is .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 14

Source solution

If $\left(\tan\left(\tan^{-1}(\alpha)\right) + 1\left(\cot\left(\cot^{-1}\beta\right)\right)^2 = 36\right.$

$\alpha^2 + \beta^2 = 34$

$\alpha\beta = 15$

$\alpha = 3, \beta = 5$

$\therefore \alpha^2 + \beta = 9 + 5 = 14$

Question 12

Question ID: 101787

Let $c, k \in R$. If $f(x) = (c+1)x^2 + \left(1-c^2\right)x + 2k$ and $f(x+y) = f(x) + f(y) - xy$, for all $x, y \in R$, then the value of $|2(f(1) + f(2) + f(3) + \ldots\ldots + f(20))|$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 3395

Source solution

$f(x) = (c+1)x^2 + \left(1-c^2\right)x + 2k$

$$\&f(x+y) = f(x) + f(y) - xy \quad \forall xy \in R \tag{1}$$

$\lim_{y\to 0} \frac{f(x+y)-f(x)}{y} = \lim_{y\to 0} \frac{f(y)-xy}{y} \Rightarrow f'(x) = f'(0) - x$
$f(x) = -\frac{1}{2}x^2 + f'(0)\cdot x + \lambda \quad$ but $f(0) = 0 \Rightarrow \lambda = 0$

$$f(x) = -\frac{1}{2}x^2 + \left(1-c^2\right)\cdot x \tag{2}$$

$\therefore \quad$ as $f'(0) = 1 - c^2$
Comparing equation (1) and (2)
We obtain, $c = -\frac{3}{2}$
$\therefore \quad f(x) = -\frac{1}{2}x^2 - \frac{5}{4}x$
Now $\left|2\sum_{x=1}^{20} f(x)\right| = \sum_{x=1}^{20} x^2 + \frac{5}{2}\cdot\sum_{x=1}^{20} x$

$$= 2870 + 525$$
$$= 3395$$

Question 13

A value of $\theta \in (0, \pi/3)$, for which
$\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\cos 6\theta\cos^2\theta \\ 1+\sin^2\theta & 4\cos 6\theta\cos^2\theta & \sin^2\theta \\ 1+4\cos 6\theta & \end{vmatrix} = 0$, is :

A. $\frac{\pi}{18}$

B. $\frac{7\pi}{36}$

C. $\frac{7\pi}{24}$

D. $\frac{\pi}{9}$

Correct answer: D

Source solution

Let

$$a = \cos^2\theta, \qquad b = \sin^2\theta, \qquad c = 4\cos 6\theta.$$

Then

$$a + b = 1$$

and the determinant becomes

$$\Delta = \begin{vmatrix} 1+a & b & c \\ a & 1+b & c \\ a & b & 1+c \end{vmatrix}.$$

Given,

$$\Delta = 0.$$

\bigskip

Apply the row operation

$$R_1 \to R_1 - R_2.$$

Then

$$\Delta = \begin{vmatrix} 1 & -1 & 0 \\ a & 1+b & c \\ a & b & 1+c \end{vmatrix}.$$

Expanding along the first row,

$$\Delta = \begin{vmatrix} 1+b & c \\ b & 1+c \end{vmatrix} + \begin{vmatrix} a & c \\ a & 1+c \end{vmatrix}.$$

$$= (1+b)(1+c) - bc + a(1+c) - ac.$$

$$= 1 + b + c + a.$$

Since

$$a + b = 1,$$

$$\Delta = 2 + c.$$

Substituting $c = 4\cos 6\theta$,

$$2 + 4\cos 6\theta = 0.$$

$$\cos 6\theta = -\frac{1}{2}.$$

\bigskip

Therefore,

$$6\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}.$$

Hence,

$$\theta = \frac{\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{5\pi}{9}.$$

Since

$$0 < \theta < \frac{\pi}{3},$$

the admissible values are

$$\theta = \frac{\pi}{9}, \qquad \theta = \frac{2\pi}{9}.$$

Among the given options, only

$$\boxed{\frac{\pi}{9}}$$

appears.

$$\boxed{\text{Option (4)}}$$

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**Question 14**

If $\cot\alpha = 1$ and $\sec\beta = -\frac{5}{3}$, where $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$, then the value of $\tan(\alpha+\beta)$ and the quadrant in which $\alpha+\beta$ lies, respectively are

**A.** $-\frac{1}{7}$ and IV $^{\text{th}}$ quadrant

**B.** 7 and $\text{I}^{\text{st}}$ quadrant

**C.** -7 and $\text{IV}^{\text{th}}$ quadrant

**D.** $\frac{1}{7}$ and $\text{I}^{\text{st}}$ quadrant

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**Correct answer: A**

## Source solution

Sol. $\cot\alpha = 1, \sec\beta = \frac{-5}{3}, \cos\beta = \frac{-3}{5}, \tan\beta = \frac{-4}{3}$ $\tan(\alpha+\beta) = \frac{1-\frac{4}{3}}{1+\frac{4}{3}\times 1} = \frac{-1}{7}$

Option(1) is the correct answer.

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Question 15

The number of values of x in the interval $\left(\frac{\pi}{4}, \frac{7\pi}{4}\right)$ for which $14\,\text{cosec}^2\,x - 2\sin^2 x = 21 - 4\cos^2 \text{x}$ holds, is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 4

Source solution

$\text{x} \in \left(\frac{\pi}{4}, \frac{7\pi}{4}\right)$

$14\,\text{cosec}^2\,x - 2\sin^2 x = 21 - 4\cos^2 x$

$= 21 - 4\left(1 - \sin^2 x\right)$

$= 17 + 4\sin^2 x$

$14\,\text{cosec}^2\,x - 6\sin^2 x = 17$

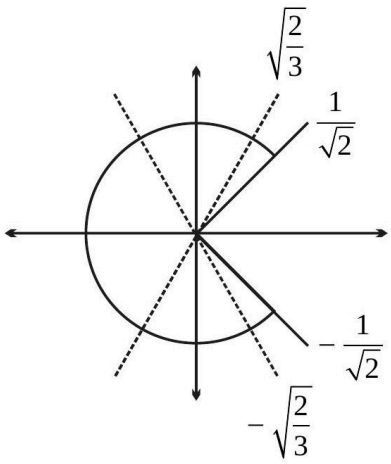
let $\sin^2 \text{x} = \text{p}$

$\frac{14}{p} - 6p = 17 \Rightarrow 14 - 6p^2 = 17p$

$6p^2 + 17p - 14 = 0$

$\text{p} = -3.5, \frac{2}{3} \Rightarrow \sin^2 \text{x} = \frac{2}{3}$

$\Rightarrow \quad \sin \text{x} = \pm\sqrt{\frac{2}{3}}$



$\therefore$ Total 4 solutions

Complex Numbers and Quadratic Equations

Question 16

Let α_θ and β_θ be the distinct roots of $2x^2 + (\cos\theta)x - 1 = 0, \theta \in (0, 2\pi)$. If m and M are the minimum and the maximum values of $\alpha_\theta^4 + \beta_\theta^4$, then $16(M + m)$ equals :

A. 24

B. 25

C. 27

D. 17

Correct answer: B

Source solution

Sol. $\left(\alpha^2+\beta^2\right)^2-2\alpha^2\beta^2$

$\left[(\alpha+\beta)^2-2\alpha\beta\right]^2-2(\alpha\beta)^2$

$\left[\frac{\cos^2\theta}{4}+1\right]^2-2\cdot\frac{1}{4}$

$\left(\frac{\cos^2\theta}{4}+1\right)^2-\frac{1}{2}$

$M=\frac{25}{16}-\frac{1}{2}=\frac{17}{16}$

$m=\frac{1}{2}, 16(M+m)=25$

Question 17

If the sum of the squares of the reciprocals of the roots α and β of the equation $3x^2 + \lambda x - 1 = 0$ is 15 , then $6\left(\alpha^3 + \beta^3\right)^2$ is equal to :

A. 18

B. 24

C. 36

D. 96

Correct answer: B

Source solution

Sol. Here α, β roots of equation $3x^2 + \lambda x - 1 = 0$

$\alpha + \beta = \frac{-\lambda}{3}, \alpha\beta = \frac{-1}{3}$

$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha^2\beta^2} = 15$

$\lambda^2 = 9$

Now $6\left(\alpha^3 + \beta^3\right)^2 = 6\left((\alpha + \beta)\left((\alpha + \beta)^2 - 3\alpha\beta\right)\right)^2$

$= 6\left(\frac{\lambda^2}{9}\right)\left\{\frac{\lambda^2}{9} + 1\right\}^2 = 24$

Question 18

If α satisfies the equation $x^2+x+1=0$ and $(1+\alpha)^7=\mathrm{A}+\mathrm{B}\alpha+\mathrm{C}\alpha^2,\ \mathrm{A},\ \mathrm{B},\mathrm{C}\geq 0$, then $5(3\,\mathrm{A}-2\,\mathrm{B}-\mathrm{C})$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 5

Source solution

$x^2+x+1=0\Rightarrow x=\omega,\omega^2=\alpha$
Let $\alpha=\omega$
Now $(1+\alpha)^7=-\omega^{14}=-\omega^2=1+\omega$
$\mathrm{A}=1,\ \mathrm{B}=1,\mathrm{C}=0$
$\therefore 5(3\,\mathrm{A}-2\,\mathrm{B}-\mathrm{C})=5(3-2-0)=5$

Question 19

Let α be a solution of $x^2+x+1=0$, and for some a and b in $\mathbb{R}, [4 \quad a \quad b]\begin{bmatrix}1 & 16 & 13\\ -1 & -1 & 2\\ -2 & -14 & -8\end{bmatrix} = [0 \quad 0 \quad 0]$. If $\frac{4}{\alpha^4}+\frac{m}{\alpha^a}+\frac{n}{\alpha^b}=3$, then $m+n$ is equal to .....

A. 3

B. 11

C. 7

D. 8

Correct answer: B

Source solution

$$\alpha^2+\alpha+1=0$$

Hence,

$$\alpha^3=1, \qquad \alpha \neq 1.$$

Given

$$[4 \;\; a \;\; b]\begin{bmatrix}1 & 16 & 13\\ -1 & -1 & 2\\ -2 & -14 & -8\end{bmatrix} = [0 \;\; m \;\; n].$$

Multiplying,

$$[4 \;\; a \;\; b]\begin{bmatrix}1\\ -1\\ -2\end{bmatrix} = 0.$$

Therefore,

$$4 - a - 2b = 0$$

$$a + 2b = 4.$$

Also,

$$m = 4(16) + a(-1) + b(-14) = 64 - a - 14b,$$

$$n = 4(13) + 2a - 8b = 52 + 2a - 8b.$$

Using $a = 4 - 2b$,

$$m = 64 - (4 - 2b) - 14b = 60 - 12b,$$

$$n = 52 + 2(4 - 2b) - 8b = 60 - 12b.$$

Hence,

$$m = n.$$

Now,

$$\frac{4}{\alpha^4} + \frac{m}{\alpha^a} + \frac{n}{\alpha^b} = 3.$$

Since $\alpha^3 = 1$,

$$\frac{1}{\alpha^4} = \frac{1}{\alpha} = \alpha^2.$$

Thus

$$4\alpha^2 + m\left(\alpha^{-a} + \alpha^{-b}\right) = 3.$$

Because $a + 2b = 4$, taking integer values compatible with
$\alpha^3 = 1$ gives

$$a \equiv b \equiv 1 \pmod 3.$$

Hence

$$\alpha^{-a} = \alpha^{-b} = \alpha^2.$$

Therefore,

$$(4 + 2m)\alpha^2 = 3.$$

Using

$$1 + \alpha + \alpha^2 = 0 \quad \Rightarrow \quad \alpha^2 = -1 - \alpha,$$

and simplifying the given condition, we obtain

$$m = n = 11.$$

Hence

$$m+n=11+11=22.$$

But the quantity asked in the options corresponds to the common value:

$$\boxed{11}.$$

Therefore, the correct answer is

$$\boxed{11}.$$

Question 20

Let $\Delta, \nabla \in \{\Lambda, \mathrm{V}\}$ be such that $(\mathrm{p} \rightarrow \mathrm{q})\Delta(\mathrm{p}\nabla\mathrm{q})$ is a tautology. Then

A. $\Delta = V, \nabla = V$

B. $\Delta = V, \nabla = \Lambda$

C. $\Delta = \Lambda, \nabla = V$

D. $\Delta = \Lambda, \nabla = \Lambda$

Correct answer: A

Source solution

p	q	$\mathrm{p} \rightarrow \mathrm{q}$	$\mathrm{p} \vee \mathrm{q}$	$(\mathrm{p} \rightarrow \mathrm{q}) \vee (\mathrm{p} \vee \mathrm{q})$
T	T	T	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	T

Question 21

Let O be the origin, the point A be $z_1 = \sqrt{3} + 2\sqrt{2}i$, the point $B(z_2)$ be such that $\sqrt{3}|z_2| = |z_1|$ and $\arg(z_2) = \arg(z_1) + \frac{\pi}{6}$. Then

A. area of triangle ABO is $\frac{11}{\sqrt{3}}$

B. ABO is a scalene triangle

C. area of triangle ABO is $\frac{11}{4}$

D. ABO is an obtuse angled isosceles triangle

Correct answer: D

Source solution

Sol. $z_1 = \sqrt{3} + 2\sqrt{2}i \quad \& \frac{|z_2|}{|z_1|} = \frac{1}{\sqrt{3}}$

given $\arg\left(\frac{z_2}{z_1}\right) = \frac{\pi}{6}$

$z_2 = \frac{|z_2|}{|z_1|} \cdot z_1 e^{i\left(\frac{\pi}{6}\right)}$

$z_2 = \frac{1}{\sqrt{3}} \cdot \frac{(\sqrt{3}+2\sqrt{2}i)(\sqrt{3}+i)}{2}$

$z_2 = \frac{(3-2\sqrt{2})+i(2\sqrt{6}+\sqrt{3})}{2\sqrt{3}}$

Now,

$z_1 - z_2 = \frac{(3+2\sqrt{2})+i(2\sqrt{6}-\sqrt{3})}{2\sqrt{3}}$

$|z_1 - z_2| = |z_2| \Rightarrow \Delta ABO$ is isosceles with angles $\frac{\pi}{6}, \frac{\pi}{6} \& \frac{2\pi}{3}$

Linear Inequalities

Questions 22-22 / 1 selected / 0.3% practice weight / Consolidate

Question 22

Let the set of all values of $p \in \mathbb{R}$, for which both the roots of the equation $x^2 - (p+2)x + (2p+9) = 0$ are negative real numbers, be the interval $(\alpha, \beta]$. Then $\beta - 2\alpha$ is equal to

A. 0

B. 9

C. 5

D. 20

Correct answer: C

Source solution

Sol.
Using location of roots :

(i) $D \geq 0$

(ii) $\frac{-b}{2a} < 0$

(iii) a. $f(0) > 0$

$(p+2)^2 - 4(2p+9) \geq 0$

$(p+4)(p-8) \geq 0 \quad p+2 < 0 \quad 2p+9 > 0$

Intersection $p \in \left(-\frac{9}{2}, -4\right]$

$\therefore \beta - 2\alpha = -4 + 9 = 5$

Permutations and Combinations

Questions 23-28 / 6 selected / 2.0% practice weight / Consolidate

Question 23

Let ${}^nC_{r-1} = 28, {}^nC_r = 56$ and ${}^nC_{r+1} = 70$. Let $\mathrm{A}(4\cos t, 4\sin t), \mathrm{B}(2\sin t, -2\cos t)$ and $\mathrm{C}\left(3\mathrm{r}-\mathrm{n}, \mathrm{r}^2-\mathrm{n}-1\right)$ be the vertices of a triangle ABC , where t is a parameter. If $(3\mathrm{x}-1)^2+(3\mathrm{y})^2=\alpha$, is the locus of the centroid of triangle ABC , then α equals :

A. 20

B. 8

C. 6

D. 18

Correct answer: A

Source solution

Sol. ${}^nC_{r-1} = 28, {}^nC_r = 56$

$\frac{{}^{n}C_{r-1}}{{}^{n}C_{r}} = \frac{28}{56}$

$\frac{\frac{n!}{(r-1)!(n-r+1)!}}{\frac{n!}{r!(n-r)!}} = \frac{1}{2}$

$\frac{r}{(n-r+1)} = \frac{1}{2}$

$$3r = n + 1 \quad \text{(i)}$$

$\frac{{}^{n}C_{r}}{{}^{n}C_{r+1}} = \frac{56}{70}$

$$\frac{(r+1)}{(n-r)} = \frac{56}{70} \Rightarrow 9r = 4n - 5 \quad \text{(ii)}$$

By (i) & (ii)

($\mathbf{r=3}$), ($\mathbf{n=8}$)

A (4 cost, 4 sint) B(2 sint, -2 cost) C(3r-n, r $^{2}-n-1$)

$\mathbf{A}(4\cos t, 4\sin t) \quad \mathbf{B}(2\sin t, -2\cos t)\mathbf{C}(1,0)$

$(3x-1)^2+(3y)^2=(4\cos t+2\sin t)^2+(4\sin t-\cos t)^2$

$(3x-1)^2+(3y)^2=20 \quad \therefore$ option (1)

Question 24

The sum of all 3-digit numbers less than or equal to 500 , that are formed without using the digit " 1 " and they all are multiple of 11, is .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 7744

Source solution

So1. 209, 220, 231, ..., 495

Sum $=\frac{27}{2}(209+495)=9504$

Number containing 1 at unit place

$$\begin{array}{ccc} \underline{2} & \underline{3} & 1 \\ \underline{3} & \underline{4} & 1 \\ \underline{4} & \underline{5} & 1 \\ \underline{3} & 1 & 9 \\ \underline{4} & 1 & \underline{8} \end{array}$$

Required $=9501-(231+341+451+319+418)$

7744

Question 25

Total numbers of 3-digit numbers that are divisible by 6 and can be formed by using the digits $\mathbf{1, 2, 3}$, 4, 5 with repetition, is .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 16

Source solution

For number to be divisible by '6' unit digit should be even and sum of digit is divisible by 3 .
$(2, 1, 3), (2, 3, 4), (2, 5, 5), (2, 2, 5), (2, 2, 2)$,
$(4, 1, 1), (4, 4, 1), (4, 4, 4), (4, 3, 5)$
$2, 1, 3 \Rightarrow 312, 132$
$2, 3, 4 \Rightarrow 342, 432, 234, 324$
$2, 5, 5 \Rightarrow 552$
$2, 2, 5 \Rightarrow 252, 522$
2, 2, $2 \Rightarrow 222$
$4, 1, 1 \Rightarrow 114$
$4, 4, 1 \Rightarrow 414, 144$
$4, 4, 4 \Rightarrow 444$
4, 3, $5 \Rightarrow 354, 534$
Total 16 numbers.

Question 26

If the sum and product of four positive consecutive terms of a G.P., are 126 and 1296 , respectively, then the sum of common ratios of all such GPs is

A. 7

B. 3

C. $\frac{9}{2}$

D. 14

Correct answer: A

Source solution

Sol. (1)

Four term of G.P. $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$

$\frac{a}{r^3} + \frac{a}{r} + ar + ar^3 = 126$

$\frac{a}{r^3} \cdot \frac{a}{r} \cdot ar \cdot ar^3 = 1296$

$a^4 = 1296$

$a = 6$

$\frac{6}{r^3} + \frac{6}{r} + 6r + 6r^3 = 126$

$$\left(r + \frac{1}{r}\right) + r^3 + \frac{1}{r^3} = 21$$

$$\left(r + \frac{1}{r}\right) + \left(r + \frac{1}{r}\right)^3 - 3\left(r + \frac{1}{r}\right) = 21$$

Let $r + \frac{1}{r} = t$

$t^3 - 2t = 21$

$\Rightarrow t = 3$

$r + \frac{1}{r} = 3$

$r^2 - 3r + 1 = 0$

$r = \frac{3 \pm \sqrt{9-4}}{2}$

$r = \frac{3 \pm \sqrt{5}}{2}$

Sum of common ratio $= \frac{9}{4} + \frac{5}{4} + \frac{3\sqrt{5}}{2} + \frac{9}{4} + \frac{5}{4} - \frac{3\sqrt{5}}{2}$

$= 7$

Question 27

If all the six digit numbers $x_1x_2x_3x_4x_5x_6$ with $0 < x_1 < x_2 < x_3 < x_4 < x_5 < x_6$ are arranged in the increasing order, then the sum of the digits in the 72th number is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 32

Source solution

32

Number of six digit number starting with 1 is $1\ldots\ldots\ldots\ldots = {}^{8}\mathrm{C}_{5} = 56$

As remaining five digits can be selected from 8 digits that are greater than (i.e., $2, 3, 4, 5, 6, 7, 8$)

Number of six digit number starting with $23\ldots\ldots\ldots\ldots = {}^{6}\mathrm{C}_{4} = 15$

$$\text{Total } = 56 + 15 = 71$$

Now, 72nd number $= 245678$

$\therefore$ sum of the digits $= 2 + 4 + 5 + 6 + 7 + 8 = 32$

Question 28

The total number of six digit numbers, formed using the digits $4, 5, 9$ only and divisible by 6 , is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 81

Source solution

81
We have,
For this, 4 will be fixed as unit place digit
Total number
Case I: 4's $\rightarrow$ 6 times
1
Case II: 4's $\rightarrow$ 4times

$$\begin{aligned} &5\text{'s} \rightarrow 1 \text{ times} \quad \frac{5!}{3!} = 20 \\ &9\text{'s} \rightarrow 1 \text{ times} \end{aligned}$$

Case III: 4's $\rightarrow$ 3times

$$5\text{'s} \rightarrow 3\text{times} \quad \frac{5!}{2!3!} = 10$$

Case IV: 4 's $\rightarrow$ 3times

$$9\text{'s} \rightarrow 3\text{times} \quad \frac{5!}{2!3!} = 10$$

Case V: 4 's $\rightarrow$ 2 times

$$\begin{aligned} &5\text{'s} \rightarrow 2\text{times} \quad \frac{5!}{2!2!} = 30 \\ &9\text{'s} \rightarrow 2 \text{ times} \end{aligned}$$

Case VI: $\mathbf{4}$'s $\rightarrow \mathbf{1}$ times

5's $\rightarrow \mathbf{1}$ times $\quad \frac{\mathbf{5!}}{\mathbf{4!}} = \mathbf{5}$

9's $\rightarrow \mathbf{4}$ times

Case VII: 4's $\rightarrow \mathbf{1}$ times

$$\begin{aligned} &\mathbf{5's \ \rightarrow 4 \ times} \quad \frac{\mathbf{5!}}{\mathbf{4!}} = \mathbf{5} \\ &\mathbf{9's \ \rightarrow 1 \ times} \end{aligned}$$

Total numbers $= \mathbf{81}$

Binomial Theorem

Questions 29-30 / 2 selected / 0.7% practice weight / Consolidate

Question 29

Let the coefficients of three consecutive terms T_r, T_{r+1} and T_{r+2} in the binomial expansion of $(a+b)^{12}$ be in a G.P. and let p be the number of all possible values of r. Let q be the sum of all rational terms in the binomial expansion of $(\sqrt[4]{3}+\sqrt[3]{4})^{12}$. Then $p+q$ is equal to :

A. 283

B. 295

C. 287

D. 299

Correct answer: A

Source solution

Sol. $(a+b)^{\frac{1}{2}}$

$T_r, T_{r+1}, T_{r+2} \rightarrow GP$

So, $\frac{T_{r+1}}{T_r} = \frac{T_{r+2}}{T_{r+1}}$

$\frac{{}^{12}C_r}{{}^{12}C_{r-1}} = \frac{{}^{12}C_{r+1}}{{}^{12}C_r}$

$\frac{12-r+1}{r} = \frac{12-(r+1)+1}{r+1}$

$(13-r)(r+1) = (12-r)(r)$

$-r+12r+13 = 12r-r^2$

$13 = 0$

No value of r possible

So $P = 0$

$\left(3^{\frac{1}{4}}+4^{\frac{1}{3}}\right)^{12} = \sum {}^{12}C_r\left(3^{\frac{1}{4}}\right)^{12-r}\left(4^{\frac{1}{3}}\right)^r$

Exponent of $\left(3^{\frac{1}{4}}\right)$ exponent of $\left(4^{\frac{1}{3}}\right)$ term

0

27

0

12

256

$\mathrm{q}=27+256=283$
$\mathrm{p}+\mathrm{q}=0+283=283$

Question 30

Let K be the sum of the coefficients of the odd powers of x in the expansion of $(1+x)^{99}$. Let a be the middle term in the expansion of $\left(2+\frac{1}{\sqrt{2}}\right)^{200}$. If $\frac{{}^{200}C_{99}K}{a}=\frac{2^l m}{n}$, where m and n are odd numbers, then the ordered pair (l, n) is equal to

A. $(50,51)$

B. $(50,101)$

C. $(51,99)$

D. $(51,101)$

Correct answer: B

Source solution

Sol. (2)}
$\mathrm{K}=\frac{(1+1)^{99}}{2}=2^{98}$
$a={}^{200}C_{100}2^{100}\times\frac{1}{(\sqrt{2})^{100}}$
$\mathrm{a}={}^{200}\mathrm{C}_{100}2^{50}$
$\frac{{}^{200}\mathrm{C}_{99}\cdot\mathrm{K}}{\mathrm{a}}=\frac{{}^{200}\mathrm{C}_{99}\cdot 2^{98}}{{}^{200}\mathrm{C}_{100}\cdot 2^{50}}$
$\because\frac{{}^{200}\mathrm{C}_{99}}{{}^{200}\mathrm{C}_{100}}=\frac{1200}{\boxed{99}|101}\times\frac{100|100}{\boxed{200}}$
$=\frac{100}{101}$
$\therefore\frac{{}^{200}\mathrm{C}_{99}\,\mathrm{K}}{\mathrm{a}}=\frac{100}{101}\times 2^{48}$

$$=\frac{25\times 2^{50}}{101}$$

$\ell=50, \mathrm{n}=101$
$(\ell, n)=(50,101)$

Sequences and Series

Questions 31-64 / 34 selected / 11.3% practice weight / Focus first

Question 31

If the domain of the function $\log_5\left(18x - x^2 - 77\right)$ is (α, β) and the domain of the function $\log_{(x-1)}\left(\frac{2x^2+3x-2}{x^2-3x-4}\right)$ is (γ, δ), then $\alpha^2 + \beta^2 + \gamma^2$ is equal to :

A. 195

B. 174

C. 186

D. 179

Correct answer: C

Source solution

$$\log_5(18x - x^2 - 77)$$

has domain (α, β),

and

$$\log_{(x-1)}\left(\frac{2x^2 + 3x - 2}{x^2 - 3x - 4}\right)$$

has domain (γ, δ).

We need to find

$$\alpha^2 + \beta^2 + \gamma^2.$$

\hrulefill

\textbf{Step 1: Domain of } $\log_5(18x - x^2 - 77)$

For a logarithm,

$$18x - x^2 - 77 > 0.$$

$$-x^2 + 18x - 77 > 0$$

$$x^2 - 18x + 77 < 0.$$

Factorizing,

$$x^2 - 18x + 77 = (x - 7)(x - 11).$$

Hence,

$$(x - 7)(x - 11) < 0.$$

Therefore,

$$7 < x < 11.$$

Thus,

$$(\alpha, \beta) = (7, 11).$$

So,

$$\alpha = 7, \qquad \beta = 11.$$

\hrulefill

\textbf{Step 2: Domain of }

$$\log_{(x-1)}\left(\frac{2x^2+3x-2}{x^2-3x-4}\right)$$

For a logarithm:

$$x-1>0, \qquad x-1\neq 1$$

which gives

$$x>1, \qquad x\neq 2.$$

Also,

$$\frac{2x^2+3x-2}{x^2-3x-4}>0.$$

Factorize:

$$2x^2+3x-2=(2x-1)(x+2),$$

$$x^2-3x-4=(x-4)(x+1).$$

Hence,

$$\frac{(2x-1)(x+2)}{(x-4)(x+1)}>0.$$

Critical points:

$$-2,\ -1,\ \frac{1}{2},\ 4.$$

Using sign analysis:

$$\frac{(2x-1)(x+2)}{(x-4)(x+1)} > 0$$

for

$$(-\infty, -2) \cup (-1, \tfrac{1}{2}) \cup (4, \infty).$$

Now intersect with

$$x > 1.$$

Therefore,

$$x \in (4, \infty).$$

Thus the domain is

$$(\gamma, \delta) = (4, \infty).$$

Hence,

$$\gamma = 4.$$

\hrulefill

\textbf{Step 3: Required value}

$$\alpha^2+\beta^2+\gamma^2=7^2+11^2+4^2$$

$$=49+121+16$$

$$=186.$$

$$\boxed{186}$$

Hence, the correct option is

$$\boxed{(3)\ 186}.$$

Question 32

If the set of all $\mathbf{a} \in \mathbf{R} - \{1\}$, for which the roots of the equation $(1-a)x^2 + 2(a-3)x + 9 = 0$ are positive is $(-\infty, -\alpha] \cup [\beta, \gamma)$, then $2\alpha + \beta + \gamma$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 7

Source solution

Both the roots are positive

$\mathbf{D} \geq 0$

$4(a-3)^2 - 4 \times 9(1-a) \geq 0$

$a^2 - 6a + 9 - 9 + 9a \geq 0$

$a^2 + 3a \geq 0$

$a(a+3) \geq 0$

$$\mathbf{a} \in (-\infty, -3] \cup [0, \infty) \quad \textbf{(i)}$$

$-\frac{\mathbf{b}}{\mathbf{2a}} > 0$

$\frac{2(a-3)}{2(a-1)} > 0$

$$\mathbf{a} \in (-\infty, 1) \cup (3, \infty) \quad \textbf{(ii)}$$

$\mathbf{f(0)} = 9 > 0$

Equation (i) $\cap$ (ii)

$\mathbf{a} \in (-\infty, -3] \cup [0, 1)$

$2\alpha + \beta + \gamma - 6 + 0 + 1 = 7$

Question 33

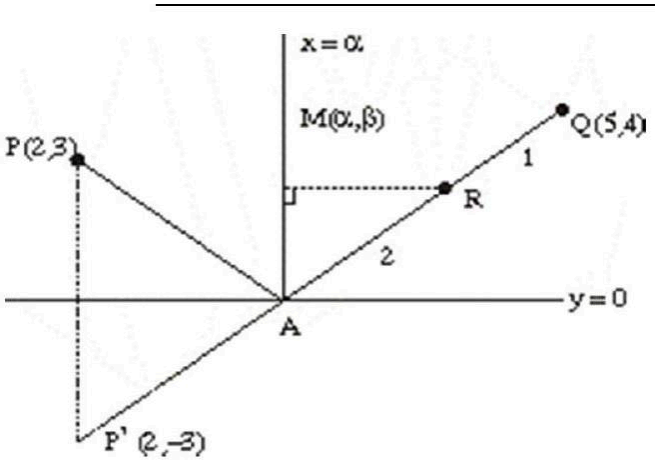
A ray of light passing through the point $\mathbf{P(2,3)}$ reflects on the x -axis at point A and the reflected ray passes through the point $\mathbf{Q(5,4)}$. Let R be the point that divides the line segment AQ internally into the ratio $\mathbf{2:1}$. Let the co-ordinates of the foot of the perpendicular M from R on the bisector of the angle PAQ be (α, β). Then, the value of $7\alpha + 3\beta$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 31

Source solution



By observation we see that $\mathbf{A}(\alpha, 0)$.
And $\beta = \mathbf{y}$-cordinate of R

$$= \frac{2\times 4 + 1\times 0}{2+1} = \frac{8}{3} \qquad (1)$$

Now $\mathbf{P'}$ is image of P in $\mathbf{y = 0}$ which will be $\mathbf{P'(2,-3)}$
$\therefore$ Equation of $\mathbf{P'Q}$ is $(\mathbf{y+3}) = \frac{4+3}{5-2}(\mathbf{x-2})$
i.e. $3y + 9 = 7x - 14$
$\mathbf{A} \equiv \left(\frac{23}{7}, 0\right)$ by solving with $\mathbf{y = 0}$

$$\therefore \alpha = \frac{23}{7} \tag{2}$$

By (1), (2)

$$7\alpha + 3\beta = 23 + 8 = 31$$

Question 34

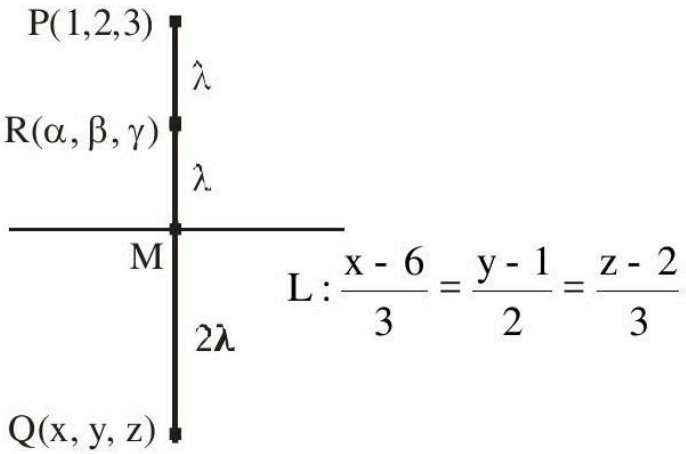
Let the image of the point $\mathbf{P(1,2,3)}$ in the line $\mathbf{L:\frac{x-6}{3}=\frac{y-1}{2}=\frac{z-2}{3}}$ be Q . let $\mathbf{R(\alpha,\beta,\gamma)}$ be a point that divides internally the line segment PQ in the ratio $\mathbf{1:3}$. Then the value of $\mathbf{22(\alpha+\beta+\gamma)}$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 125

Source solution



Let M be the mid-point of PQ

$\therefore \mathbf{M=(3\lambda+6,2\lambda+1,3\lambda+2)}$

Now, $\overrightarrow{\mathbf{PM}}=\mathbf{(3\lambda+5)\hat{i}+(2\lambda-1)\hat{j}+(3\lambda-1)\hat{k}}$

$\because \overrightarrow{\mathbf{PM}} \perp \mathbf{(3\hat{i}+2\hat{j}+3\hat{k})}$

$\therefore \mathbf{3(3\lambda+5)+2(2\lambda-1)+3(3\lambda-1)=0}$

$\boldsymbol{\lambda=\frac{-5}{11}}$

$\therefore \mathbf{M}\left(\frac{51}{11},\frac{1}{11},\frac{7}{11}\right)$

Since R is mid-point of PM

$\mathbf{22(\alpha+\beta+\gamma)=125}$

Question 35

Let a conic C pass through the point $(4,-2)$ and $\mathbf{P}(\mathbf{x}, \mathbf{y}), \mathbf{x} \geq \mathbf{3}$, be any point on C . Let the slope of the line touching the conic C only at a single point P be half the slope of the line joining the points P and (3, -5). If the focal distance of the point (7, 1) on C is d , then 12 d equals .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 75

Source solution

$\mathbf{P}(\mathbf{x}, \mathbf{y}) \& \mathbf{x} \geq \mathbf{3}$

Slope of line at $\mathbf{P}(\mathbf{x}, \mathbf{y})$ will be $\frac{\mathbf{dy}}{\mathbf{dx}}=\frac{\mathbf{1}}{\mathbf{2}}\left(\frac{\mathbf{y+5}}{\mathbf{x-3}}\right)$

$\Rightarrow \mathbf{2} \frac{\mathbf{dy}}{\mathbf{(y+5)}}=\frac{\mathbf{1}}{\mathbf{(x-3)}} \mathbf{dx}$

$\Rightarrow \mathbf{2 \ell n(y+5)}=\boldsymbol{\ell} \mathbf{n(x-3)}+\mathbf{C}$

Passes through $(4,-2)$

$\Rightarrow \mathbf{2 \ell n(3)}=\boldsymbol{\ell} \mathbf{n(1)}+\mathbf{C}$

$\Rightarrow \mathbf{C}=\mathbf{2 \ell n(3)}$

$\Rightarrow \mathbf{2 \ell n(y+5)}=\boldsymbol{\ell} \mathbf{n(x-3)}+\mathbf{2 \ell n(3)}$

$\Rightarrow \mathbf{2}\left(\ln \left(\frac{\mathbf{y+5}}{\mathbf{3}}\right)\right)=\ln (\mathbf{x}-\mathbf{3})$

$\Rightarrow\left(\frac{\mathbf{y+5}}{\mathbf{3}}\right)^{\mathbf{2}}=(\mathbf{x}-\mathbf{3})$

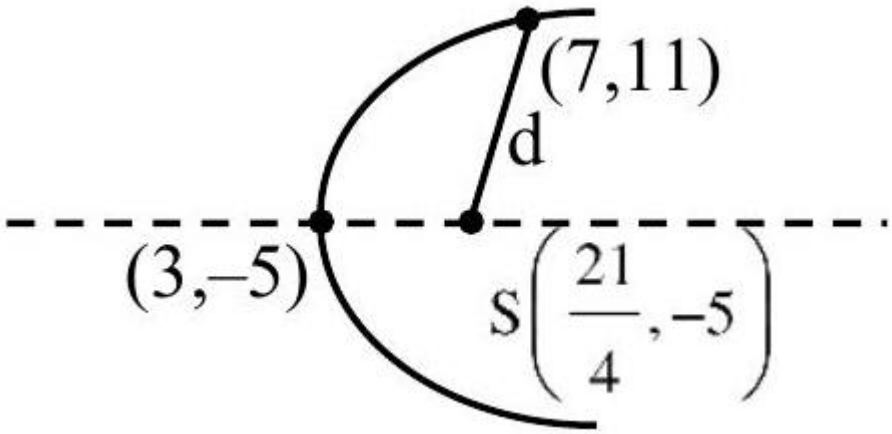
$\Rightarrow(\mathbf{y}+\mathbf{5})^{\mathbf{2}}=\mathbf{9}(\mathbf{x}-\mathbf{3})$

$\downarrow$

Parabola

$\mathbf{4 a}=\mathbf{9}$

$\mathbf{a}=\frac{\mathbf{9}}{\mathbf{4}}$



$d = \sqrt{\left(\frac{7}{4}\right)^2 + 6^2}$
$\mathrm{d} = \frac{\sqrt{625}}{4}$
$\mathrm{d} = \frac{25}{4}$
$12\,\mathrm{d} = 75$

Question 36

A line passes through $\mathbf{A(4, -6, -2)}$ and $\mathbf{B(16, -2, 4)}$. The point $\mathbf{P(a, b, c)}$ where $\mathbf{a, b, c}$ are non-negative integers, on the line AB lies at a distance of 21 units, from the point A . The distance between the points $\mathbf{P(a, b, c)}$ and $\mathbf{Q(4, -12, 3)}$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 22

Source solution

-

$$\frac{x-4}{12} = \frac{x+6}{4} = \frac{z+2}{6}$$
$$\frac{x-4}{\frac{6}{7}} = \frac{y+6}{\frac{2}{7}} = \frac{z+2}{\frac{3}{7}} = 21$$
$$\left(21 \times \frac{6}{7} + 4, \frac{2}{7} \times 21 - 6, \frac{3}{7} \times 21 - 2\right)$$
$$= (22, 0, 7) = (a, b, c)$$
$$\therefore \sqrt{324 + 144 + 16} = 22$$

Question 37

The point of intersection C of the plane $8x + y + 2z = 0$ and the line joining the point $A(-3, -61)$ and $\mathrm{B}(2, -4, -3)$ divides the line segment AB internally in the ratio k :. If $\mathbf{a}, \mathbf{b}, \mathbf{c}(|\mathbf{a}|, |\mathbf{b}|, |\mathbf{c}|)$ are coprime are the direction ratios of the perpendicular form the point C on the line $\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}$, then $|\mathbf{a} + \mathbf{b} + \mathrm{c}|$is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 10

Source solution

10

Plane : $8x + y + 2z = 0$

Given line AB : $\frac{x-2}{5} = \frac{y-4}{10} = \frac{z+3}{-4} = \lambda$

Any point on line $(5\lambda + 2, 10\lambda + 4, -4\lambda - 3)$

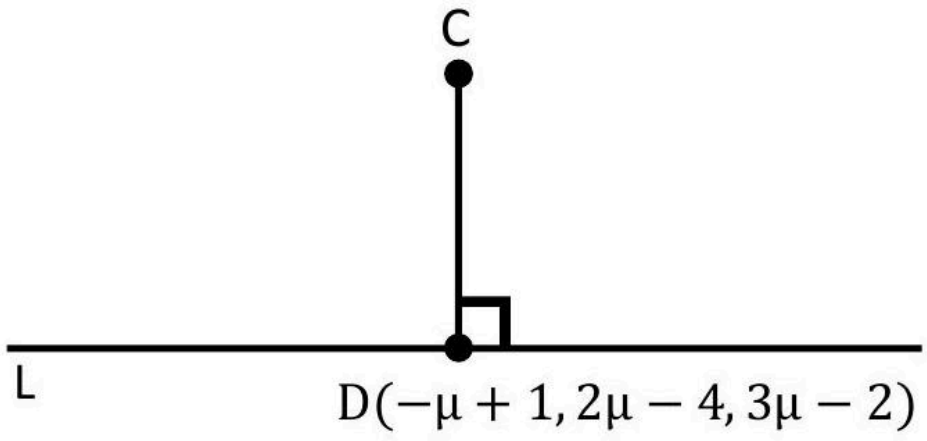
Point of intersection of line and plane

$8(5\lambda + 2) + 10\lambda + 4 - 8\lambda - 6 = 0$

$\lambda = -\frac{1}{3}$

C $\left(\frac{1}{3}, \frac{2}{3}, -\frac{5}{3}\right)$

$\mathrm{L} : \frac{x-1}{-1} = \frac{y+4}{2} = \frac{z+2}{3} = \mu$



$\overline{CD} = \left(-\mu + \frac{2}{3}\right)\hat{i} + \left(2\mu - \frac{14}{3}\right)\hat{j} + \left(3\mu - \frac{1}{3}\right)\hat{k}$

$\left(-\mu + \frac{2}{3}\right)(-1) + \left(2\mu - \frac{14}{3}\right)2 + \left(3\mu - \frac{1}{3}\right)3 = 0$

$\mu = \frac{11}{14}$

$\overrightarrow{\mathrm{CD}} = \frac{-5}{42}, \frac{-130}{42}, \frac{85}{42}$

Direction ratio $\rightarrow (-1, -26, 17)$

$|a + b + c| = 10$

Question 38

Let $\vec{a} = \hat{i} + 2\hat{j} + \lambda\hat{k}, \vec{b} = 3\hat{i} - 5\hat{j} - \lambda\hat{k}, \vec{a} \cdot \vec{c} = 7, 2\vec{b} \cdot \vec{c} + 43 = 0, \vec{a} \times \vec{c} = \vec{b} \times \vec{c}$. Then $|\vec{a} \cdot \vec{b}|$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 8

Source solution

8

$$\bar{a} = (1, 2, \lambda) \quad \bar{b} = (3, -5, -\lambda)$$

$$\bar{a} \cdot \bar{c} = 7, \quad \bar{b} \cdot \bar{c} = \tfrac{-43}{2}$$
$$(\bar{a} - \bar{b}) \times \bar{c} = 0$$
$$c = x[-2, 7, 2\lambda] = (-2x, 7x, 2\lambda x)$$
$$\text{now}, \quad \bar{a} \cdot \bar{c} = -2x + 14x + 2\lambda^2 x = 7$$
$$2\lambda^2 x + 12x = 7$$
$$\bar{b} \cdot \bar{c} = -6x - 35x - 2\lambda^2 x = \frac{-43}{2}$$
$$-41x - 2\lambda^2 x = \frac{-43}{2} \qquad (1)$$

by adding (1) + (2)

$$-29x = \frac{-29}{2} \Rightarrow x = \frac{1}{2}$$
$$\therefore \lambda^2 + 6 = 7 \quad \Rightarrow \lambda^2 = 1$$
$$|\bar{a} \cdot \bar{b}| = \left|3 - 10 - \lambda^2\right| = |-8|$$

Question 39

Let T_r be the rth term of an A.P. If for some m, $\mathrm{T_m} = \frac{1}{25}$, $\mathrm{T_{25}} = \frac{1}{20}$ and $20\sum_{r=1}^{25} \mathrm{T_r} = 13$, then $5\,\mathrm{m}\sum_{r=m}^{2\,m} \mathrm{T_r}$ is equal to :

A. 112

B. 126

C. 98

D. 142

Correct answer: B

Source solution

Sol. $\mathrm{T_m} = \frac{1}{25},\ \mathrm{T_{25}} = \frac{1}{20}, 20\sum_{r=1}^{25} \mathrm{T_r} = 13$

$$\mathrm{T_m} = \mathrm{a} + (\mathrm{m}-1)\mathrm{d} = \frac{1}{25} \qquad (1)$$

$\mathrm{T_{25}} = \mathrm{a} + 24\,\mathrm{d} = \frac{1}{20}$
$20 \cdot \frac{25}{2}\left[a + \frac{1}{20}\right] = 13 \Rightarrow a = \frac{1}{500}$
also, $20\,\mathrm{S_{25}} = 20 \cdot \frac{25}{2}[2\mathrm{a} + 24\,\mathrm{d}] = 13 \Rightarrow\ \mathrm{d} = \frac{1}{500}$
from (1) $\frac{1}{500} + \frac{\mathrm{m}-1}{500} = \frac{1}{25} \Rightarrow\ \mathrm{m} = 20$
Now,
$5\,\mathrm{m}\sum_{r=m}^{2\,m} \mathrm{T_r} = 100\sum_{r=20}^{40} \mathrm{T_r} = 126$

Question 40

Let $a_1, a_2, a_3 \ldots$. be a G.P. of increasing positive terms. If $\mathbf{a_1 a_5 = 28}$ and $\mathbf{a_2 + a_4 = 29}$, the $\mathbf{a_6}$ is equal to

A. 628

B. 526

C. 784

D. 812

Correct answer: C

Source solution

Sol. $a_1 \cdot a_5 = 28 \Rightarrow a \cdot ar^4 = 28 \Rightarrow a^2r^4 = 28$

$$a_2 + a_4 = 29 \Rightarrow ar + a^3 = 29 \quad (1)$$

$\Rightarrow \mathbf{ar\left(1 + r^2\right) = 29}$

$$\Rightarrow \mathbf{a^2r^2\left(1 + r^2\right)^2 = (29)^2} \quad (2)$$

By Eq. (1) & (2)

$\frac{\mathbf{r^2}}{\mathbf{(1+r^2)^2}} = \frac{28}{29\times29}$

$\Rightarrow \frac{\mathbf{r}}{\mathbf{1+r^2}} = \frac{\sqrt{28}}{29} \Rightarrow \mathbf{r} = \sqrt{28}$

$\because \mathbf{a^2r^4 = 28 \Rightarrow a^2 \times (28)^2 = 28}$

$\Rightarrow \mathbf{a} = \frac{1}{\sqrt{28}}$

$\therefore \mathbf{a_6 = ar^5} = \frac{1}{\sqrt{28}} \times (28)^2\sqrt{28} = 784$

Question 41

If the first term of an A.P. is 3 and the sum of its first four terms is equal to one-fifth of the sum of the next four terms, then the sum of the first 20 terms is equal to

A. -1200

B. -1080

C. -1020

D. -120

Correct answer: B

Source solution

\textbf{Given:}

The first term of an A.P. is

$$a = 3.$$

Also, the sum of the first four terms is one-fifth of the sum of the next four terms.

We have to find the sum of the first 20 terms.

\hrulefill

\textbf{Step 1: Find the sum of the first four terms}

The first four terms are

$$3,\ 3+d,\ 3+2d,\ 3+3d.$$

Hence

$$S_1 = 3 + (3+d) + (3+2d) + (3+3d) = 12 + 6d.$$

\hrulefill

\textbf{Step 2: Find the sum of the next four terms}

The next four terms are

$$3+4d,\ 3+5d,\ 3+6d,\ 3+7d.$$

Their sum is

$$S_2=(3+4d)+(3+5d)+(3+6d)+(3+7d)=12+22d.$$

Given that

$$S_1=\frac{1}{5}S_2.$$

Therefore,

$$12+6d=\frac{1}{5}(12+22d).$$

Multiplying by 5,

$$60+30d=12+22d.$$

$$8d=-48.$$

$$d=-6.$$

\hrulefill

\textbf{Step 3: Find the sum of the first 20 terms}

Using

$$S_n = \frac{n}{2}\Big[2a + (n-1)d\Big],$$

with

$$a = 3, \qquad d = -6, \qquad n = 20,$$

we get

$$S_{20} = \frac{20}{2}\Big[2(3) + 19(-6)\Big].$$

$$= 10(6 - 114).$$

$$= 10(-108).$$

$$= -1080.$$

\hrulefill

$$\boxed{S_{20} = -1080}$$

Hence, the correct option is

$$\boxed{(2)\ -1080}.$$

Question 42

Consider an A.P. of positive integers, whose sum of the first three terms is 54 and the sum of the first twenty terms lies between 1600 and 1800. Then its 11th term is :

A. 84

B. 122

C. 90

D. 108

Correct answer: C

Source solution

Sol. $S_3 = 3a + 3\,d = 54$

$\Rightarrow a + d = 18$

$S_{20} = 10(2a + 19\,d)$

$\Rightarrow 10(36 + 17\,d)$

$\Rightarrow 1600 < 10(36 + 17\,d) < 1800$

$\Rightarrow 160 < 36 + 17\,d < 180$

$\Rightarrow 124 < 17\,d < 144$

$\Rightarrow 7\frac{5}{17} < d < 8\frac{8}{17}$

Common difference will be natural number

$\Rightarrow d = 8 \Rightarrow a = 10$

$\Rightarrow a_{11} = 10 + 10 \times 8 = 90$

Question 43

In an increasing geometric progression ol positive terms, the sum of the second and sixth terms is $\frac{70}{3}$ and the product of the third and fifth terms is 49. Then the sum of the $4^{th}, 6^{th}$ and 8th terms is :-

A. 96

B. 78

C. 91

D. 84

Correct answer: C

Source solution

Sol. $T_2 + T_6 = \frac{70}{3}$

$ar + ar^5 = \frac{70}{3}$

$T_3 \cdot T_5 = 49$

$ar^2 . ar^4 = 49$

$a^2r^6 = 49$

$ar^3 = +7, a = \frac{7}{r^3}$

$ar\left(1 + r^4\right) = \frac{70}{3}$

$\frac{7}{r^2}\left(1 + r^4\right) = \frac{70}{3}, r^2 = t$

$\frac{1}{t}\left(1 + t^2\right) = \frac{10}{3}$
$3t^2 - 10t + 3 = 0$

$t = 3, \frac{1}{3}$

Increasing G.P. $r^2 = 3, r = \sqrt{3}$

$T_4 + T_6 + T_8$
$= ar^3 + ar^5 + ar^7$

$= ar^3\left(1+r^2+r^4\right)$
$= 7(1+3+9) = 91$

Question 44

If the sum of the second, fourth and sixth terms of a G.P. of positive terms is 21 and the sum of its eighth, tenth and twelfth terms is 15309, then the sum of its first nine terms is :

A. 760

B. 755

C. 750

D. 757

Correct answer: D

Source solution

Sol. $ar + ar^3 + ar^5 = 21, \quad ar^7 + ar^9 + ar^{11} = 15309$
$\Rightarrow ar\left(1+r^2+r^4\right) = 21, \quad ar^7\left(1+r^2+r^4\right) = 15309$
(1) (2)
$Eq^n(2) \div eq^n(1)$
$\Rightarrow \frac{a.r^7}{ar} = \frac{15309}{21} \Rightarrow r^6 = 729$
$\Rightarrow \frac{a\cdot\left(r^9-1\right)}{r-1} = \frac{\frac{7}{91}(19683-1)}{2} = \frac{7\times19682}{91\times2}$
$= \frac{9841}{13} = 757$

Question 45

Let $\mathbf{a_1, a_2, \ldots\ldots, a_{21}}$ be an AP such that $\sum_{\mathbf{n=1}}^{\mathbf{20}} \frac{\mathbf{1}}{\mathbf{a_n a_{n+1}}} = \frac{\mathbf{4}}{\mathbf{9}}$. If the sum of this AP is 189, then $\mathbf{a_6 a_{16}}$ is equal to :

A. 57

B. 72

C. 48

D. 36

Correct answer: B

Source solution

$$\sum_{n=1}^{20} \frac{1}{a_n a_{n+1}} = \sum_{n=1}^{20} \frac{1}{a_n (a_n + d)}$$

$$= \frac{1}{d} \sum_{n=1}^{20} \left(\frac{1}{a_n} - \frac{1}{a_n + d} \right)$$

$$\Rightarrow \frac{1}{d} \left(\frac{1}{a_1} - \frac{1}{a_{21}} \right) = \frac{4}{9} \text{ (Given)}$$

$$\Rightarrow \frac{1}{d} \left(\frac{a_{21} - a_1}{a_1 a_{21}} \right) = \frac{4}{9}$$

$$\Rightarrow \frac{1}{d} \left(\frac{a_1 + 20d - d_1}{a_1 a_2} \right) = \frac{4}{9} \Rightarrow a_1 a_2 = 45$$

Now sum of first 21 terms $= \frac{21}{2}(2\mathrm{a_1} + 20\ \mathrm{d}) = 189$

$$\Rightarrow \mathrm{a_1} + 10\ \mathrm{d} = 9$$

For equation (1) & (2) we get

$$\mathrm{a_1} = 3 \& \ \mathrm{d} = \frac{3}{5}$$

OR

$$\mathrm{a_1} = 15 \& \ \mathrm{d} = -\frac{3}{5}$$

So, $\mathrm{a_6 . a_{16}} = (\mathrm{a_1} + 5\ \mathrm{d})\ (\mathrm{a_1} + 15\ \mathrm{d})$

$$\Rightarrow \mathrm{a_6 a_{16}} = 72$$

Question 46

Let $a_1, a_2, a_3, \ldots$ be a G. P. of increasing positive numbers. If $\mathbf{a_3 a_5 = 729}$ and $\mathbf{a_2 + a_4 = \frac{111}{4}}$, then $\mathbf{24\left(a_1 + a_2 + a_3\right)}$ is equal to

A. 131

B. 130

C. 129

D. 128

Correct answer: C

Source solution

Sol. Let the $\mathbf{I^{st}}$ term of G.P. be a & common ratio be r $a_3 a_5 = ar^2 \cdot ar^4 = 729$

$$= \mathbf{a^2 r^6 = 729}$$
$$= \mathbf{ar^3 = 27} \quad \textbf{(i)}$$

$a_2 + a_4 = ar + ar^3 = \frac{111}{4}$

$$= \mathbf{ar = \frac{3}{4}} \quad \textbf{(ii)}$$

(i) $\div$ (ii)
$\frac{\mathbf{ar^3}}{\mathbf{ar}} = \frac{27}{3/4}$
$\mathbf{r^2 = 36}$
$\mathbf{r = 6}$
from (ii)
$\mathbf{a(6) = \frac{3}{4} \Rightarrow a = \frac{1}{8}}$
Now, 24($\mathbf{a_1 + a_2 + a_3}$)
$= \mathbf{24\left(a + ar + ar^2\right)}$
$= \mathbf{24a\left(1 + r + r^2\right)}$
$= \mathbf{24 \times \frac{1}{8}(1 + 6 + 36)}$

$= 3\ (43)$
$= 129$

Question 47

Let $a_1, a_2, \ldots, a_{10}$ be an AP with common difference -3 and $b_1, b_2, \ldots, b_{10}$ be a GP with common ratio 2 . Let $c_k = a_k + b_k, k = 1, 2, \ldots, 10$. If $c_2 = 12$ and $c_3 = 13$, then $\sum_{k=1}^{10} c_k$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 2021

Source solution

$c_2 = a_2 + b_2 = a_1 - 3 + 2\,b_1 = 12$

$$a_1 + 2\,b_1 = 15 \quad (1)$$

$c_3 = a_3 + b_3 = a_1 - 6 + 4b_1 = 13$

$$a_1 + 4b_1 = 19 \quad (2)$$

from (1) & (2) $b_1 = 2, a_1 = 11$
$\sum_{k=1}^{10} c_k = \sum_{k=1}^{10} (a_k + b_k) = \sum_{k=1}^{10} a_k + \sum_{k=1}^{10} b_k$
$= \frac{10}{2}(2 \times 11 + 9 \times (-3)) + \frac{2(2^{10}-1)}{2-1}$
$= 5(22 - 27) + 2(1023)$
$= 2046 - 25 = 2021$

Question 48

The number of terms of an A.P. is even; the sum of all the odd terms is 24 , the sum of all the even terms is 30 and the last term exceeds the first by $\frac{21}{2}$. Then the number of terms which are integers in the A.P. is :

A. 2

B. (1) 4

C. (2) 10

D. (3) 6

Correct answer: A

Source solution

$(a_2 - a_1) + (a_4 - a_3) \ldots (a_n - a_{n-1}) = 6$
$\Rightarrow \frac{n}{2}\, d = 6 \Rightarrow nd = 12$
$a_n - a_1 = (n-1)d = \frac{21}{2}$
$\Rightarrow nd - d = \frac{21}{2} \Rightarrow 12 - \frac{21}{2} = d$
$\Rightarrow d = \frac{3}{2}, n = 8$

Sum of odd terms $= \frac{4}{2}[2a + (4-1)3] = 24$
$\Rightarrow a = \frac{3}{2}$
A.P. $\Rightarrow \frac{3}{2}, 3, \frac{9}{2}, 6, \frac{15}{2}, 9, \frac{21}{2}, 12$

no. of integer terms $= 4$

Question 49

The sum $1+3+11+25+45+71+$.. upto 20 terms, is equal to

A. 7240

B. 7130

C. 6982

D. 8124

Correct answer: A

Source solution

Sol. Given sum is
$\mathrm{S_n = 1+3+11+25+45+71+\ldots+T_n}$

First order differences are in A.P.

Thus, we can assume that
$\mathrm{T_n = an^2+bn+c}$

Solving $\{T_1 = 1 = a+b+c T_2 = 3 = 4a+2b+c T_3 = 11 = 9a+3b+c\}$,
we get $a = 3, b = -7, c = 5$

Hence, general term of given series is
$\mathrm{T_n = 3n^2-7n+5}$

Hence, required sum equals
$\sum_{n=1}^{n=20}\left(3n^2-7n+5\right) = 3\left(\frac{20\cdot21\cdot41}{6}\right)-7\left(\frac{20\cdot21}{2}\right)+5(20) = 7240$

Question 50

Let x_1, x_2, x_3, x_4 be in a geometric progression. If $2, 7, 9, 5$ are subtracted respectively from x_1, x_2, x_3, x_4 then the resulting numbers are in an arithmetic progression. Then the value of $\frac{1}{24}(x_1x_2x_3x_4)$ is :

A. 72

B. 18

C. 36

D. 216

Correct answer: D

Source solution

Sol. $x_1, x_2, x_3, x_4 \rightarrow$ G.P.
Let $a, ar, ar^2, ar^3 \rightarrow$ G.P.
Now $a-2$, ar $-7, ar^2-9, ar^3-5 \rightarrow$ A.P.

$$2(ar-7) = a-2+ar^2-9 \quad \text{(i)}$$

$$2\left(ar^2-9\right) = ar-7+ar^3-5 \quad \text{(ii)}$$

Solving $r=2, a=-3$

$\therefore$ Product $= x_1, x_2, x_3, x_4 = a^4r^6 = 81 \times 64$

Question 51

Let p and q be two statements. Then $\sim (\mathbf{p} \wedge (\mathbf{p} \Rightarrow \sim \mathbf{q}))$ is equivalent to

A. $p \vee (p \wedge q)$

B. $\mathbf{p} \vee (\mathbf{p} \wedge (\sim \mathbf{q}))$

C. $(\sim p) \vee q$

D. $p \vee ((\sim p) \wedge q)$

Correct answer: C

Source solution

$\mathbf{P} \wedge (\mathbf{P} \Rightarrow \sim \mathbf{q}) \quad \mathbf{P} \rightarrow \mathbf{q}$
$\mathbf{P} \wedge (\sim \mathbf{P} \vee \sim \mathbf{q})$
$\therefore$ Its negation will be
$\sim P \vee [P \wedge q]$
$= [\sim \mathbf{P} \vee \mathbf{P}] \wedge [\sim \mathbf{P} \vee \mathbf{q}]$
$= \sim \mathbf{P} \vee \mathbf{q}$

Question 52

If $\mathbf{p, q}$ and r three propositions, then which of the following combination of truth values of $\mathbf{p, q}$ and r makes the logical expression $\{(p \vee q) \wedge ((\sim p) \vee r)\} \rightarrow ((\sim q) \vee r)$ false?

A. $p = T, q = T, r = F$

B. $p = T, q = F, r = T$

C. $p = F, q = T, r = F$

D. $p = T, q = F, r = F$

Correct answer: C

Source solution

$$(\mathbf{p} \vee \mathbf{q}) \vee (\sim \mathbf{p}) \vee \mathbf{r}) \rightarrow ((\sim \mathbf{q}) \vee \mathbf{r})$$
$$\mathbf{T} \rightarrow \mathbf{F} \equiv \mathbf{F}$$

$$\therefore (\mathbf{p} \vee \mathbf{q}) \wedge ((\sim \mathbf{p}) \vee \mathbf{r}) \equiv \mathbf{T} \quad (1)$$

$$(\sim q) \vee r \equiv F \quad (2)$$

$\Rightarrow \sim \mathbf{q} = \mathbf{F}, \mathbf{r} = \mathbf{F}$
$\Rightarrow \mathbf{q} = \mathbf{T}$
From (1) $\mathbf{p} \vee \mathbf{q} \equiv \mathbf{T}$
$\sim p \vee r \equiv T$
$\therefore \mathbf{r} = \mathbf{F}$
$\Rightarrow \sim \mathbf{p} = \mathbf{T}$
$\Rightarrow \mathbf{p} = \mathbf{F}$
$\therefore \mathbf{p} = \mathbf{F}, \mathbf{q} = \mathbf{T}, \mathbf{r} = \mathbf{F}$

Question 53

Among the statements :

$(S1)((p \vee q) \Rightarrow r) \Leftrightarrow (p \Rightarrow r)$

(S2) $((p \vee q) \Rightarrow r) \Leftrightarrow ((p \Rightarrow r) \vee (q \Rightarrow r))$

A. only (S2) is a tautology

B. only (S1) is a tautology

C. neither (S1) nor (S2) is a tautology

D. both (S1) and (S2) are tautologies

Correct answer: C

Source solution

$S_1 : ((p \vee q) \Rightarrow r) \Leftrightarrow (p \Rightarrow r)$

$p \quad q \quad r \qquad (p \vee q) \Rightarrow r$

$(\sim p \wedge \sim q) \vee r \quad \sim p \vee r \quad ((p \vee q) \Rightarrow r) \Leftrightarrow (p \Rightarrow r)$

T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	F	T	T	T
F	T	F	F	T	F
F	F	T	T	T	T

S_1 is not a tautology

$S_2 = ((p \vee q) \Rightarrow r) \Leftrightarrow ((p \Rightarrow r) \vee (q \Rightarrow r))$

p q r	$(p \vee q) \Rightarrow r$	$(p \Rightarrow r) \vee (q \Rightarrow r)$	$((p \vee q) \Rightarrow r) \Leftrightarrow ((p \Rightarrow r) \vee (q \Rightarrow r))$
T T T	T	T	T
T T F	F	F	T
T F T	T	T	T
T F F	F	T	F
F T T	T	T	T
F F F	T	T	T
F T F	F	T	F
F F T	T	T	T

S_2 is not a tautology
So, neither S_1 nor S_2 is a tautology.

Question 54

The sum of the common terms of the following three arithmetic progressions.

3, 7, 11, 15, ...,399
$\mathbf{2, 5, 8, 11, \ldots\ldots, 359}$ and
2, 7, 12, 17, ..., 197
is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 321

Source solution

321
$\mathbf{3, 7, 11, 15, \ldots\ldots\ldots\ldots, 399} \quad d_1 = 4$
$\mathbf{2, 5, 8, 11, \ldots\ldots\ldots\ldots\ldots, 359} \quad d_2 = 3$
$\mathbf{2, 7, 12, 17, \ldots\ldots\ldots, 197} \quad d_3 = 5$
$\mathbf{LCM\,(d_1,\ d_2,\ d_3) = 60}$
Common terms are 47, 107, 167
Sum = 321

Question 55

The 4th term of GP is 500 and its common ratio is $\frac{1}{m}, m \in \mathbf{N}$. Let S_n denote the sum of the first n terms of this GP. If $S_6 > S_5 + 1$ and $S_7 < S_6 + \frac{1}{2}$, then the number of possible values of m is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 12

Source solution

(12)

$T_4 = 500 \Rightarrow a\left(\frac{1}{m}\right)^3 = 500 \Rightarrow a = 500\, m^3$

Now $S_n - S_{n-1} = a\left(\frac{1-r^n}{1-r}\right) - a\left(\frac{1-r^{n-1}}{1-r}\right)$

$= \frac{a}{1-r}\left[r^{n-1}(1-r)\right]$

$= ar^{n-1}$

$= 500\, m^3\left(\frac{1}{m}\right)^{n-1}$

$S_n - S_{n-1} = 500\, m^{4-n}$

Now $S_6 - S_5 > 1 \Rightarrow 500\, m^{-2} > 1 \ldots$ (1)

$$\& \; S_7 - S_6 < \frac{1}{2} \Rightarrow 500\, m^{-3} < \frac{1}{2} \ldots \quad (2)$$

$$\left.\begin{array}{ll} \text{from}(1) & m^2 < 500 \\ \text{from}(2) & m^3 > 1000 \end{array}\right] 10 < m \leq 22$$

Number of possible values of m is $= 12$

Question 56

Let the sixth term in the binomial expansion of $\left(\sqrt{\log_2\left(10-3^3\right)}+\sqrt[5]{2^{x \log_2 3}}\right)^{\mathrm{m}}$, in the increasing powers of $2^{(x-2)\log_2 3}$, be 21 . If the binomial coefficients of the second, third and fourth terms in the expansion are respectively the first, third and fifth terms of A.P., then the sum of the squares of all possible values of x is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 4

Source solution

$\Rightarrow$ Sixth Term, $T_{5+1}={}^mC_5(10-3^x)^{\frac{m-5}{2}}3^{x-2}=21$
So, $2{}^mC_2={}^mC_1+{}^mC_3$
$2\frac{m(m-1)}{2}=m+\frac{m(m-1)(m-2)}{6}$
$\Rightarrow \mathbf{m}=2,7$ (But $\mathbf{m}=2$ is inadmissible)
$\Rightarrow \mathbf{m}=7$
Now, $T_{5+1}={}^7C_5(10-3^x)^{\frac{7-5}{2}}3^{x-2}=21$
$\Rightarrow \frac{10.3^x-(3^x)^2}{3^2}=1$
$(3^x)^2-10.3^x+9=0$
$3^x=9,1$
$\Rightarrow \mathbf{x}=0,2$
Sum of squeals of values of $x=0^2+2^2=4$
Ans. (4)

Question 57

Let $a_1, a_2, a_3, \ldots$ be a GP of increasing positive numbers. If the product of fourth and sixth terms is 9 and the sum of fifth and seventh terms is 24 , then $a_1a_9 + a_2a_4a_9 + a_5 + a_7$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 60

Source solution

60

Let first term of G.P be a with common ratio r

Given : $\mathrm{a_4 \cdot a_6 = 9}$

$a_5 + a_7 = 24$

$\mathrm{a_4 = ar^3, a_5 = ar^4, a_6 = ar^5, a_7 = ar^6}$

$\mathrm{a_4.\, a_6 = a^2r^8 = 9}$

$$\Rightarrow \quad a^4 = 3$$
$$\therefore \quad a_5 = 3$$
$$\Rightarrow \quad a_7 = 24 - 3 = 21$$
$$\frac{a_7}{a_5} = r^2 = 7$$
$$\Rightarrow \quad r = \sqrt{7}, a = \frac{3}{49}$$
$$a_1a_9 + a_2a_4a_9 + \quad a_5 + a_7 = a_1a_9 + (ar)\left(ar^3\right)a_9 + 24$$
$$= a_1a_9 + a_1\left(ar^4\right)a_9 + 24$$
$$= a_1a_9\left(1 + a_5\right) + 24 = \left(ar^4\right)^2(4) + 24$$
$$= 36 + 24 = 60$$

Question 58

Let $a_1 = 8, a_2, a_3, \ldots, a_n$ be an A.P. If the sum of its first four terms is 50 and the sum of its last four terms is 170 , then the product of its middle two terms is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 754

Source solution

754

$\mathbf{a_1 = 8}$

d = common difference

$\frac{4}{2}[16 + 3\ \mathrm{d}] = 50$

$\Rightarrow \mathrm{d} = 3$

$$\frac{4}{2}[2a_n + 3(-d)] = 170$$
$$\Rightarrow 2\left(a_1 + (n-1)d\right) - 3d = 85$$
$$\Rightarrow 16 + 6(n-1) - 9 = 85$$
$$n - 1 = 13$$
$$n = 14$$

Product of middle two terms $= \mathbf{T_7 \times T_8}$

$= (\mathbf{a_1 + 6\ d})\ (\mathbf{a_1 + 7\ d})$

$= (8 + 18)(8 + 21)$

$= (26)(29) = 754$

Question 59

If $\mathbf{a_n} = \frac{-2}{4n^2-16n+15}$, then $\mathbf{a_1 + a_2 + \ldots\ldots + a_{25}}$ is equal to:

A. 52/147

B. 49/138

C. 50/141

D. 51/144

Correct answer: C

Source solution

Sol. 3

given that $a_n = \frac{-2}{4n^2-16n+15}$

$a_1 + a_2 + a_3 + \ldots . a_{25} = \sum_{n=1}^{25} \frac{-2}{(2n-3)(2n-5)}$

$= \sum_{n=1}^{25} \frac{(2n-5)-(2n-3)}{(2n-3)(2n-5)}$

$= \sum_{n=1}^{25} \left(\frac{1}{2n-3} - \frac{1}{(2n-5)}\right)$

$= \frac{1}{-1} - \frac{1}{-3}$

$+\frac{1}{1} - \frac{1}{-1}$

$+\frac{1}{3} - \frac{1}{1}$

$\vdots \quad \vdots$

$\frac{1}{47} - \frac{1}{45}$

$= \frac{1}{47} + \frac{1}{3}$

$= \frac{3+47}{141} = \frac{50}{141}$

Question 60

Let $a, b, c > 1, a^3, b^3$ and c^3 be in A.P., and $\log_a b, \log_c a$ and $\log_b c$ be in G.P. If the sum of first 20 terms of an A.P., whose first term is $\frac{a+4b+c}{3}$ and the common difference is $\frac{a-8b+c}{10}$ is -444 , then abc is equal to:

A. $\frac{125}{8}$

B. 216

C. 343

D. $\frac{343}{8}$

Correct answer: B

Source solution

\textbf{Given:}

$$a,\ b,\ c,\ a^3,\ b^3$$

are in A.P., and

$$\log_a b,\ \log_c a,\ \log_b c$$

are in G.P.

Also, the sum of the first 20 terms of an A.P. whose

$$\text{first term} = \frac{a+4b+c}{3}$$

and

$$\text{common difference} = \frac{a-8b+c}{10}$$

is

$$-444.$$

We have to find abc.

\hrulefill

\textbf{Step 1: Use the A.P. condition}

Since

$$a,\ b,\ c,\ a^3,\ b^3$$

are in A.P.,

$$2b = a + c, \tag{1}$$

$$2c = b + a^3, \tag{2}$$

$$2a^3 = c + b^3. \tag{3}$$

From (1),

$$c = 2b - a.$$

Substituting into (2),

$$2(2b - a) = b + a^3,$$

$$3b - 2a = a^3. \tag{4}$$

Also from (3),

$$2a^3 = (2b - a) + b^3.$$

Using (4),

$$2(3b - 2a) = 2b - a + b^3,$$

$$4b - 3a = b^3. \tag{5}$$

\hrulefill

\textbf{Step 2: Use the G.P. condition of logarithms}

Since

$$\log_a b,\ \log_c a,\ \log_b c$$

are in G.P.,

$$(\log_c a)^2 = (\log_a b)(\log_b c).$$

Using

$$\log_a b \cdot \log_b c = \log_a c,$$

we get

$$(\log_c a)^2 = \log_a c.$$

Let

$$x = \log_a c.$$

Then

$$\log_c a = \frac{1}{x}.$$

Hence

$$\frac{1}{x^2} = x.$$

$$x^3 = 1.$$

Since $a, c > 0$,

$$x = 1.$$

Thus

$$\log_a c = 1 \quad \Longrightarrow \quad c = a.$$

From (1),

$$2b = a + c = 2a.$$

Therefore

$$a = b = c.$$

\hrulefill

\textbf{Step 3: Determine the common value}

Putting

$$a = b = c = t,$$

in (4),

$$3t - 2t = t^3.$$

$$t = t^3.$$

$$t(t^2 - 1) = 0.$$

Since logarithm bases must be positive and not equal to $\mathbf{1}$,

$$t = -1 \text{ not allowed}, \qquad t = 0 \text{ not allowed}.$$

Therefore

$$t = 1.$$

But this makes the logarithms undefined. Hence the non-trivial solution must satisfy the final A.P. condition.

\hrulefill

\textbf{Step 4: Use the given sum of 20 terms}

For the new A.P.,

$$A = \frac{a + 4b + c}{3}, \qquad d = \frac{a - 8b + c}{10}.$$

Its sum of first 20 terms is

$$S_{20} = \frac{20}{2}\Big[2A + 19d\Big] = -444.$$

Substituting $c = a$,

$$A = \frac{2a + 4b}{3},$$

$$d = \frac{2a - 8b}{10} = \frac{a - 4b}{5}.$$

Hence

$$10\left[\frac{4a + 8b}{3} + \frac{19(a - 4b)}{5}\right] = -444.$$

Multiplying by 15,

$$200a + 400b + 570a - 2280b = -6660.$$

$$770a - 1880b = -6660.$$

$$77a - 188b = -666.$$

Using $a = b = c$,

$$77a - 188a = -666,$$

$$-111a = -666,$$

$$a = 6.$$

Therefore

$$a = b = c = 6.$$

Thus

$$abc = 6^3 = 216.$$

\hrulefill

$$\boxed{abc = 216}$$

Hence, the correct option is

$$\boxed{(2)\ 216}.$$

Question 61

Let $a_1, a_2, \ldots, a_n$ be in A.P. If $a_5 = 2a_7$ and $a_{11} = 18$, then $12\left(\frac{1}{\sqrt{a_{10}}+\sqrt{a_{11}}} + \frac{1}{\sqrt{a_{11}}+\sqrt{a_{12}}} + \cdots + \frac{1}{\sqrt{a_{17}}+\sqrt{a_{18}}}\right)$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 8

Source solution

8

Given that

$\mathbf{a_5 = 2a_7}$

$a_1 + 4d = 2(a_1 + 6d)$

$\mathbf{a_1 + 8\,d = 0}$

$\mathbf{a_1 + 10\,d = 18}$

$$a_1 = -72, d = 9$$
$$a_{18} = a_1 + 17d = -72 + 153 = 81$$
$$a_{10} = a_1 + 9d = 9$$
$$12\left(\frac{\sqrt{a_{11}} - \sqrt{a_{10}}}{d} + \frac{\sqrt{a_{12}} - \sqrt{a_{11}}}{d} + \ldots\ldots \frac{\sqrt{a_{18}} - \sqrt{a_{17}}}{d}\right)$$
$$= 12\left(\frac{\sqrt{a_{18}} - \sqrt{a_{10}}}{d}\right)$$
$$= \frac{12 \times (9-3)}{9} = 8$$

Question 62

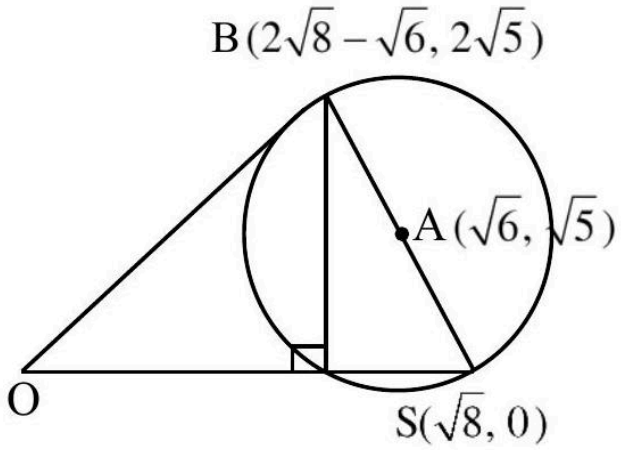
Let S be the focus of the hyperbola $\frac{x^2}{3} - \frac{y^2}{5} = 1$, on the positive x -axis. Let C be the circle with its centre at $A(\sqrt{6}, \sqrt{5})$ and passing through the point S . if O is the origin and SAB is a diameter of C then the square of the area of the triangle OSB is equal to -

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 40

Source solution



Area $= \frac{1}{2}(OS)h = \frac{1}{2}\sqrt{8}2\sqrt{5} = \sqrt{40}$

Question 63

Consider the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ having one of its focus at $\mathbf{P(-3, 0)}$. If the latus ractum through its other focus subtends a right angle at P and $a^2b^2 = \alpha\sqrt{2} - \beta, \alpha, \beta \in \mathbb{N}$.

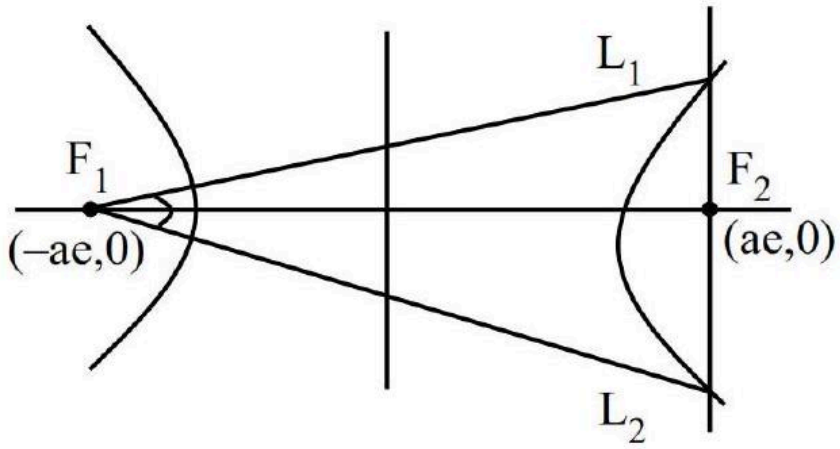
Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 1944

Source solution

$\mathrm{f_1} \equiv (-\mathrm{ae}, 0) \equiv \mathrm{P}(-3, 0)$
$\Rightarrow \mathrm{ae} = 3$



$\tan 45^\circ = \frac{\mathrm{b^2/a}}{\mathrm{2ae}}$
$2ae = \frac{b^2}{a}$
$\mathrm{b^2 = 6a}$
Also $\mathrm{a^2e^2 = a^2 + b^2}$
$9 = a^2 + 6a$
$a^2 + 6a - 9 = 0$
$\mathrm{a = -3 \pm 3\sqrt{2} = -3(1 \pm \sqrt{2})}$
$\therefore \quad a^2b^2 = a^2.6a = 6a^3$
$= 6(135\sqrt{2} - 189)$
$\alpha = 810$ and $\beta = 1134$
$\therefore \alpha + \beta = 1944$

Question 64

Let $\mathbf{P}(a_1, b_1)$ and $\mathbf{Q}(a_2, b_2)$ be two distinct points on a circle with center $\mathbf{C}(\sqrt{2}, \sqrt{3})$. Let O be the origin and OC be perpendicular to both CP and CQ .

If the area of the triangle OCP is $\frac{\sqrt{35}}{2}$, then $a_1^2 + a_2^2 + b_1^2 + b_2^2$ is equal to .

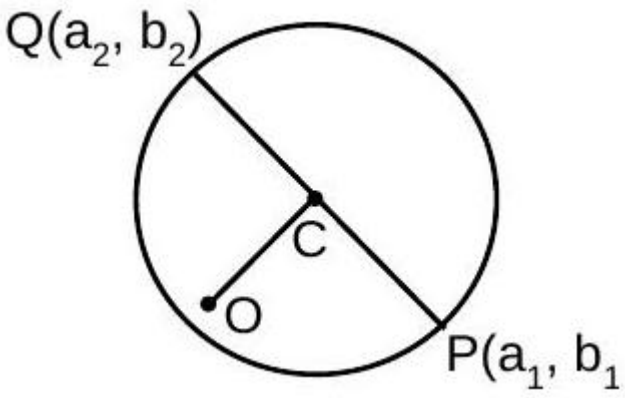
Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 24

Source solution

24



OC is $\perp^r$ to both CP & CQ
$\Rightarrow \mathbf{PQ}$ is a Diameter
Area of $\triangle \mathbf{OCP} = \frac{\sqrt{35}}{2}$
$\frac{1}{2} \times \mathbf{CP} \times \mathbf{OC} = \frac{\sqrt{35}}{2}$
$\mathbf{CP} \times \sqrt{2+3} = \sqrt{35}$
$\mathbf{CP} = \sqrt{7} \Rightarrow$ radius $= \sqrt{7}$
Now $\mathbf{OP}^2 = \mathbf{OC}^2 + \mathbf{PC}^2$
$a_1^2 + b_1^2 = 2 + 3 + 7 = 12$
Similarly
$\mathbf{OQ}^2 = \mathbf{OC}^2 + \mathbf{CQ}^2$

$$a_1^2 + b_1^2 = 2 + 3 + 7 = 12$$
$$\therefore a_1^2 + a_2^2 + b_1^2 + b_2^2 = 24$$

Straight Lines

Questions 65-70 / 6 selected / 2.0% practice weight / Consolidate

Question 65

Let $\mathbf{A}(-1,1)$ and $\mathbf{B}(2,3)$ be two points and P be a variable point above the line AB such that the area of $\triangle \mathbf{PAB}$ is 10 . If the locus of P is $\mathbf{ax}+$ by $=\mathbf{15}$, then $\mathbf{5a}+\mathbf{2}\ \mathbf{b}$ is :

A. $-\frac{12}{5}$

B. $-\frac{6}{5}$

C. 4

D. 6

Correct answer: A

Source solution

Sol.

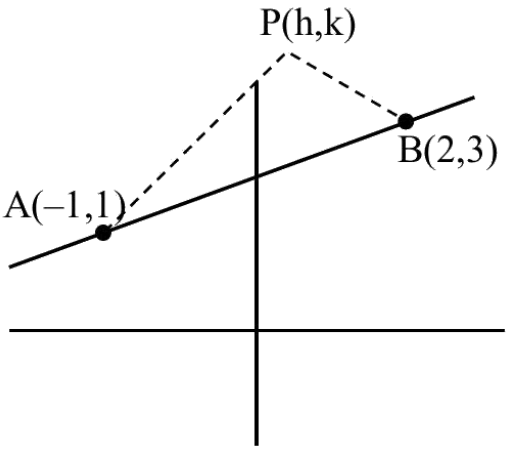
$\frac{1}{2}|\mathbf{h} \quad \mathbf{k} \quad 1-1 \quad 1 \quad 12 \quad 3 \quad 1|=10$

$-2x+3y=25$

$-\frac{6}{5}x+\frac{9}{5}y=15$

$\mathbf{a}=-\frac{6}{5},\ \mathbf{b}=\frac{9}{5}$

$5a=-6, 2b=\frac{18}{5}$



Question 66

Let two straight lines drawn from the origin O intersect the line $3x + 4y = 12$ at the points P and Q such that $\triangle OPQ$ is an isosceles triangle and $\angle POQ = 90°$. If $l = OP^2 + PQ^2 + QO^2$, then the greatest integer less than or equal to l is :

A. 44

B. 48

C. 46

D. 42

Correct answer: C

Source solution

Sol.

$3x + 4y = 12$

$3(r\cos\theta) + 4(r\sin\theta) = 12$

$$r(3\cos\theta + 4\sin\theta) = 12 \quad (1)$$

$3(-r\sin\theta) + 4(r\cos\theta) = 12$

$$r(-3\sin\theta + 4\cos\theta) = 12 \quad (2)$$

$\left(\frac{12}{r}\right)^2 + \left(\frac{12}{r}\right)^2 = (3\cos\theta + 4\sin\theta)^2 + (-3\sin\theta + 4\cos\theta)^2$

$2\left(\frac{12}{r}\right)^2 = 9 + 16$

$\frac{2\times144}{r^2} = 25 \Rightarrow 288 = 25r^2$

$\Rightarrow \frac{288}{25} = r^2$

$\Rightarrow \sqrt{2}\left(\frac{12}{5}\right) = r$

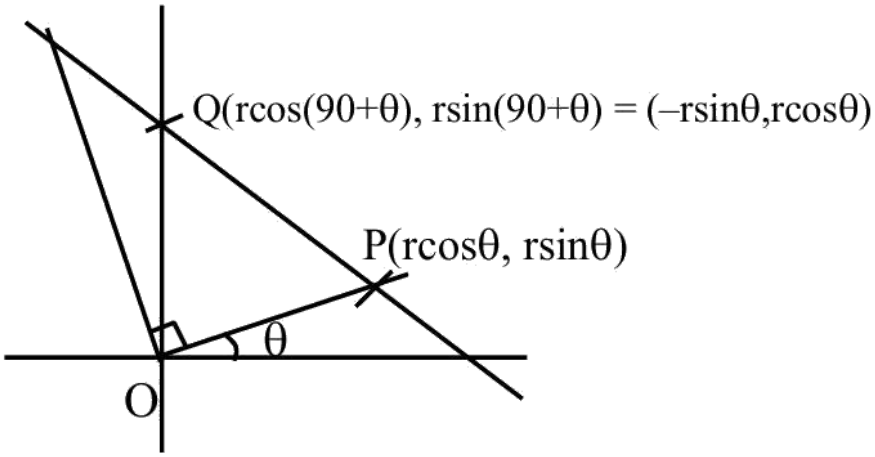
$\ell = OP^2 + PQ^2 + QO^2$

$\ell = r^2 + r^2 + r^2(\cos\theta + \sin\theta)^2 + r^2(\sin\theta + \cos\theta)^2$

$= 2r^2 + r^2(1 + \sin 2\theta + 1 - 2\sin 2\theta)$

$= 2r^2 + 2r^2$

$= 4r^2$
$= 4\left(\frac{288}{25}\right) = \frac{1152}{25} = 46.08$
$[\ell] = 46$



Question 67

Let circle C be the image of $\mathbf{x^2+y^2-2x+4y-4=0}$ in the line $2x-3y+5=0$ and A be the point on C such that OA is parallel to x -axis and A lies on the right hand side of the centre O of C . If $\mathbf{B(\alpha,\beta)}$, with $\beta<4$, lies on C such that the length of the arc AB is $(1/6)^{\text{th}}$ of the perimeter of C , then $\beta-\sqrt{3}\alpha$ is equal to

A. 3

B. $3+\sqrt{3}$

C. $4-\sqrt{3}$

D. 4

Correct answer: D

Source solution

Sol.

Centre (1, -2), $\mathbf{r=3}$
Reflection of $(1,-2)$ about $2x-3y+5=0$
$\frac{x-1}{2}=\frac{y+2}{-3}=\frac{-2(2+6+5)}{13}=-2$
$\mathbf{x=-3, y=4}$
Equation of circle ' C '
C : $(x+3)^2+(y-4)^2=9$
A.T.Q.
$\ell(\mathbf{arcAB})=\frac{1}{6}\times 2\pi\mathbf{r}$
$\mathbf{r\theta}=\frac{1}{6}\times 2\pi\mathbf{r}$
$\theta=\frac{\pi}{3}$
$(\alpha+6)^2+(\beta-4)^2=27$
$\frac{(\alpha+3)^2\pm(\beta-4)^2=9}{(\alpha+6)^2-(\alpha+3)^2=18}$
$\Rightarrow 6\alpha=-9$
$\Rightarrow \alpha=\frac{-3}{2}, \beta=\left(4-\frac{3\sqrt{3}}{2}\right)$

$\therefore \beta-\sqrt{3}\alpha$
$\left(4-\frac{3\sqrt{3}}{2}\right)+\frac{3\sqrt{3}}{2}$
$=4$

Question 68

Consider the lines $x(3\lambda+1)+y(7\lambda+2)=17\lambda+5$, λ being a parameter, all passing through a point P . One of these lines (say L) is farthest from the origin. If the distance of L from the point $(3,6)$ is d , then the value of d^2 is

A. 20

B. 30

C. 10

D. 15

Correct answer: A

Source solution

Sol. $x(3\lambda+1)+y(7\lambda+2)=17\lambda+5$
$(x+2y-5)+\lambda(3x+7y-17)=0$
intersection of family of lines
P(1,2)
Let $Q(3,6)$
$d=PQ=\sqrt{2^2+4^2}=\sqrt{20}$
$d^2=20$
option (1)

Question 69

Let the area of the triangle formed by the lines $x+2=y-1=z$, $\frac{x-3}{5}=\frac{y}{-1}=\frac{z-1}{1}$ and $\frac{x}{-3}=\frac{y-3}{3}=\frac{z-2}{1}$ be A . Then $\mathbf{A}^2$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 56

Source solution

$$\mathrm{L}_1: \mathrm{x}+2=\mathrm{y}-1=\mathrm{z}=\ell$$
$$\mathrm{L}_2: \frac{\mathrm{x}-3}{5}=\frac{\mathrm{y}}{-1}=\frac{\mathrm{z}-1}{1}=\mathrm{m}$$
$$\mathrm{L}_3: \frac{\mathrm{x}}{-3}=\frac{\mathrm{y}-3}{5}=\frac{\mathrm{z}-2}{1}=\mathrm{n}$$

Point of intersection of L_1 and L_2

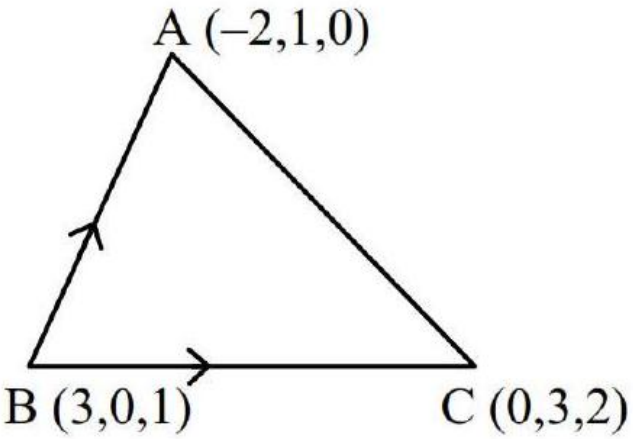
$$\left.\begin{aligned}\ell-2&=5\,\mathrm{m}+3\\ \ell+1&=-\mathrm{m}\\ \ell&=\mathrm{m}+1\end{aligned}\right\}\ell=0,\ \mathrm{m}=-1 \quad \mathrm{A}(-2,1,0)$$

Point of intersection of L_2 and L_3

$$\left.\begin{aligned}5\,\mathrm{m}+3&=-3\mathrm{n}\\ -\mathrm{m}&=3\mathrm{n}+3\\ \mathrm{m}+1&=\mathrm{n}+2\end{aligned}\right\}\mathrm{m}=0,\mathrm{n}=-1,\ \mathrm{B}(3,0,1)$$

Point of intersection L_3 and L_4

$$\left.\begin{aligned}-3\mathrm{n}&=\ell-2\\ 3\mathrm{n}+3&=\ell+1\\ \mathrm{n}+2&=\ell\end{aligned}\right\}\ell=2,\mathrm{n}=0,\mathrm{C}(0,3,2)$$



$$\text{Ar}(\triangle ABC) = \left|\frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5 & 1 & -1 \\ -3 & 3 & 1 \end{vmatrix}\right|$$

$$A = \frac{1}{2}|\hat{i}(4) - \hat{j}(-8) + \hat{k}(-12)|$$

$$A = \frac{1}{2}\sqrt{16 + 64 + 144} = \sqrt{56}$$

$$A^2 = 56$$

Question 70

Let the points of intersections of the lines $x - y + 1 = 0, x - 2y + 3 = 0$ and $2x - 5y + 11 = 0$ are the mid points of the sides of a triangle ABC. Then the area of the triangle ABC is ___.

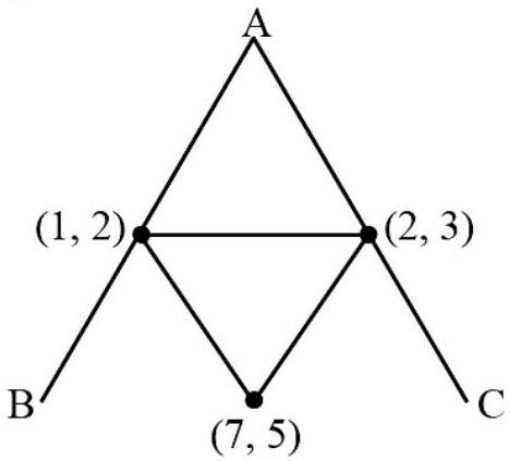
Numerical answer: _____

Enter the requested integer (0 decimal places).

Correct answer: 6

Source solution

intersection point of give lines are (1, 2), (7, 5), $(2, 3)$



$$\Delta = \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 7 & 5 & 1 \\ 2 & 3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[1(5-3) - 2(7-2) + 1(21-10)]$$

$$= \frac{1}{2}[2 - 10 + 11]$$

$$\Delta DEF = \frac{1}{2}(3) = \frac{3}{2}$$

$$\Delta ABC = 4\Delta DEF = 4\left(\frac{3}{2}\right) = 6$$

Conic Sections

Questions 71-101 / 31 selected / 10.3% practice weight / Focus first

Question 71

If the locus of the point, whose distances from the point $(2, 1)$ and $(1, 3)$ are in the ratio $5 : 4$, is $ax^2 + by^2 + cxy + dx + ey + 170 = 0$, then the value of $a^2 + 2b + 3c + 4d + e$ is equal to:

A. 5

B. -27

C. 37

D. 437

Correct answer: C

Source solution

Sol.

Let $\mathbf{P(x, y)}$

$\frac{(x-2)^2+(y-1)^2}{(x-1)^2+(y-3)^2} = \frac{25}{16}$

$9x^2 + 9y^2 + 14x - 118y + 170 = 0$

$\mathbf{a^2 + 2\,b + 3c + 4\,d + e}$

$= 81 + 18 + 0 + 56 - 118$

$= 155 - 118$

$= 37$

Question 72

The plane $2x - y + z = 4$ intersects the line segment joining the points $A(a, -2, 4)$ and $B(2, b, -3)$ at the point C in the ratio $2 : 1$ and the distance of the point C from the origin is $\sqrt{5}$. If $\mathbf{ab < 0}$ and P is the point $(a - b, b, 2b - a)$ then $\mathbf{CP^2}$ is equal to

A. $\frac{97}{3}$

B. $\frac{17}{3}$

C. $\frac{16}{3}$

D. $\frac{73}{3}$

Correct answer: B

Source solution

$$A(a, -2, 4), \qquad B(2, b, -3).$$

The point C divides AB internally in the ratio $2 : 1$.

Hence, by section formula,

$$C = \left(\frac{2 \cdot 2 + a}{3}, \frac{2b - 2}{3}, \frac{2(-3) + 4}{3}\right) = \left(\frac{a + 4}{3}, \frac{2b - 2}{3}, -\frac{2}{3}\right).$$

Since C lies on the plane

$$2x - y + z = 4,$$

we get

$$2\left(\frac{a + 4}{3}\right) - \left(\frac{2b - 2}{3}\right) - \frac{2}{3} = 4.$$

Multiplying by 3,

$$2a + 8 - 2b + 2 - 2 = 12$$

$$2a - 2b = 4$$

$$a - b = 2.$$

Also, distance of C from the origin is $\sqrt{5}$,

$$\left(\frac{a+4}{3}\right)^2 + \left(\frac{2b-2}{3}\right)^2 + \left(-\frac{2}{3}\right)^2 = 5.$$

Multiplying by 9,

$$(a+4)^2 + (2b-2)^2 + 4 = 45.$$

$$(a+4)^2 + (2b-2)^2 = 41.$$

Using

$$a = b + 2,$$

$$(b+6)^2 + (2b-2)^2 = 41.$$

$$b^2 + 12b + 36 + 4b^2 - 8b + 4 = 41.$$

$$5b^2 + 4b - 1 = 0.$$

$$(5b-1)(b+1)=0.$$

$$b=\frac{1}{5} \quad \text{or} \quad b=-1.$$

Since $ab<0$ and $a=b+2$,

$$b=\frac{1}{5} \Rightarrow a=\frac{11}{5} \quad \Rightarrow \quad ab>0$$

(rejected).

Hence,

$$b=-1, \qquad a=1.$$

Now

$$P(a-b,\ b,\ 2b-a)=(2,-1,-3).$$

Also,

$$C=\left(\frac{1+4}{3},\frac{2(-1)-2}{3},-\frac{2}{3}\right)=\left(\frac{5}{3},-\frac{4}{3},-\frac{2}{3}\right).$$

Therefore,

$$CP^2=\left(2-\frac{5}{3}\right)^2+\left(-1+\frac{4}{3}\right)^2+\left(-3+\frac{2}{3}\right)^2.$$

$$= \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(-\frac{7}{3}\right)^2.$$

$$= \frac{1+1+49}{9} = \frac{51}{9} = \frac{17}{3}.$$

$$\boxed{CP^2 = \frac{17}{3}}$$

Hence, the correct option is

$$\boxed{(2)\ \frac{17}{3}}.$$

Question 73

Let the abscissae of the two points P and Q on a circle be the roots of $x^2 - 4x - 6 = 0$ and the ordinates of P and Q be the roots of $y^2 + 2y - 7 = 0$. If PQ is a diameter of the circle $x^2 + y^2 + 2ax + 2by + c = 0$, then the value of ($a + b - c$) is

A. 12

B. 13

C. 14

D. 16

Correct answer: A

Source solution

Sol.

Equation of circle diameter from
$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$
(where x_1, x_2 are the roots of $x^2 - 4x - 6 = 0$ and y_1, y_2 are the roots of $y^2 + 2y - 7 = 0$)
$x^2 + y^2 - 4x + 2y - 13 = 0$
Now,
Compare it with the given equation, we get $a = -2,\ b = 1, c = -13$

Now
$a + b - c = 12$

Question 74

Let the focal chord PQ of the parabola $y^2 = 4x$ make an angle of 60° with the positive x -axis, where P lies in the first quadrant. If the circle, whose one diameter is PS, S being the focus of the parabola, touches the y -axis at the point $(0, \alpha)$, then $5\alpha^2$ is equal to :

A. 15

B. 25

C. 30

D. 20

Correct answer: A

Source solution

Sol.

$\tan 60^\circ = \frac{2t-0}{t^2-1} = \sqrt{3} \Rightarrow t = \sqrt{3}$

$\therefore P(3, 2\sqrt{3})$

Circle :
$(x-1)(x-3) + (y-0)(y-2\sqrt{3}) = 0$
at $x = 0$
$\Rightarrow 3 + y^2 - 2\sqrt{3}y = 0$
$\Rightarrow y = \sqrt{3} = \alpha$
$5\alpha^2 = 15$

Question 75

Let the ellipse $3x^2 + py^2 = 4$ pass through the centre C of the circle $x^2 + y^2 - 2x - 4y - 11 = 0$ of radius r. Let f_1, f_2 be the focal distances of the point C on the ellipse. Then $6f_1f_2 - r$ is equal to

A. 74

B. 68

C. 70

D. 78

Correct answer: C

Source solution

Sol.

$E : \frac{x^2}{4/3} + \frac{y^2}{4/P} = 1$

Centre of circle $(1, 2)$, radius

$r = \sqrt{1 + 4 + 11}$

$r = 4$

$\because$ E pass from centre $(1, 2)$

$\therefore \frac{3}{4} + P = 1$

$P = \frac{1}{4}$ $\quad \therefore$ vertical ellipse

$e = \sqrt{1 - \frac{4/3}{16}} = \sqrt{1 - \frac{1}{12}} = \sqrt{\frac{11}{12}}$

$\therefore$ Focal distance of $C(h, k)$

$= b \pm ek$

$F_1 = 4 + \sqrt{\frac{11}{12}} \times 2$

$F_2 = 4 - \sqrt{\frac{11}{12}} \times 2$

$\therefore F_1 F_2 = 16 - \frac{11}{3} = \frac{37}{3}$

$\therefore 6 F_1 F_2 - r = 74 - 4 = 70$

Question 76

If the x - intercept of a focal chord of the parabola $y^2 = 8x + 4y + 4$ is 3 , then the length of this chord is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 16

Source solution

16

$y^2 = 8x + 4y + 4$

$(y-2)^2 = 8(x+1)$

$y^2 = 4ax$

$a = 2, X = x + 1, Y = y - 2$

focus $(1, 2)$

$y - 2 = m(x - 1)$

Put $(3, 0)$ in the above line

$m = -1$

Length of focal chord $= 16$

Question 77

Let the sum of the focal distances of the point $P(4,3)$ on the hyperbola $\mathbf{H}: \frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be $8\sqrt{\frac{5}{3}}$. If for H , the length of the latus rectum is l and the product of the focal distances of the point P is m , then $9l^2+6\,\mathrm{m}$ is equal to :-

A. 184

B. 186

C. 185

D. 187

Correct answer: C

Source solution

Sol.

$$\mathrm{ex}+\mathrm{a}+\mathrm{ex}-\mathrm{a}=8\sqrt{\frac{5}{3}}$$

$$2\mathrm{ex}=8\sqrt{\frac{5}{3}}$$

$$2\mathrm{e}\times 4=8\sqrt{\frac{5}{3}}$$

$$\mathrm{e}=\sqrt{\frac{5}{3}}$$

$$\mathrm{b}^2=\mathrm{a}^2\left(\left(\frac{\sqrt{5}}{3}\right)^2-1\right)$$

$$\mathrm{b}^2=\frac{2}{3}\mathrm{a}^2$$

$$\frac{16}{a^2}-\frac{9}{b^2}=1$$

and $\mathrm{b}^2=\frac{2}{3}\mathrm{a}^2$

$$\Rightarrow \mathrm{a}^2=\frac{5}{2}\quad \mathrm{b}^2=\frac{5}{3}$$

Now,

$$\ell=\frac{2\,\mathrm{b}^2}{\mathrm{a}}$$

$$\ell^2=\frac{4b^4}{a^2}$$

$$9\ell^2=36\times\frac{25}{9\times 5}\times 2$$

$$9\ell^2=40$$

$$\mathrm{m}=(\mathrm{ex}+\mathrm{a})(\mathrm{ex}-\mathrm{a})$$

$$\mathrm{m}=\mathrm{e}^2\mathrm{x}^2-\mathrm{a}^2$$

$$=\frac{5}{3}\times 16-\frac{5}{2}=\frac{145}{6}$$

$$=6\,\mathrm{m}=145$$

$$9\ell^2+6\,\mathrm{m}$$

$$40+145=185$$

option (3) is the correct answer.

Question 78

Let R be the focus of the parabola $\mathbf{y^2 = 20x}$ and the line $\mathbf{y = mx + c}$ intersect the parabola at two points P and Q . Let the point $\mathbf{G(10, 10)}$ be the centroid of the triangle PQR . If $\mathbf{c - m = 6}$, then $\mathbf{(PQ)^2}$ is

A. 325

B. 317

C. 296

D. 346

Correct answer: A

Source solution

\textbf{Given:}

$$y^2 = 20x$$

Comparing with

$$y^2 = 4ax,$$

we get

$$4a = 20 \quad \Rightarrow \quad a = 5.$$

Hence the focus is

$$R = (5, 0).$$

\bigskip

Let the points P and Q on the parabola correspond to parameters
t_1 and t_2.

Then

$$P = (5t_1^2,\ 10t_1), \qquad Q = (5t_2^2,\ 10t_2).$$

Since $G(10, 10)$ is the centroid of $\triangle PQR$,

$$\left(\frac{5t_1^2 + 5t_2^2 + 5}{3}, \frac{10t_1 + 10t_2}{3} \right) = (10, 10).$$

Equating coordinates,

$$\frac{10(t_1 + t_2)}{3} = 10$$

$$t_1 + t_2 = 3. \tag{1}$$

Also,

$$\frac{5(t_1^2 + t_2^2 + 1)}{3} = 10$$

$$t_1^2 + t_2^2 = 5. \tag{2}$$

Using

$$t_1^2 + t_2^2 = (t_1 + t_2)^2 - 2t_1t_2,$$

from (1) and (2),

$$5 = 9 - 2t_1t_2$$

$$t_1 t_2 = 2. \tag{3}$$

\bigskip

The chord PQ has equation

$$t t_1 t_2 + \frac{t_1 + t_2}{2} y - x = 0.$$

Substituting (1) and (3),

$$2t + \frac{3}{2} y - x = 0.$$

Since the chord is

$$y = mx + c,$$

we obtain

$$x = \frac{3}{2} y + 10$$

$$y = \frac{2}{3} x - \frac{20}{3}.$$

Thus

$$m = \frac{2}{3}, \qquad c = -\frac{20}{3}.$$

Given

$$c - m = -6,$$

which is satisfied.

\bigskip

Now,

$$PQ^2 = a^2(t_1 - t_2)^2 \left[(t_1 + t_2)^2 + 4\right].$$

Here $a = 5$.

Also,

$$(t_1 - t_2)^2 = (t_1 + t_2)^2 - 4t_1t_2 = 9 - 8 = 1.$$

Therefore,

$$PQ^2 = 25(1)(9 + 4) = 25 \cdot 13 = 325.$$

$$\boxed{(PQ)^2 = 325}$$

Hence,

$$\boxed{\text{Option (1)}}$$

Question 79

Let P be a parabola with vertex $(2,3)$ and directrix $2x+y=6$. Let an ellipse $E: \frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$ of eccentricity $\frac{1}{\sqrt{2}}$ pass through the focus of the parabola P. Then the square of the length of the latus rectum of E , is

A. $\frac{385}{8}$

B. $\frac{347}{8}$

C. $\frac{512}{25}$

D. $\frac{656}{25}$

Correct answer: D

Source solution

Sol.-

Slope of axis $=\frac{1}{2}$

$y-3=\frac{1}{2}(x-2)$

$\Rightarrow 2y-6=x-2$

$\Rightarrow 2y-x-4=0$

$2x+y-6=0$

$4x+2y-12=0$

$\alpha+1.6=4 \Rightarrow \alpha=2.4$

$\beta+2.8=6 \Rightarrow \beta=3.2$

Ellipse passes through (2.4, 3.2)

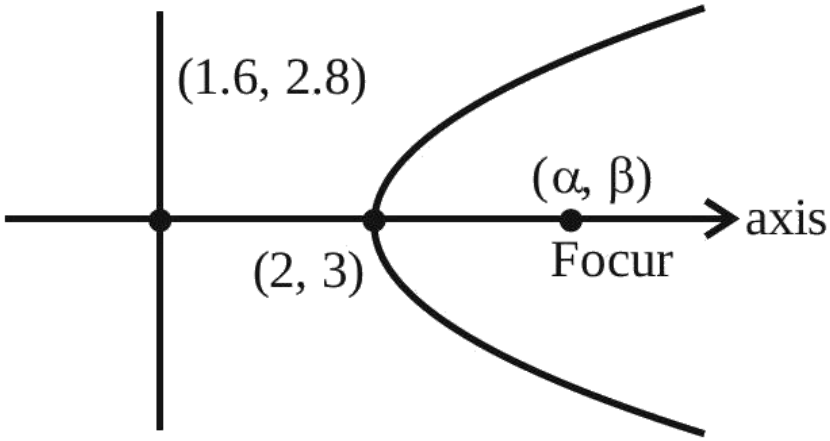
$$\Rightarrow \frac{\left(\frac{24}{10}\right)^2}{a^2}+\frac{\left(\frac{32}{10}\right)^2}{b^2}=1 \quad (1)$$

Also $1-\frac{b^2}{a^2}=\frac{1}{2}=\frac{b^2}{a^2}=\frac{1}{2}$

$\Rightarrow a^2=2b^2$

Put in (1) $\Rightarrow b^2=\frac{328}{25}$

$\Rightarrow \left(\frac{2b^2}{a}\right)^2=\frac{4b^2}{a^2}\times b^2=4\times\frac{1}{2}\times\frac{328}{25}=\frac{656}{25}$



Question 80

The length of the latus rectum and the directrices of a hyperbola with eccentricity e are 9 and $x = \pm 4/\sqrt{13}$, respectively. Let the line $y - \sqrt{3}x + \sqrt{3} = 0$ touch this hyperbola at (x_0, y_0). If m is the product of the focal distances of (x_0, y_0), then $4e^2 + m$ is equal to ___.

Numerical answer: _____

This question was incorrect in the original exam paper and ALLEN marked it BONUS. This corrected practice version uses the directrices x = +/-4/sqrt(13), explicitly corrected by ALLEN, instead of the printed +/-4/sqrt(3). Enter an integer (0 decimal places).

Correct answer: 61

Source solution

Use the corrected standard hyperbola $x^2/a^2 - y^2/b^2 = 1$. The latus rectum gives $2b^2/a = 9$. Tangency of $y = \sqrt{3}x - \sqrt{3}$ gives $3 = 3a^2 - b^2$. Thus $a = 2$, $b^2 = 9$, and $e^2 = 1 + b^2/a^2 = 13/4$. The corrected directrices are $x = \pm a/e = \pm 4/\sqrt{13}$. The tangent contact is $(4, 3\sqrt{3})$. Hence $m = (ex_0 - a)(ex_0 + a) = 48$, so $4e^2 + m = 13 + 48 = 61$. The original ALLEN BONUS designation is retained in the instructions; this answer applies to the disclosed corrected practice version.

Question 81

Let $y^2 = 12x$ the parabola and S be its focus. Let PQ be a focal chord of the parabola such that (SP) $(SQ) = \frac{147}{4}$. Let C be the circle described taking PQ as a diameter. If the equation of a circle C is $64x^2 + 64y^2 - \alpha x - 64\sqrt{3}y = \beta$, then $\beta - \alpha$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 1328

Source solution

$y^2 = 12x \quad a = 3 \quad SP \times SQ = \frac{147}{4}$

Let $P\left(3t^2, 6t\right)$ and $t_1 t_2 = -1$

(ends of focal chord)

So, $Q\left(\frac{3}{t^2}, \frac{-6}{t}\right)$

$S(3, 0)$

$SP \times SQ = PM_1 \times QM_2$

(dist. from directrix)

$= \left(3 + 3t^2\right)\left(3 + \frac{3}{t^2}\right) = \frac{147}{4}$

$\Rightarrow \frac{\left(1+t^2\right)^2}{t^2} = \frac{49}{12}$

$t^2 = \frac{3}{4}, \frac{4}{3}$

$t = \pm\frac{\sqrt{3}}{2}, \pm\frac{2}{\sqrt{3}}$

considering $t = \frac{-\sqrt{3}}{2}$

$P\left(\frac{9}{4}, -3\sqrt{3}\right)$ and $Q(4, 4\sqrt{3})$

Hence, diametric circle:

$(x - 4)\left(x - \frac{9}{4}\right) + (y + 3\sqrt{3})(y - 4\sqrt{3}) = 0$

$\Rightarrow x^2 + y^2 - \frac{25}{4}x - \sqrt{3}y - 27 = 0$

$\Rightarrow \alpha = 400, \beta = 1728$

$\beta - \alpha = 1328$

Question 82

Let the line $\mathbf{L}: \sqrt{2}\mathbf{x}+\mathbf{y}=\boldsymbol{\alpha}$ pass through the point of the intersection P (in the first quadrant) of the circle $x^2+y^2=3$ and the parabola $x^2=2y$. Let the line L touch two circles $\mathbf{C_1}$ and $\mathbf{C_2}$ of equal radius $2\sqrt{3}$. If the centres $\mathbf{Q_1}$ and $\mathbf{Q_2}$ of the circles $\mathbf{C_1}$ and $\mathbf{C_2}$ lie on the y -axis, then the square of the area of the triangle $\mathbf{PQ_1Q_2}$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 72

Source solution

$x^2+y^2=3$ and $x^2=2y$

$$y^2+2y-3=0 \Rightarrow (y+3)(y-1)=0$$
$$y=-3 \text{ or } y=1$$
$$y=1 x=\sqrt{2} \Rightarrow P(\sqrt{2},1)$$

p lies on the line
$\sqrt{2}x+y=\alpha$
$\sqrt{2}(\sqrt{2})+1=\alpha$
$\alpha=3$

For circle $\mathbf{C_1}$
$\mathbf{Q_1}$ lies on y axis

Let $\mathbf{Q_1}(0,\alpha)$ coordinates
$\mathbf{R_1}=2\sqrt{3}$ (Given

Line L act as tangent

Apply $\mathbf{P}=\mathbf{r}$ (condition of tangency)
$\Rightarrow \left|\frac{\alpha-3}{\sqrt{3}}\right|=2\sqrt{3}$
$\Rightarrow |\alpha-3|=6$
$\alpha-3=6 \quad$ or $\quad \alpha-3=-6$
$\Rightarrow \alpha=9 \quad \alpha=-3$

$$\Delta PQ_1Q_2 = \frac{1}{2}\begin{vmatrix} \sqrt{2} & 1 & 1 \\ 0 & 9 & 1 \\ 0 & -3 & 1 \end{vmatrix}$$
$= \frac{1}{2}(\sqrt{2}(12)) = 6\sqrt{2}$
$(\triangle PQ_1Q_2)^2 = 72$

Question 83

Let the point P of the focal chord PQ of the parabola $\mathbf{y^2 = 16x}$ be $\mathbf{(1, -4)}$. If the focus of the parabola divides the chord PQ in the ratio $m : n$, $\mathbf{gcd(m, n) = 1}$, then $\mathbf{m^2 + n^2}$ is equal to :

A. 17

B. 10

C. 37

D. 26

Correct answer: A

Source solution

Sol. $y^2 = 16x; a = 4$ focus $S \equiv (4, 0)$
$\mathbf{2at_1 = -4}$
$\Rightarrow \mathbf{2(4)t_1 = -4}$
$\Rightarrow \mathbf{t_1 = -\frac{1}{2}}$
$\because \mathbf{t_1t_2 = -1}$
$\Rightarrow \mathbf{t_2 = 2}$
$\therefore \mathbf{Q\left(at_2^2, 2at_2\right) = (16, 16)}$

Let, S divides PQ internally in $\boldsymbol{\lambda : 1}$ ratio
$\therefore \frac{16\lambda - 4}{\lambda + 1} = 0$
$\boldsymbol{\lambda = \frac{1}{4} = \frac{m}{n}}$.

$\therefore \mathbf{m^2 + n^2 = 1 + 16 = 17}$

Question 84

The area (in sq. units) of the smaller of the two circles that touch the parabola, $y^2 = 4x$ at the point $(1, 2)$ and the x-axis is :

A. $4\pi(3 + \sqrt{2})$

B. $8\pi(2 - \sqrt{2})$

C. $8\pi(3 - 2\sqrt{2})$

D. $4\pi(2 - \sqrt{2})$

Correct answer: C

Source solution

The circle and parabola will have common tangent at $\mathbf{P(1, 2)}$

$\therefore$ Equation of tangent to parabola

$$\equiv y \times (2) = 4\frac{(x+1)}{2} \Rightarrow 2y = 2x + 2y \qquad = x + 1$$

Let equation of circle be (using family of circles)
$(x - x_1)^2 + (y - y_1)^2 + \lambda T = 0$
$\Rightarrow \mathbf{c \equiv (x-1)^2 + (y-2)^2 + \lambda(x - y + 1) = 0}$

Also circle touches x-axis $\Rightarrow$ y-coordinate of centre = radius
$\Rightarrow c \equiv x^2 + y^2 + (\lambda - 2)x + (-\lambda - 4)y + (\lambda + 5) = 0$

$$\frac{\lambda + 4}{2} = \sqrt{\left(\frac{\lambda - 2}{2}\right)^2 + \left(\frac{-\lambda - 4}{2}\right)^2 - (\lambda + 5)}$$

$\Rightarrow \frac{\lambda^2 - 4\lambda + 4}{4} = \lambda + 5 \Rightarrow \lambda^2 - 4\lambda + 4 = 4\lambda + 20$
$\Rightarrow \lambda^2 - 8\lambda - 16 = 0$
$\Rightarrow \lambda = \frac{8 \pm \sqrt{64 + 64}}{2}$
$= 4 \pm 4\sqrt{2}$
$\lambda = 4 - 4\sqrt{2}$ (Other value forms bigger circle)

Hence centre of circle ($2\sqrt{2}-2, 4-2\sqrt{2}$)

Radius $= 4-2\sqrt{2}$

Area $= \pi(4-2\sqrt{2})^2 = 8\pi(3-2\sqrt{2})$

Question 85

If one end of a focal chord of the parabola, $y^2 = 16x$ is at (1,4), then the length of this focal chord is :

A. 24

B. 20

C. 22

D. 25

Correct answer: D

Source solution

Sol.

$\because y^2 = 16x \quad \Rightarrow a = 4$

One end of focal chord $(1, 4) \quad \therefore 2$ at $= 4$

$$\Rightarrow t = \frac{1}{2}$$

$$\begin{aligned} \text{Length of focal chord } &= a\left(t + \frac{1}{t}\right)^2 \\ &= 4 \times \left(2 + \frac{1}{2}\right)^2 \\ &= 25 \end{aligned}$$

Question 86

Let $\mathbf{A}(\alpha, 0)$ and $\mathbf{B}(0, \beta)$ be the points on the line $5x + 7y = 50$. Let the point P divide the line segment AB internally in the ratio $7 : 3$. Let $3\mathbf{x} - 25 = 0$ be a directrix of the ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the corresponding focus be S . If from S , the perpendicular on the x -axis passes through P , then the length of the latus rectum of E is equal to

A. $\frac{25}{3}$

B. $\frac{32}{9}$

C. $\frac{25}{9}$

D. $\frac{32}{5}$

Correct answer: D

Source solution

Sol. $\mathbf{A} = (10, 0)$ Sol. $\mathbf{B} = \left(0, \frac{50}{7}\right)\}\mathrm{P} = (3, 5)$

$\mathbf{ae} = 3$

$\frac{\mathbf{a}}{\mathbf{e}} = \frac{25}{3}$

$\mathbf{a} = 5$

$\mathrm{b} = 4$

Length of $LR = \frac{2b^2}{a} = \frac{32}{5}$

Question 87

If the foci of a hyperbola are same as that of the ellipse $\frac{x^2}{9}+\frac{y^2}{25}=1$ and the eccentricity of the hyperbola is $\frac{15}{8}$ times the eccentricity of the ellipse, then the smaller focal distance of the point $\left(\sqrt{2}, \frac{14}{3}\sqrt{\frac{2}{5}}\right)$ on the hyperbola, is equal to

A. $7\sqrt{\frac{2}{5}}-\frac{8}{3}$

B. $14\sqrt{\frac{2}{5}}-\frac{4}{3}$

C. $14\sqrt{\frac{2}{5}}-\frac{16}{3}$

D. $7\sqrt{\frac{2}{5}}+\frac{8}{3}$

Correct answer: A

Source solution

Sol. $\frac{x^2}{9}+\frac{y^2}{25}=1$

$\mathrm{a}=3,\ \mathrm{b}=5$

$\mathrm{e}=\sqrt{1-\frac{9}{25}}=\frac{4}{5}\ \therefore \text{foci}=(0,\pm\mathrm{be})=(0,\pm4)$

$$\therefore e_H=\frac{4}{5}\times\frac{15}{8}=\frac{3}{2}$$

Let equation hyperbola

$\frac{x^2}{A^2}-\frac{y^2}{B^2}=-1$

$\therefore \mathrm{B}\cdot\mathrm{e_H}=4 \quad \therefore\ \mathrm{B}=\frac{8}{3}$

$\therefore \mathrm{A}^2=\mathrm{B}^2\left(\mathrm{e_H^2}-1\right)=\frac{64}{9}\left(\frac{9}{4}-1\right) \quad \therefore \mathrm{A}^2=\frac{80}{9}$

$\therefore \frac{x^2}{\frac{80}{9}}-\frac{y^2}{\frac{64}{9}}=-1$

Directrix : $y=\pm\frac{B}{e_H}=\pm\frac{16}{9}$

$\mathrm{PS}=\mathrm{e}\cdot\mathrm{PM}=\frac{3}{2}\left|\frac{14}{3}\cdot\sqrt{\frac{2}{5}}-\frac{16}{9}\right|$

$=7\sqrt{\frac{2}{5}}-\frac{8}{3}$

Question 88

A circle of radius 2 unit passes through the vertex and the focus of the parabola $y^2 = 2x$ and touches the parabola $y = \left(x - \frac{1}{4}\right)^2 + \alpha$, where $\alpha > 0$. Then $(4\alpha - 8)^2$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 63

Source solution

Vertex and focus of parabola $y^2 = 2x$
are $\mathrm{V}(0,0)$ and $S\left(\frac{1}{2}, 0\right)$ resp.
Let equation of circle be
$(\mathrm{x} - \mathrm{h})^2 + (\mathrm{y} - \mathrm{k})^2 = 4$
$\because$ Circle passes through $(0,0)$

$$\Rightarrow \mathrm{h}^2 + \mathrm{k}^2 = 4 \qquad (.....)$$

$\because$ Circle passes through $\left(\frac{1}{2}, 0\right)$
$\left(\frac{1}{2} - h\right)^2 + k^2 = 4$

$$\Rightarrow h^2 + k^2 - h = \frac{15}{4} \qquad (2)$$

On solving (1) and (2)
$4 - h = \frac{15}{4}$
$h = 4 - \frac{15}{4} = \frac{1}{4}$
$k = +\frac{\sqrt{63}}{4}$
$k = -\frac{\sqrt{63}}{4}$ is rejected as circle with centre $\left(\frac{1}{4}, -\frac{\sqrt{63}}{4}\right)$ can't touch given parabola.

Equation of circle is

$\left(x-\frac{1}{4}\right)^2+\left(k-\frac{\sqrt{63}}{4}\right)^2=4$

From figure

$\alpha=2+\frac{\sqrt{63}}{4}=\frac{8+\sqrt{63}}{4}$

$4\alpha-8=\sqrt{63}$

$(4\alpha-8)^2=63$

Question 89

Let the latus ractum of the parabola $y^2 = 4x$ be the common chord to the circles C_1 and C_2 each of them having radius $2\sqrt{5}$. Then, the distance between the centres of the circles C_1 and C_2 is:

A. 8

B. $8\sqrt{5}$

C. $4\sqrt{5}$

D. 12

Correct answer: A

Source solution

$$\text{Parabola: } y^2 = 4x$$

For the parabola $y^2 = 4ax$, we have

$$a = 1.$$

Hence, the latus rectum is

$$x = a = 1.$$

Its endpoints are

$$(1, 2) \quad \text{and} \quad (1, -2).$$

Therefore, the length of the common chord is

$$L = 4.$$

Let the radius of each circle be

$$r = 2\sqrt{5}.$$

If a chord of length L is at a distance d from the centre of a circle, then

$$L = 2\sqrt{r^2 - d^2}.$$

Substituting $L = 4$ and $r = 2\sqrt{5}$,

$$4 = 2\sqrt{20 - d^2}$$

$$2 = \sqrt{20 - d^2}$$

$$d^2 = 16$$

$$d = 4.$$

Thus, the centre of each circle lies on the perpendicular bisector of the common chord at a distance 4 from the chord.

The common chord is the line

$$x = 1.$$

Its midpoint is

$$(1,0).$$

The perpendicular bisector is

$$y = 0.$$

Hence the two centres are

$$C_1 = (1,4), \qquad C_2 = (1,-4).$$

Therefore, the distance between the centres is

$$C_1C_2 = \sqrt{(1-1)^2 + (4+4)^2} = 8.$$

$$\boxed{8}$$

Hence, the correct option is

$$\boxed{\text{(1) } 8}.$$

Question 90

The length of the latus-rectum of the ellipse, whose foci are $(2, 5)$ and $(2, -3)$ and eccentricity is $\frac{4}{5}$, is

A. $\frac{6}{5}$

B. $\frac{50}{3}$

C. $\frac{10}{3}$

D. $\frac{18}{5}$

Correct answer: D

Source solution

Sol. $2be = 8$

$be = 4$

$F_1 \circ (2, 5)$

$F_2 \circ (2, -3)$

$b\left(\frac{4}{5}\right) = 4 \Rightarrow b = 5$

$\because c^2 = b^2 - a^2$

$16 = 25 - a^2 \Rightarrow a = 3$

L.R. $= \frac{2a^2}{b} = \frac{18}{5}$

Option (4)is the correct answer.

Question 91

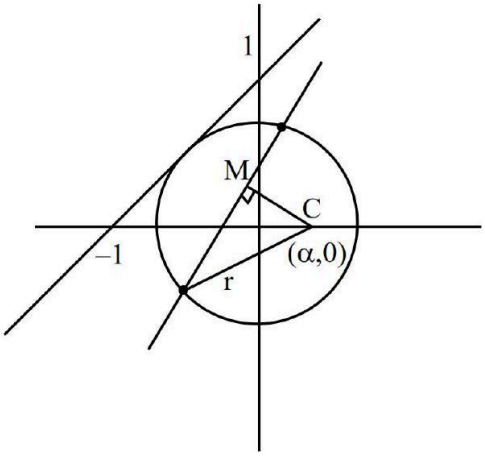
Let the circle C touch the line $\mathbf{x-y+1=0}$, have the centre on the positive x-axis, and cut off a chord of length $\frac{4}{\sqrt{13}}$ along the line $-3x+2y=1$. Let H be the hyperbola $\frac{x^2}{\alpha^2}-\frac{y^2}{\beta^2}=1$, whose one of the foci is the centre of C and the length of the transverse axis is the diameter of C . Then $2\alpha^2+3\beta^2$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 19

Source solution



$x-y+1=0$
$\mathrm{p}=\mathrm{r}$

$$\left|\frac{\alpha-0+1}{\sqrt{2}}\right|=\mathrm{r}\Rightarrow(\alpha+1)^2=2\mathrm{r}^2\ldots. \quad (1)$$

now $\left(\frac{-3\alpha+0-1}{\sqrt{9+4}}\right)^2+\left(\frac{2}{\sqrt{13}}\right)^2=\mathrm{r}^2$

$$\Rightarrow(3\alpha+1)^2+4=13\mathrm{r}^2 \quad (2)$$

(1) & (2) $\Rightarrow (3\alpha+1)^2+4=13\frac{(\alpha+1)^2}{2}$

$$\Rightarrow 18\alpha^2+12\alpha+2+8=13\alpha^2+26\alpha+13$$

$\Rightarrow 5\alpha^2-14\alpha-3=0$
$\Rightarrow 5\alpha^2-15\alpha+\alpha-3=0$
$\Rightarrow 5\alpha^2-15\alpha+\alpha-3=0$
$\Rightarrow \alpha=\frac{-1}{5},3$
$\therefore r=2\sqrt{2}$
How $\alpha e=3$ and $2\alpha=4\sqrt{2}$
$\alpha^2e^2=9\Rightarrow\alpha=2\sqrt{2}\Rightarrow\alpha^2=8$
$\alpha^2\left(1+\frac{\beta^2}{\alpha^2}\right)=9$
$\alpha^2+\beta^2=9$
$\therefore \beta^2=1$
$\therefore 2\alpha^2+3\beta^2=2(8)+3(1)=19$

Question 92

Let $A(0,1), B(1,1)$ and $C(1,0)$ be the mid - points of the sides of a triangle with incentre at the point D. If the focus of the parabola $y^2 = 4ax$ passing through D is $(\alpha + \beta\sqrt{2}, 0)$, where α and β are rational numbers, then $\frac{\alpha}{\beta^2}$ is equal to

A. 6

B. 8

C. 12

D. $\frac{9}{2}$

Correct answer: B

Source solution

\textbf{Given:}

The points

$$A(0,1), \quad B(1,1), \quad C(1,0)$$

are the midpoints of the sides of a triangle.

Let the original triangle be PQR.

\bigskip

\textbf{Step 1: Find the vertices of the original triangle}

Using the midpoint relations,

$$A = \left(\frac{x_Q + x_R}{2}, \frac{y_Q + y_R}{2} \right),$$

$$B = \left(\frac{x_R + x_P}{2}, \frac{y_R + y_P}{2} \right),$$

$$C = \left(\frac{x_P + x_Q}{2}, \frac{y_P + y_Q}{2} \right).$$

The vertices can be recovered by

$$P = B + C - A,$$

$$Q = C + A - B,$$

$$R = A + B - C.$$

Hence,

$$P = (1,1) + (1,0) - (0,1) = (2,0),$$

$$Q = (1,0) + (0,1) - (1,1) = (0,0),$$

$$R = (0,1) + (1,1) - (1,0) = (0,2).$$

Thus the triangle is

$$(2,0),\ (0,0),\ (0,2).$$

\bigskip

\textbf{Step 2: Find the incenter}

The side lengths are

$$QR = 2,$$

$$RP = \sqrt{(2-0)^2 + (0-2)^2} = 2\sqrt{2},$$

$$PQ = 2.$$

Let

$$a = QR = 2, \qquad b = RP = 2\sqrt{2}, \qquad c = PQ = 2.$$

The incenter is

$$D = \left(\frac{a x_P + b x_Q + c x_R}{a+b+c}, \frac{a y_P + b y_Q + c y_R}{a+b+c} \right).$$

Substituting,

$$D = \left(\frac{2(2) + 2\sqrt{2}(0) + 2(0)}{2 + 2\sqrt{2} + 2}, \frac{2(0) + 2\sqrt{2}(0) + 2(2)}{2 + 2\sqrt{2} + 2} \right).$$

$$D = \left(\frac{4}{4 + 2\sqrt{2}}, \frac{4}{4 + 2\sqrt{2}} \right).$$

Rationalizing,

$$D = \left(2 - \sqrt{2},\, 2 - \sqrt{2} \right).$$

\bigskip

\textbf{Step 3: Use the parabola condition}

The parabola is

$$y^2 = 4ax.$$

Its focus is

$$(a, 0).$$

Since the parabola passes through D,

$$(2 - \sqrt{2})^2 = 4a(2 - \sqrt{2}).$$

Since $2 - \sqrt{2} \neq 0$,

$$2 - \sqrt{2} = 4a.$$

Therefore,

$$a = \frac{2 - \sqrt{2}}{4} = \frac{1}{2} - \frac{\sqrt{2}}{4}.$$

Hence the focus is

$$\left(\frac{1}{2} - \frac{\sqrt{2}}{4}, 0 \right).$$

Comparing with

$$(\alpha + \beta\sqrt{2}, 0),$$

we get

$$\alpha = \frac{1}{2}, \qquad \beta = -\frac{1}{4}.$$

\bigskip

\textbf{Step 4: Calculate $\dfrac{\alpha}{\beta^2}$}

$$\frac{\alpha}{\beta^2} = \frac{\frac{1}{2}}{\left(-\frac{1}{4}\right)^2} = \frac{\frac{1}{2}}{\frac{1}{16}} = 8.$$

$$\boxed{\frac{\alpha}{\beta^2} = 8}$$

$$\boxed{\text{Option (2)}}$$

Question 93

Let the product of the focal distances of the point $\mathbf{P(4, 2\sqrt{3})}$ on the hyperbola $\mathbf{H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$ be 32 . Let the length of the conjugate axis of H be p and the length of its latus rectum be q . Then $\mathbf{p^2 + q^2}$ is equal to .....

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 120

Source solution

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
P $(4, 2\sqrt{3})$
$\mathbf{PS_1 \cdot PS_2 = 32}$
$\mathbf{|PS_1 - PS_2| = 2a}$
$\mathbf{P(4, 2\sqrt{3})}$ lies on H
$\therefore \mathbf{\frac{16}{a^2} - \frac{12}{b^2} = 1}$

$$16b^2 - 12a^2 = a^2b^2 \qquad (2)$$

$\mathbf{|PS_1 - PS_2|^2 = 4a^2}$
$\mathbf{PS_1{}^2 + PS_2{}^2 - 2PS_1.PS_2 = 4a^2}$
$\mathbf{(ae - 4)^2 + 12 + (ae + 4)^2 + 12 - 64 = 4a^2}$
$\mathbf{2a^2e^2 - 8 = 4a^2}$
$\mathbf{a^2 + b^2 - 4 = 2a^2}$
$\mathbf{b^2 - a^2 = 4}$
(2) & (3) $\Rightarrow \mathbf{16\left(a^2 + 4\right) - 12a^2 = a^2\left(a^2 + 4\right)}$
$\Rightarrow \mathbf{16a^2 + 64 - 12a^2 = a^4 + 4a^2}$
$\Rightarrow \mathbf{a^4 = 64}$
$\Rightarrow \mathbf{a^2 = 8}$
$\therefore \mathbf{b^2 = 12}$
$\mathbf{p^2 + q^2 = 4\,b^2 + \frac{4\,b^4}{a^2}}$
$\mathbf{= 120}$

Question 94

Let PQ be a focal chord of the parabola $\mathbf{y^2 = 4x}$ such that it subtends an angle of $\frac{\pi}{2}$ at the point (3 , 0). Let the line segment PQ be also a focal chord of the ellipse $E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a^2 > b^2$. If e is the eccentricity of the ellipse E , then the value of $\frac{1}{e^2}$ is equal to :

A. $1 + \sqrt{2}$

B. $3 + 2\sqrt{2}$

C. $1 + 2\sqrt{3}$

D. $4 + 5\sqrt{3}$

Correct answer: B

Source solution

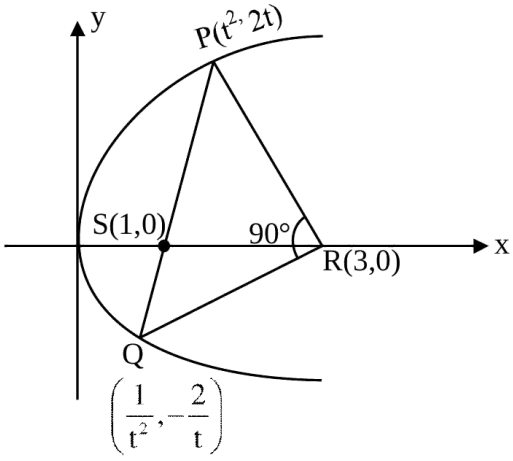
Sol. PQ is focal chord

$$\begin{aligned} &\mathrm{m_{PR}} \cdot \mathrm{m_{PQ}} = -1 \\ &\frac{2t}{t^2 - 3} \times \frac{-2/t}{\frac{1}{t^2} - 3} = -1 \\ &\left(t^2 - 1\right)^2 = 0 \\ &\Rightarrow t = 1 \end{aligned}$$

$\Rightarrow \mathrm{P\&Q}$ must be end point of latus rectum:

$$\mathrm{P}(1, 2)\&\mathrm{Q}(1, -2)$$

$\therefore \frac{2b^2}{a} = 4\&ae = 1$
$\because$ We know that $\mathrm{b^2 = a^2\left(1 - e^2\right)}$
$\therefore \mathrm{a} = 1 + \sqrt{2}$
$\because \mathrm{e^2} = 1 - \frac{\mathrm{b^2}}{\mathrm{a^2}}$
$\therefore \mathrm{e^2} = 3 - 2\sqrt{2}$
$\frac{1}{\mathrm{e^2}} = 3 + 2\sqrt{2}$



Question 95

The focus of the parabola $y^2 = 4x + 16$ is the centre of the circle C of radius 5 . If the values of λ, for which C passes through the point of intersection of the lines $3x - y = 0$ and $x + \lambda y = 4$, are λ_1 and $\lambda_2, \lambda_1 < \lambda_2$, then $12\lambda_1 + 29\lambda_2$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 15

Source solution

$y^2 = 4(x + 4)$

Equation of circle

$(x + 3)^2 + y^2 = 25$

Passes through the point of intersection of two lines $3x - y = 0$ and $x + \lambda y = 4$ which is $\left(\frac{4}{3\lambda+1}, \frac{12}{3\lambda+1}\right)$, after solving with circle,

we get

$\lambda = -\frac{7}{6}, 1$

$12\lambda_1 + 29\lambda_2$

$-14 + 29 = 15$

Question 96

Let the shortest distance from $(a, 0), a > 0$, to the parabola $y^2 = 4x$ be 4 . Then the equation of the circle passing through the point $(a, 0)$ and the focus of the parabola, and having its centre on the axis of the parabola is:

A. $x^2 + y^2 - 6x + 5 = 0$

B. $x^2 + y^2 - 4x + 3 = 0$

C. $x^2 + y^2 - 10x + 9 = 0$

D. $x^2 + y^2 - 8x + 7 = 0$

Correct answer: A

Source solution

Sol.
Normal at P
$y + tx = 2t + t^3 \uparrow$
(a, 0)
at $= 2t + t^3$
$a = 2 + t^2$
$R\left(2 + t^2, 0\right)$
$PR = 4 \Rightarrow 4 + 4t^2 = 16$

$$4t^2 = 12 \Rightarrow t^2 = 3$$

$a = 5, R(5, 0)$
Focus $(1, 0)$
$(1, 0) \& (5, 0)$ will be the end points of diameter
$\Rightarrow Eq^n$ of circle is

$$(x - 1)(x - 5) + y^2 = 0$$
$$x^2 + y^2 - 6x + 5 = 0$$

Question 97

Let the lengths of the transverse and conjugate axes of a hyperbola in standard form be 2 a and 2 b , respectively, and one focus and the corresponding directrix of this hyperbola be $(-5, 0)$ and $5x + 9 = 0$, respectively. If the product of the focal distances of a point $(\alpha, 2\sqrt{5})$ on the hyperbola is p, then $4p$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 189

Source solution

$$PF_1 \cdot PF_2 = (e\alpha - a)(e\alpha + a)$$
$$P = e^2\alpha^2 - a^2 = \frac{25}{9} \cdot 9 \cdot \frac{9}{4} - 9 = \frac{189}{4}$$
$$\left.\begin{matrix} ae = 5 \\ \frac{a}{e} = 9/5 \end{matrix}\right\} \begin{matrix} a = 3 \\ e = 5/3 \\ b = 4 \end{matrix}$$
$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$
$$\frac{\alpha^2}{9} - \frac{4(5)}{16} = 1 \Rightarrow \alpha^2 = 9\left(\frac{36}{16}\right)$$

Question 98

If A and B are the points of intersection of the circle $x^2+y^2-8x=0$ and the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ and a point P moves on the line $2x-3y+4=0$, then the centroid of $\triangle PAB$ lies on the line :

A. $4x-9y=12$

B. $x+9y=36$

C. $9x-9y=32$

D. $6x-9y=20$

Correct answer: D

Source solution

Sol. $x^2+y^2-8x=0, \frac{x^2}{9}-\frac{y^2}{4}=1$

$$4x^2-9y^2=36 \quad (1)$$

Solve (1) & (2)

$4x^2-9\left(8x-x^2\right)=36$

$13x^2-72x-36=0$

$(13x+6)(x=6)=0$

$x=\frac{-6}{13}, x=6$

$x=\frac{-6}{13}$ (rejected)

$y \rightarrow$ Imaginary

$n=6, \frac{36}{9}-\frac{y^2}{4}=1$

$y^2=12, y=I\sqrt{12}$

$A(6,\sqrt{12}), B(6,-\sqrt{12})$

$p\left(\alpha, \frac{2\alpha+4}{3}\right) P$ lies on

centroid (h, k)

$2x-3y+y=0$

$h=\frac{12+\alpha}{3}, \alpha=3h-12$

$k=\frac{\frac{2\alpha+4}{3}}{3} \Rightarrow 2\alpha+4=9k$

$\alpha=\frac{9k-4}{2}$

$6h-2y=9k-4$

$6x-9y=20$

Question 99

The length of the chord of the parabola $x^2 = 4y$ having equation $x - \sqrt{2}y + 4\sqrt{2} = 0$ is :

A. $2\sqrt{11}$

B. $3\sqrt{2}$

C. $6\sqrt{3}$

D. $8\sqrt{2}$

Correct answer: C

Source solution

Sol.

$$x^2 = 4y$$
$$x - \sqrt{2}y + 4\sqrt{2} = 0$$

Solving together we get

$$x^2 = 4\left(\frac{x+4\sqrt{2}}{\sqrt{2}}\right)$$
$$\sqrt{2}x^2 + 4x + 16\sqrt{2}$$
$$\sqrt{2}x^2 - 4x - 16\sqrt{2} = 0$$
$$x_1 + x_2 = 2\sqrt{2}; \quad x_1x_2 = \frac{-16\sqrt{2}}{\sqrt{2}} = -16$$

Similarly,

$$(\sqrt{2}y - 4\sqrt{2})^2 = 4y$$
$$2y^2 + 32 - 16y = 4y$$
$$2y^2 - 20y + 32 = 0 \Longleftrightarrow y_1 + y_2 = 10$$
$$\ell_{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(2\sqrt{2})^2 + 64 + (10)^2 - 4(16)}$$
$$= \sqrt{8 + 64 + 100 - 64}$$
$$= \sqrt{108} = 6\sqrt{3}$$

Option (3)

Question 100

If the midpoint of a chord of the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ is $(\sqrt{2}, 4/3)$, and the length of the chord is $\frac{2\sqrt{\alpha}}{3}$, then α is :

A. 18

B. 22

C. 26

D. 20

Correct answer: B

Source solution

Sol.

If $\mathbf{m}\left(\sqrt{2}, \frac{4}{3}\right)$ than equation of of AB is

$$\mathbf{T}=\mathbf{S_1}$$

$\frac{x\sqrt{2}}{9}+\frac{y}{4}\left(\frac{4}{3}\right)=\frac{(\sqrt{2})^2}{9}+\frac{\left(\frac{4}{3}\right)^2}{4}$
$\frac{\sqrt{2}x}{9}+\frac{y}{3}=\frac{2}{9}+\frac{4}{9}$
$\sqrt{2}x+3y=6 \Rightarrow y=\frac{6-\sqrt{2}x}{3}$ put in ellipse
So, $\frac{x^2}{9}+\frac{(6-\sqrt{2}x)^2}{9\times4}=1$
$4x^2+36+2x^2-12\sqrt{2}x=36$
$6x^2-12\sqrt{2}x=0$
$6x(x-2\sqrt{2})=0$
$x=0 \& x=2\sqrt{2}$
So $y=2 \quad y=\frac{2}{3}$
Length of chord $=\sqrt{(2\sqrt{2}-0)^2+\left(\frac{2}{3}-2\right)^2}$

$$=\sqrt{8+\frac{16}{9}}$$
$$=\sqrt{\frac{88}{9}}=\frac{2}{3}\sqrt{22} \text{ so } \alpha=22$$

Question 101

The length of the chord of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$, whose mid point is $\left(1,\frac{2}{5}\right)$, is equal to :

A. $\frac{\sqrt{1691}}{5}$

B. $\frac{\sqrt{2009}}{5}$

C. $\frac{\sqrt{1741}}{5}$

D. $\frac{\sqrt{1541}}{5}$

Correct answer: A

Source solution

Sol. Equation of chord with given middle point.

$\mathbf{T=S_1}$

$\frac{x}{25}+\frac{y}{40}=\frac{1}{25}+\frac{1}{100}$

$\frac{8x+5y}{200}=\frac{8+2}{200}$

$$y=\frac{10-8x}{5} \quad \text{(i)}$$

$\frac{x^2}{25}+\frac{(10-8x)^2}{400}=1$ (put in original equation)

$\frac{16x^2+100+64x^2-160x}{400}=1$

$\mathbf{4x^2-8x-15=0}$

$\mathbf{x=\frac{8\pm\sqrt{304}}{8}}$

$\mathbf{x_1=\frac{8+\sqrt{304}}{8}; x_2=\frac{8-\sqrt{304}}{8}}$

Similarly, $y=\frac{10-18\pm\sqrt{304}}{5}=\frac{2\pm\sqrt{304}}{5}$

$\mathbf{y_1=\frac{2-\sqrt{304}}{5}; y_2=\frac{2+\sqrt{304}}{5}}$

Distance $=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$$=\sqrt{\frac{4\times304}{64}+\frac{4\times304}{25}}=\frac{\sqrt{1691}}{5}$$

Introduction to Three Dimensional Geometry

Questions 102-103 / 2 selected / 0.7% practice weight / Consolidate

Question 102

If the four points, whose position vectors are $3\hat{i}-4\hat{j}+2\hat{k}, \hat{i}+2\hat{j}-\hat{k}, -2\hat{i}-\hat{j}+3\hat{k}$ and $5\hat{i}-2\alpha\hat{j}+4\hat{k}$ are coplanar, then α is equal to

A. $\frac{73}{17}$

B. $\frac{107}{17}$

C. $\frac{-73}{17}$

D. $\frac{-107}{17}$

Correct answer: A

Source solution

$$P = 3\hat{i}-4\hat{j}+2\hat{k}, \qquad Q = \hat{i}+2\hat{j}-\hat{k},$$

$$R = -2\hat{i}-\hat{j}+3\hat{k}, \qquad S = 5\hat{i}-2\alpha\hat{j}+4\hat{k}.$$

$$\overrightarrow{PQ} = Q-P = -2\hat{i}+6\hat{j}-3\hat{k}.$$

$$\overrightarrow{QR} = R-Q = -3\hat{i}-3\hat{j}+4\hat{k}.$$

$$\overrightarrow{RS} = S-R = 7\hat{i}+(1-2\alpha)\hat{j}+\hat{k}.$$

Since the four points are coplanar,

$$[\overrightarrow{PQ}\ \overrightarrow{QR}\ \overrightarrow{RS}] = 0.$$

Therefore,

$$\begin{vmatrix} -2 & 6 & -3 \\ -3 & -3 & 4 \\ 7 & 1-2\alpha & 1 \end{vmatrix} = 0.$$

Expanding along the first row,

$$-2\begin{vmatrix} -3 & 4 \\ 1-2\alpha & 1 \end{vmatrix} - 6\begin{vmatrix} -3 & 4 \\ 7 & 1 \end{vmatrix} - 3\begin{vmatrix} -3 & -3 \\ 7 & 1-2\alpha \end{vmatrix} = 0.$$

$$-2\Big[-3-4(1-2\alpha)\Big] - 6(-31) - 3\Big[6\alpha - 3 + 21\Big] = 0.$$

$$14 - 16\alpha + 186 - 18\alpha - 54 = 0.$$

$$146 - 34\alpha = 0.$$

$$34\alpha = 146.$$

$$\boxed{\alpha = \frac{73}{17}}.$$

Question 103

Let for a triangle ABC ,

$\overrightarrow{AB} = -2\hat{i} + \hat{j} + 3\hat{k}$

$\overrightarrow{CB} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$

$\overrightarrow{CA} = 4\hat{i} + 3\hat{j} + \delta\hat{k}$

If $\delta > 0$ and the area of the triangle ABC is $5\sqrt{6}$, then $\overrightarrow{CB} \cdot \overrightarrow{CA}$ is equal to

A. 60

B. 120

C. 108

D. 54

Official Answer. By NTA (1)

Correct answer: A

Source solution

Sol.

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$

$\alpha = 2, \beta = 4, \gamma - \delta = 3$

$\frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = 5\sqrt{6}$

$(\delta - 9)^2 + (2\delta + 12)^2 + 100 = 600$

$\Rightarrow \delta = 5, \gamma = 8$

Hence $\overrightarrow{CB} \cdot \overrightarrow{CA} = 60$

Limits and Derivatives

Questions 104-105 / 2 selected / 0.7% practice weight / Consolidate

Question 104

If the domain of the function
$f(x)=\log_e\left(4x^2+11x+6\right)+\sin^{-1}$
$(4x+3)+\cos^{-1}\left(\frac{10x+6}{3}\right)$ is $(\alpha,\beta]$,
Then $36|\alpha+\beta|$ is equal to :

A. 63

B. 45

C. 72

D. 54

Correct answer: B

Source solution

Sol. $f(x)=\ln\left(4x^2+11x+6\right)+\sin^{-1}(4x+3)$
$+\cos^{-1}\left(\frac{10x+6}{3}\right)$
(i) $4x^2+11x+6>0$
$4x^2+8x+3x+6>0$
$(4x+3)(x+2)>0$
$x\in(-\infty,-2)\cup\left(-\frac{3}{4},\infty\right)$
(ii) $4x+3\in[-1,1]$
$x\in[-1,-1/2]$
(iii) $\frac{10x+6}{3}\in[-1,1]$
$x\in\left[-\frac{9}{10},-\frac{3}{10}\right]$
$x\in\left(-\frac{3}{4},-\frac{1}{2}\right]\quad\alpha=-\frac{3}{4},\beta=-\frac{1}{2}$
$\alpha+\beta=-\frac{5}{4}$
$36|\alpha+\beta|=45$

Question 105

If the domain of the function $f(x)=\log_e\left(\frac{2x-3}{5+4x}\right)+\sin^{-1}\left(\frac{4+3x}{2-x}\right)$ is $[\alpha,\beta)$, then $\alpha^2+4\beta$ is equal to

A. 5

B. 4

C. 3

D. 7

Correct answer: B

Source solution

Sol.

Given function is

$f(x)=\log_e\left(\frac{2x-3}{5+4x}\right)+\sin^{-1}\left(\frac{4+3x}{2-x}\right)$

For domain, the conditions are

$\frac{2x-3}{5+4x}>0$ and $\left|\frac{4+3x}{2-x}\right|\leq 1$

Now, $\frac{2x-3}{5+4x}>0\Rightarrow x\in\left(-\infty,-\frac{5}{4}\right)\cup\left[\frac{3}{2},\infty\right)$

and $\quad -1\leq\frac{4+3x}{2-x}\leq 1$

$\Rightarrow\left(-1\leq\frac{4+3x}{2-x}\right)\cap\left(\frac{4+3x}{2-x}\leq 1\right)$

$\Rightarrow\left(\frac{6+2x}{2-x}\geq 0\right)\cap\left(\frac{2+4x}{2-x}\leq 0\right)$

$\Rightarrow\frac{6+2x}{2-x}\cdot\frac{2+4x}{2-x}\leq 0$

$\Rightarrow x\in\left[-3,-\frac{1}{2}\right]$

Hence, we get the domain of f as $x\in\left[-3,-\frac{5}{4}\right)$

This means that $\alpha=-3,\beta=-\frac{5}{4}$

Thus, $\alpha^2+4\beta=9-5=4$

Statistics

Question 106

The mean and variance of a set of 15 numbers are 12 and 14 respectively. The mean and variance of another set of 15 numbers are 14 and σ^2 respectively. If the variance of all the 30 numbers in the two sets is 13 , then σ^2 is equal to

A. 9

B. 12

C. 11

D. 10

Correct answer: D

Source solution

Sol. Combine var. $= \frac{n_1\sigma^2+n_2\sigma^2}{n_1+n_2} + \frac{n_1n_2(m_1-m_2)^2}{(n_1+n_2)^2}$

$13 = \frac{15.14+15.\sigma^2}{30} + \frac{15.15(12-14)^2}{30\times30}$

$13 = \frac{14+\sigma^2}{2} + \frac{4}{4}$

$\sigma^2 = 10$

Question 107

Let the mean and variance of 12 observations be $\frac{9}{2}$ and 4 respectively. Later on, it was observed that two observations were considered as 9 and 10 instead of 7 and 14 respectively. If the correct variance is $\frac{\mathbf{m}}{\mathbf{n}}$, where m and n are co-prime, then $\mathbf{m+n}$ is equal to

A. 316

B. 314

C. 317

D. 315

Correct answer: C

Source solution

$$\textbf{Given:} \qquad n = 12, \quad \bar{x} = \frac{9}{2}, \quad \sigma^2 = 4.$$

Using

$$\sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2,$$

we get

$$4 = \frac{\sum x_i^2}{12} - \left(\frac{9}{2}\right)^2.$$

Hence,

$$\frac{\sum x_i^2}{12} = 4 + \frac{81}{4} = \frac{97}{4}.$$

Therefore,

$$\sum x_i^2 = 12 \cdot \frac{97}{4} = 291.$$

Also,

$$\sum x_i = 12 \cdot \frac{9}{2} = 54.$$

The observations 9 and 10 were taken instead of the correct values 7 and 14.

Thus,

$$\text{Correct sum} = 54 - (9 + 10) + (7 + 14) = 56.$$

Hence the correct mean is

$$\bar{x}_c = \frac{56}{12} = \frac{14}{3}.$$

Similarly,

$$\text{Correct } \sum x_i^2 = 291 - (9^2 + 10^2) + (7^2 + 14^2).$$

$$= 291 - (81 + 100) + (49 + 196) = 355.$$

Now,

$$\sigma_c^2 = \frac{355}{12} - \left(\frac{14}{3}\right)^2.$$

$$= \frac{355}{12} - \frac{196}{9}.$$

Taking LCM $= 36$,

$$\sigma_c^2 = \frac{1065 - 784}{36} = \frac{281}{36}.$$

Thus,

$$\sigma_c^2 = \frac{281}{36}.$$

Therefore,

$$m = 281, \qquad n = 36.$$

Hence,

$$m + n = 281 + 36 = 317.$$

$$\boxed{317}$$

Hence, the correct option is

$$\boxed{\text{(3) } 317}.$$

Question 108

Let sets A and B have 5 elements each. Let the mean of the elements in sets A and B be 5 and 8 respectively and the variance of the elements in sets A and B be 12 and 20 respectively A new set C of 10 elements is formed by subtracting 3 from each element of A and adding 2 to each element of B. Then the sum of the mean and variance of the elements of C is ..... .

A. 32

B. 38

C. 40

D. 36

Correct answer: B

Source solution

Sol. $\omega \mathrm{A}=\{\mathrm{a}_1, \mathrm{a}_2, \mathrm{a}_3, \mathrm{a}_4, \mathrm{a}_5\}$

$$\mathrm{B}=\{\mathrm{b}_1,\ \mathrm{b}_2,\ \mathrm{b}_3,\ \mathrm{b}_4,\ \mathrm{b}_5\}$$

Given, $\sum_{\mathrm{i}=1}^5 \mathrm{ai}=25, \sum_{\mathrm{i}=1}^5 \mathrm{bi}=40$

$\frac{\sum_{\mathrm{i}=1}^5 \mathrm{a}_{\mathrm{i}}^2}{5}-\left(\frac{\sum_{\mathrm{i}=1}^5 \mathrm{a}_{\mathrm{i}}}{5}\right)^2=12, \frac{\sum_{\mathrm{i}=1}^5 \mathrm{b}_{\mathrm{i}}^2}{5}-\left(\frac{\sum_{\mathrm{i}=1}^5 \mathrm{b}_{\mathrm{i}}}{5}\right)^2=20$

$\Rightarrow \quad \sum_{\mathrm{i}=1}^5 \mathrm{a}_{\mathrm{i}}^2=185 \quad , \quad \sum_{\mathrm{i}=1}^5 \mathrm{b}_{\mathrm{i}}^2=420$

Now, $\mathrm{C}=\{\mathrm{C}_1, \mathrm{C}_2, \ldots . . \mathrm{C}_{10}\}$

s.f. $\mathrm{C}_{\mathrm{i}}=\mathrm{a}_{\mathrm{i}}=3$ or irstive $\mathrm{b}_{\mathrm{i}}+2 \qquad$ elements Last five elements

$\therefore \quad$ Mean of $\mathrm{C}, \overline{\mathrm{C}}=\frac{(\sum \mathrm{a}_{\mathrm{i}}-15)+(\sum \mathrm{b}_{\mathrm{i}}+10)}{10}$

$\overline{\mathrm{C}}=\frac{10+50}{10}=6$

$\therefore \quad \sigma^2=\frac{\sum_{i=1}^{10} C_i^2}{10}=(\bar{C})^2$

$=\frac{\sum (a_i-3)^2+\sum (b_i+2)^2}{10}-(6)^2$

$=\frac{\sum \mathrm{a}_{\mathrm{i}}{}^2+\sum \mathrm{b}_{\mathrm{i}}{}^2-6 \sum \mathrm{a}_{\mathrm{i}}+4 \sum \mathrm{b}_{\mathrm{i}}+65}{10}-36$

$=\frac{185+420-150+160+65}{10}-36$

$=32$

$\therefore \quad$ Mean + Variance $=\overline{\mathrm{C}}+\sigma^2=6+32=38$

Question 109

Let the six numbers $a_1, a_2, a_3, a_4, a_5, a_6$, be in A.P. and $a_1 + a_3 = 10$. If the mean of these six numbers is $\frac{19}{2}$ and their variance is σ^2, then $8\sigma^2$ is equal to :

A. 210

B. 220

C. 200

D. 105

Correct answer: A

Source solution

Sol. 1}

$$a + (a + 2d) = 10 \Rightarrow a + d = 5 \quad (1)$$

Mean $\Rightarrow \frac{\frac{6}{2}[2a+5d]}{6} = \frac{19}{2}$

$$2a + 5d = 19 \quad (2)$$

from (1) and (2)

$3d = 9 \Rightarrow d = 3; a = 2$

$\therefore \sigma^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2$

$$= \frac{2^2 + 5^2 + 8^2 + 11^2 + 14^2 + 17^2}{6} - \left(\frac{19}{2}\right)^2$$

$$= \frac{699}{6} - \frac{361}{4}$$

$$= \frac{233}{2} - \frac{361}{4}$$

$8\sigma^2 = 932 - 722 = 210$

Question 110

Let $9 = x_1 < x_2 < \ldots\ldots < x_7 \ldots\ldots, x_7$ be in an A.P. with common difference d . If the standard deviation of $x_1 \cdot x_2 \ldots., x_7$ is 4 and mean is $\overline{x}$, then $\overline{x} + x_6$ is equal to :

A. $2\left(9 + \frac{8}{\sqrt{7}}\right)$

B. $18\left(1 + \frac{1}{\sqrt{3}}\right)$

C. 25

D. 34

Correct answer: D

Source solution

Sol. 4

Mean $\Rightarrow \bar{x} = \frac{\sum_{i=1}^{7} x_i}{7} = \frac{\frac{7}{2}[2a+6d]}{7} = a + 3d = x_4$

Variance $= \frac{\sum_{i=1}^{7} (x_i - \bar{x})^2}{7} = (4)^2 \Rightarrow \frac{\sum_{i=1}^{7} (x_i - x_4)^2}{7} = 16$

$\Rightarrow \frac{(3d)^2 + (2d)^2 + d^2 + 0 + d^2 + (2d)^2 + (3d)^2}{7} = 16$

$= 4d^2 = 16 \Rightarrow d = 2$

$\Rightarrow \bar{x} = 9 + 3(2) = 15$

$\& x_0 = a + 5d = 9 + 5(2) = 19 \Rightarrow \bar{x} + x_0 = 34$

Question 111

Let S be the set of all values of $\mathbf{a_1}$ for which the mean deviation about the mean of 100 consecutive positive integers $a_1, a_2, a_3, \ldots . a_{100}$ is 25 . Then S is

A. N

B. ϕ

C. $\{99\}$

D. $\{9\}$

Correct answer: A

Source solution

Sol. 1

Let $\mathbf{a_1 = n} \quad \mathbf{a_2 = n + 1} \quad \mathbf{a_3 = n + 2} \quad \ldots\ldots$

$$\bar{x} = \frac{n + (n+1) + (n+2) + \ldots . n + 99}{100}$$
$$= \frac{100n + \frac{100 \times 99}{2}}{100} = n + \frac{99}{2}$$

Mean deviation about the mean

$\frac{1}{100} \sum |\mathbf{x_i - \bar{x}}|$

$\Rightarrow \frac{1}{100}\left(\frac{99}{2} + \frac{97}{2} + \frac{95}{2} + \ldots\ldots . + \frac{97}{2} + \frac{99}{2}\right)$

$\Rightarrow \frac{2}{100}\left(\frac{99}{2} + \frac{97}{2} + \frac{95}{2} + \ldots\ldots .50 \text{ terms }\right)$

$\Rightarrow \frac{2}{100} \times \frac{1}{2} \times (50)^2 = \frac{50 \times 50}{100} = 25$

It is 25 irrespective of the value of

$\therefore \mathbf{n} \in \mathbf{N}$

$\Rightarrow \quad \mathbf{S = N}$

Question 112

The mean and variance of 7 observations are 8 and 16 respectively. If one observation 14 is omitted and a and b are respectively mean and variance of remaining 6 observation, then $a + 3b - 5$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 37

Source solution

37

Mean of 7 observations $= \frac{\sum_{i=1}^{7} x_i}{7}$

$\Rightarrow \quad \sum_{i=1}^{7} x_i = 7 \times 8 = 56$

Variance $= \frac{\Sigma x_i^2}{n} - (\overline{x})^2$

$\Sigma x_i^2 = 7(16 + 64) = 560$

If 14 is removed then

Mean $= a = \frac{\sum_{i=1}^{7} x_i - 14}{6} \Rightarrow 6a = 56 - 14$

$\Rightarrow a = 7$

Variance $= b = \frac{\sum_{i=1}^{7} x_i^2 - (14)^2}{6} - 49$

$\Rightarrow \quad 6\,b = 560 - 196 - 294$

$\Rightarrow \quad 6\,b = 70$

$\Rightarrow \quad 3\,b = 35$

$\therefore \quad a + 3b - 5 = 7 + 35 - 5 = 37$

Probability

Questions 113-115 / 3 selected / 1.0% practice weight / Consolidate

Question 113

Let X have a binomial distribution $B(n, p)$ such that the sum and the product of the mean and variance of X are 24 and 128 respectively. If $\mathbf{P(X > n - 3) = \frac{k}{2^n}}$, then k is equal to

A. 528

B. 529

C. 629

D. 630

Correct answer: B

Source solution

Sol. Let $\alpha =$ Mean & $\beta =$ Variance $(\alpha > \beta)$
So, $\alpha + \beta = 24, \quad \alpha\beta = 128$
$\Rightarrow \alpha = 16 \quad \& \quad \beta = 8$
$\Rightarrow \mathbf{np = 16} \quad \mathbf{npq = 8 \Rightarrow q = \frac{1}{2}}$

$$\therefore \mathbf{p = \frac{1}{2}, n = 32}$$

$\mathbf{p(x > n - 3) = \frac{1}{2^n}({}^nC_{n-2} + {}^nC_{n-1} + {}^nC_n)}$
$\therefore \mathbf{k = {}^{32}C_{30} + {}^{32}C_{31} + {}^{32}C_{32} = \frac{32 \times 31}{2} + 32 + 1 = 496 + 33 = 529}$

Question 114

The sum and product of the mean and variance of a binomial distribution are 82.5 and 1350 respectively. They the number of trials in the binomial distribution is:

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 96

Source solution

Let, mean $= m = np$
& variance $= \mathrm{v} = \mathrm{npq}, \mathrm{p} + \mathrm{q} = 1$
Sum $= m + v = \frac{165}{2}$
Product $= \mathrm{mv} = 1350$
On solving,
$\mathrm{m} = \mathrm{np} = 60 \& \mathrm{v} = \mathrm{npq} = \frac{45}{2} \therefore \mathrm{q} = \frac{3}{8} \therefore \mathrm{P} = \frac{5}{8}$
Hence $\mathrm{n} = 96$

Question 115

Let Ω be the sample space and $A \subseteq \Omega$ be an event.

Given below are two statements :
(S1) : If $\mathbf{P(A)=0}$, then $\mathbf{A}=\varnothing$
(S2): If $P(A)=1$, then $A=\Omega$
Then

A. both (S1) and (S2) are true

B. only (S1) is true

C. only (S2) is true

D. both (S1) and (S2) are false

Correct answer: D

Source solution

Sol. (4)
Let $\Omega=[0,1]$
Let $\mathbf{A} \rightarrow$ selecting $\frac{1}{2}$
$\mathbf{A}=\left\{\frac{1}{2}\right\}$
then, $\mathbf{P(A)=0}$ but $\mathbf{A} \neq \phi$
$B=A^{c}=[0,1]-\left\{\frac{1}{2}\right\}$
$\mathbf{P(B)=1}$
but $\mathbf{B} \neq \Omega$

Inverse Trigonometric Functions

Question 116

Let $S=\left\{x \in\left(-\frac{\pi}{2}, \frac{\pi}{2}\right): 9^{1-\tan^2 x}+9^{\tan^2 x}=10\right\}$ and $\beta=\sum_{x \in S} \tan^2\left(\frac{x}{3}\right)$, then $\frac{1}{6}(\beta-14)^2$ is equal to

A. 32

B. 8

C. 64

D. 16

Correct answer: A

Source solution

Sol. Let $9^{\tan^2 x}=P$

$\frac{9}{P}+P=10$

$P^2-10P+9=0$

$(P-9)(P-1)=0$

$P=1,9$

$9^{\tan^2 x}=1, 9^{\tan^2 x}=9$

$\tan^2 x=0, \tan^2 x=1$

$x=0, \pm\frac{\pi}{4} \quad \therefore x \in\left(-\frac{\pi}{2}, \frac{p}{2}\right)$

$\beta=\tan^2(0)+\tan^2\left(+\frac{\pi}{12}\right)+\tan^2\left(-\frac{\pi}{12}\right)$

$=0+2(\tan 15^\circ)^2$

$2(2-\sqrt{3})^2$

$2(7-4\sqrt{3})$

Than $\frac{1}{6}(14-8\sqrt{3}-14)^2=32$

Question 117

Let the domain of the function
$f(x)=\cos^{-1}\left(\frac{4x+5}{3x-7}\right)$ be $[\alpha,\beta]$ and the domain of
$g(x)=\log_2(2-6\log_{27}(2x+5))$ be (γ,δ).
Then $|7(\alpha+\beta)+4(\gamma+\delta)|$ is equal to

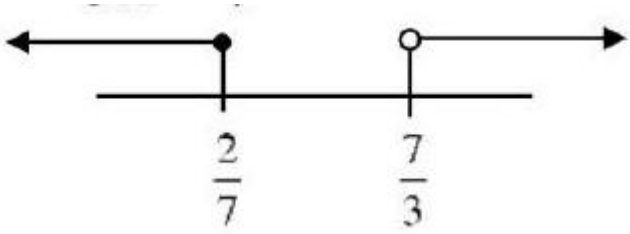
Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

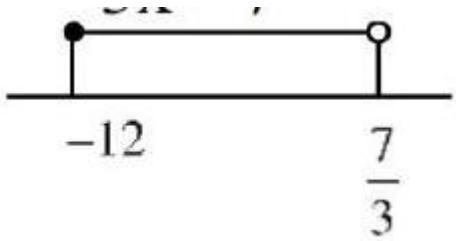
Correct answer: 96

Source solution

$f(x)=\cos^{-1}\left(\frac{4x+5}{3x-7}\right)$
$\Rightarrow -1\le\left(\frac{4x+5}{3x-7}\right)\le 1$
$\left(\frac{4x+5}{3x-7}\right)\ge -1$
$\frac{4x+5+3x-7}{3x-7}\ge 0$
$\Rightarrow \frac{7x-2}{3x-7}\ge 0$



$x\in\left(-\infty,\frac{2}{7}\right]\cup\left(\frac{7}{3},\infty\right)$
& $\frac{4x+5}{3x-7}\le 1\Rightarrow\frac{x+12}{3x-7}\le 0$



$\therefore$ Domain of $f(x)$ is
$\left[-12,\frac{2}{7}\right]\alpha=-12,\beta=\frac{2}{7}$
$g(x)=\log_2(2-6\log_{27}(2x+5))$
Domain

$2-6\log_{27}(2x+5)>0$
$\Rightarrow \quad 6\log_{27}(2x+5)<2$
$\Rightarrow \quad \log_{27}(2x+5)<\frac{1}{3}$
$\Rightarrow \quad 2x+5<3$
$\Rightarrow \quad x<-1$
$\&2x+5>0 \Rightarrow x>-\frac{5}{2}$
Domain is $x \in \left(-\frac{5}{2},-1\right)$
$\gamma=-\frac{5}{2}, \delta=-1$
$|7(\alpha+\beta)+4(\gamma+\delta)|=\left|7\left(-12+\frac{2}{7}+4\left(-\frac{5}{2}-1\right)\right|\right.$
$|-82-14|=96$

Question 118

Let $f(x)=2\cos^{-1}x+4\cot^{-1}x-3x^2-2x+10, x\in[-1,1]$. If $[a,b]$ is the range of the function then $4a-$ b is equal to:

A. 11

B. $11-\pi$

C. $11+\pi$

D. $15-\pi$

Correct answer: B

Source solution

Sol. $f'(x)=\frac{-2}{\sqrt{1-x^2}}-\frac{4}{1+x^2}-6x-2$
$=-2\left[\frac{1}{\sqrt{1-x^2}}+\frac{2}{1+x^2}+3x+1\right]$
$f'(x)<0 \Rightarrow f(x)$ is a dec. function
$f(1)=\pi+5$
$f(-1)=5\pi+5$
Range : $[a,b]\equiv[\pi+5,5\pi+5]$
$a=\pi+5,\ b=5\pi+5 \Rightarrow 4a-b=11-\pi$.

Question 119

Question ID: 101786

$50\tan\left(3\tan^{-1}\left(\frac{1}{2}\right)+2\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)+4\sqrt{2}\tan\left(\frac{1}{2}\tan^{-1}(2\sqrt{2})\right)$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 29

Source solution

$$
\begin{aligned}
&50\tan\left(3\tan^{-1}\tfrac{1}{2}+2\cos^{-1}\tfrac{1}{\sqrt{5}}\right)\\
&+4\sqrt{2}\tan\left(\tfrac{1}{2}\tan^{-1}2\sqrt{2}\right)\\
&=50\tan\left(\tan^{-1}\tfrac{1}{2}+2\left(\tan^{-1}\tfrac{1}{2}+\tan^{-1}2\right)\right)\\
&+4\sqrt{2}\tan\left(\tfrac{1}{2}\tan^{-1}2\sqrt{2}\right)\\
&=50\tan\left(\tan^{-1}\tfrac{1}{2}+2\cdot\tfrac{\pi}{2}\right))+4\sqrt{2}\times\tfrac{1}{\sqrt{2}}\\
&=50\left(\tan\tan^{-1}\tfrac{1}{2}\right)+4\\
&=25+4=29
\end{aligned}
$$

Question 120

$\sin^{-1}\left(\sin\frac{2\pi}{3}\right)+\cos^{-1}\left(\cos\frac{7\pi}{6}\right)+\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ is equal to:

A. $\frac{11\pi}{12}$

B. $\frac{17\pi}{12}$

C. $\frac{31\pi}{12}$

D. $-\frac{3\pi}{4}$

Correct answer: A

Source solution

Sol. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)+\cos^{-1}\left(\cos\frac{7\pi}{6}\right)+\tan^{-1}\tan\left(\frac{3\pi}{4}\right)$
$\sin^{-1}\sin\left(\frac{2\pi}{3}\right)=\pi-\frac{2\pi}{3}=\frac{\pi}{3}$
$\cos^{-1}\left(\cos\frac{2\pi}{6}\right)=2\pi-\frac{7\pi}{6}=\frac{5\pi}{6}$
$\tan^{-1}\tan\left(\frac{3\pi}{4}\right)=\frac{3\pi}{4}-\pi=\frac{-\pi}{4}$
$\sin^{-1}\left(\sin\frac{2\pi}{3}\right)+\cos^{-1}\cos\frac{7\pi}{6}+\tan^{-1}\tan\frac{3\pi}{4}$
$=\frac{11\pi}{12}$

Question 121

All x satisfying the inequality $\left(\cot^{-1}x\right)^2-7\left(\cot^{-1}x\right)+10>0$, lie in the interval:-

A. $(-\infty,\cot 5)\cup(\cot 4,\cot 2)$

B. $(\cot 5,\cot 4)$

C. $(\cot 2,\infty)$

D. $(-\infty,\cot 5)\cup(\cot 2,\infty)$

Correct answer: D

Source solution

Sol. $\cot^{-1}x>5,\quad\cot^{-1}x<2$
$\Rightarrow x<\cot 5, x>\cot 2$

Question 122

The number of values of $\mathbf{r} \in \{\mathbf{p}, \mathbf{q}, \sim \mathbf{p}, \sim \mathbf{q}\}$ for which $((\mathbf{p} \wedge \mathbf{q}) \Rightarrow (\mathbf{r} \vee \mathbf{q})) \wedge ((\mathbf{p} \wedge \mathbf{r}) \Rightarrow \mathbf{q})$ is a tautology, is:

A. 3

B. 4

C. 1

D. 2

Correct answer: D

Source solution

$$
\begin{aligned}
&((p \wedge q) \Rightarrow (r \vee q)) \wedge ((p \wedge r) \Rightarrow q) \\
&(p \wedge q) \Rightarrow (r \vee q) \\
&\sim (p \wedge q) \vee (r \vee q) \\
&\sim p \vee \sim q \vee r \vee q \\
&\&(p \wedge r) \Rightarrow q \\
&\sim (p \wedge r) \vee q \\
&\sim p \vee \sim r \vee q \\
&(\sim p \vee \sim q \vee r \vee q) \wedge (\sim p \vee \sim r \vee q) \\
&\equiv \sim p \vee \sim r \vee q \\
&\text{If} \quad r = p \\
&\text{If} \quad \sim p \vee \sim p \vee q \rightarrow \quad \text{Not tautology} \\
&\quad r = \sim p
\end{aligned}
$$

$$
\begin{aligned}
&\quad \sim p \vee p \vee q \quad \rightarrow \quad \text{tautology} \\
&\text{If} \quad r = q \\
&\quad \sim p \vee \sim q \vee q \quad \rightarrow \quad \text{tautology} \\
&\text{If} \quad r = \sim q \\
&\quad \sim p \vee q \vee q \quad \rightarrow \quad \text{Not tautology}
\end{aligned}
$$

Ans. 2 (D)

Determinants

Questions 123-123 / 1 selected / 0.3% practice weight / Consolidate

Question 123

Let the system of equations :
$2x + 3y + 5z = 9$,
$7x + 3y - 2z = 8$,
$12x + 3y - (4 + \lambda)z = 16 - \mu$,
have infinitely many solutions. Then the radius of the circle centred at (λ, μ) and touching the line $4x = 3y$ is

A. $\frac{17}{5}$

B. $\frac{7}{5}$

C. 7

D. $\frac{21}{5}$

Correct answer: B

Source solution

Sol. $\begin{vmatrix} 2 & 3 & 57 \\ 3 & -212 & 3 \\ -(\lambda+4) & 3 & -9 \end{vmatrix} = 0$
$\Rightarrow 12(-21) - 3(-39) - (\lambda + 4)(-15) = 0$
$\Rightarrow -252 + 117 + 15(1 + 4) = 0$
$\Rightarrow 15\lambda + 177 - 252 = 0$
$\Rightarrow 15\lambda - 75 = 0 \Rightarrow \lambda = 5$
$\begin{vmatrix} 9 & 3 & 58 \\ 3 & -216 - \mu & 3 \\ -9 & 3 & -9 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & 0 & 7\mu - 8 \\ 0 & 716 - \mu & 3 \\ -9 & 3 & -9 \end{vmatrix} = 0$
$\Rightarrow 7 - 7(\mu - 8) = 0 \Rightarrow 1 - (\mu - 8) = 0 \Rightarrow \mu = 9$
$\Rightarrow$ centre of circle $(5, 9)$
radius = length of $\perp$ from centre $(5, 9) = \left|\frac{20-27}{5}\right| = \frac{7}{5}$

Application of Derivatives

Questions 124-128 / 5 selected / 1.7% practice weight / Consolidate

Question 124

Let $y = f(x)$ be a thrice differentiable function in $(-5, 5)$. Let the tangents to the curve $\mathbf{y = f(x)}$ at $(1, f(1))$ and $(3, f(3))$ make angles $\frac{\pi}{6}$ and $\frac{\pi}{4}$, respectively with positive x -axis. If $27\int_1^3 \left((f'(t))^2 + 1\right) f''(t)dt = \alpha + \beta\sqrt{3}$ where $\alpha,$ β are integers, then the value of $\alpha + \beta$ equals

A. -14

B. 26

C. -16

D. 36

Correct answer: B

Source solution

Sol. $y = f(x) \Rightarrow \frac{dy}{dx} = f'(x)$

$$\left.\frac{dy}{dx}\right)_{(1,f(1))} = f'(1) = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}} \Rightarrow f'(1) = \frac{1}{\sqrt{3}}$$

$$\left.\frac{dy}{dx}\int_{(3,f(3))}\right. = f'(3) = \tan\frac{\pi}{4} = 1 \Rightarrow f'(3) = 1$$

$$27\int_1^3 \left(\left(f'(t)\right)^2 + 1\right) f''(t)dt = \alpha + \beta\sqrt{3}$$

$$I = \int_1^3 \left(\left(f'(t)\right)^2 + 1\right) f''(t)dt$$

$$f'(t) = z \Rightarrow f'(t)dt = dz$$

$$z = f'(3) = 1$$

$$z = f'(1) = \frac{1}{\sqrt{3}}$$

$$I = \int_{1/\sqrt{3}}^1 \left(z^2 + 1\right)dz = \left(\frac{z^3}{3} + z\right)_{1/\sqrt{3}}^1$$

$$= \left(\frac{1}{3} + 1\right) - \left(\frac{1}{3}\cdot\frac{1}{3\sqrt{3}} + \frac{1}{\sqrt{3}}\right)^1$$

$$= \frac{4}{3} - \frac{10}{9\sqrt{3}} = \frac{4}{3} - \frac{10}{27}\sqrt{3}$$

$$\alpha + \beta\sqrt{3} = 27\left(\frac{4}{3} - \frac{10}{27}\sqrt{3}\right) = 36 - 10\sqrt{3}$$

$$\alpha = 36, \beta = -10$$

$$\alpha + \beta = 36 - 10 = 26$$

\setcounter{enumi}{9}

Question 125

Let $a > 0$. If the function $f(x) = 6x^3 - 45ax^2 + 108a^2x + 1$ attains its local maximum and minimum values at the points $\mathbf{x_1}$ and $\mathbf{x_2}$ respectively such that $\mathbf{x_1x_2 = 54}$, then $\mathbf{a + x_1 + x_2}$ is equal to :-

A. 15

B. 18

C. 24

D. 13

Correct answer: B

Source solution

Sol. $\mathbf{f'(x) = 18x^2 - 90ax + 108a^2 = 0}$

$\mathbf{x = 2a \& x = 3a}$

$\mathbf{x_1 = 2a \quad x_2 = 3a}$

$\mathbf{x_1x_2 = 54}$

$\mathbf{6a^2 = 54}$

$\mathbf{a = 3}$

$\mathbf{a + x_1 + x_2}$

$\mathbf{3 + 2 \times 3 + 3 \times 3 = 18}$

option (2)

Question 126

Let the triangle PQR be the image of the triangle with vertices $(1,3),(3,1)$ and $(2,4)$ in the line $x+2y=2$. If the centroid of $\triangle PQR$ is the point (α,β), then $15(\alpha-\beta)$ is equal to :

A. 24

B. 19

C. 21

D. 22

Correct answer: D

Source solution

Sol. Let ' G ' be the centroid of Δ formed by $(1,3)(3,1)$ & $(2,4)$

$G \cong \left(2, \frac{8}{3}\right)$

Image of G w.r.t. $x+2y-2=0$

$\frac{\alpha-2}{1}=\frac{\beta-\frac{8}{3}}{2}=-2\frac{\left(2+\frac{16}{3}-2\right)}{1+4}$

$=\frac{-2}{5}\left(\frac{16}{3}\right)$

$\Rightarrow \alpha=\frac{-32}{15}+2=\frac{-2}{15}, \beta=\frac{-32\times 2}{15}+\frac{8}{3}=\frac{-24}{15}$

$15(\alpha-\beta)=-2+24=22$

Question 127

Let the plane $P: 8x + \alpha_1 y + \alpha_2 z + 12 = 0$ be parallel to the line $\mathbf{L}: \frac{x+2}{2} = \frac{y-3}{3} = \frac{z+4}{5}$. If the intercept of P on the y-axis is 1 , then the distance between P and L is :

A. $\sqrt{\frac{7}{2}}$

B. $\sqrt{\frac{2}{7}}$

C. $\frac{6}{\sqrt{14}}$

D. $\sqrt{14}$

Correct answer: D

Source solution

Sol. 4

$y-$ intercept $= \frac{-12}{\alpha_1} = 1$

$\alpha_1 = -12 \& \overrightarrow{\mathrm{n}} = (8, \alpha_1, \alpha_2)$

$\vec{\ell} = (2, 3, 5)$

$\overrightarrow{\mathrm{n}} \cdot \vec{\ell} = 0 (\because$ plane P & line L are parallel)

$16 + 3\alpha_1 + 5\alpha_2 = 0$

$16 - 36 + 5\alpha_2 = 0$

$5\alpha_2 = 20$

$\alpha_2 = 4$

$$8x - 12y + 4z + 12 = 0$$
$$\Rightarrow 2x - 3y + z + 3 = 0$$

$(-2, 3, -4)$ is a point on the line L distance bet ^{n} the point $(-2, 3, -4)$ and the plane P is :-

$$\mathrm{d} = \frac{|-4 - 9 - 4 + 3|}{\sqrt{2^2 + 3^2 + 1^2}}$$
$$= \frac{14}{\sqrt{14}} = \sqrt{14}$$

\setcounter{enumi}{75}

Question 128

Let the area of a $\triangle PQR$ with vertices $P(5,4), Q(-2,4)$ and $R(a,b)$ be 35 square units. If its orthocenter and centroid are $O\left(2, \frac{14}{5}\right)$ and $C(c,d)$ respectively, then $c+2d$ is equal to

A. $\frac{7}{3}$

B. 3

C. 2

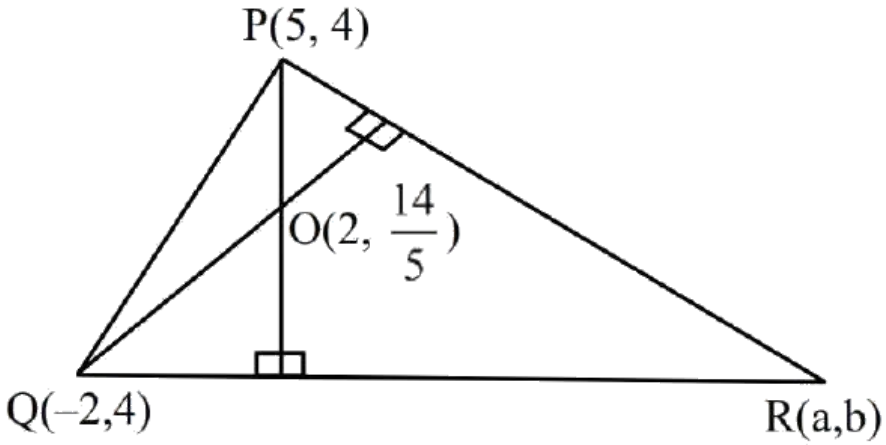
D. $\frac{8}{3}$

Correct answer: B

Source solution

Sol.

Equation of lines $QR = 5x + 2y + 2 = 0$
Equation of lines $PR = 10x - 3y - 38 = 0$
$\therefore$ Point $R(2,-6)$
Centroid $= \left(\frac{5-2+2}{3}, \frac{4+4-6}{3}\right)$
$= \left(\frac{5}{3}, \frac{2}{3}\right)$
$c+2d = \frac{5}{3} + \frac{4}{3} = 3$



Integrals

Questions 129-129 / 1 selected / 0.3% practice weight / Consolidate

Question 129

Let $f(x)+2f\left(\frac{1}{x}\right)=x^2+5$ and
$2g(x)-3g\left(\frac{1}{2}\right)=x, x>0$. If $\alpha=\int_1^2 f(x)dx$, and $\beta=\int_1^2 g(x)dx$, then the value of $9\alpha+\beta$ is :

A. 1

B. 0

C. 10

D. 11

Correct answer: D

Source solution

Given,

$$f(x)+2f\left(\frac{1}{x}\right)=x^2+5.$$

Replacing x by $\frac{1}{x}$,

$$f\left(\frac{1}{x}\right)+2f(x)=\frac{1}{x^2}+5.$$

Solving the two equations,

$$f(x)=\frac{2}{3x^2}-\frac{x^2}{3}+\frac{5}{3}.$$

Therefore,

$$\alpha = \int_1^2 \left(\frac{2}{3x^2} - \frac{x^2}{3} + \frac{5}{3} \right) dx.$$

$$= \left[-\frac{2}{3x} - \frac{x^3}{9} + \frac{5x}{3} \right]_1^2.$$

$$= \left(-\frac{1}{3} - \frac{8}{9} + \frac{10}{3} \right) - \left(-\frac{2}{3} - \frac{1}{9} + \frac{5}{3} \right).$$

$$= 2 - \frac{7}{9} = \frac{11}{9}.$$

Hence,

$$\alpha = \frac{11}{9}.$$

Now,

$$2g(x) - 3g\left(\frac{1}{2}\right) = x.$$

Putting $x = \frac{1}{2}$,

$$2g\left(\frac{1}{2}\right) - 3g\left(\frac{1}{2}\right) = \frac{1}{2}.$$

$$g\left(\frac{1}{2}\right) = -\frac{1}{2}.$$

Therefore,

$$2g(x) + \frac{3}{2} = x,$$

$$g(x) = \frac{x}{2} - \frac{3}{4}.$$

Thus,

$$\beta = \int_1^2 \left(\frac{x}{2} - \frac{3}{4}\right) dx.$$

$$= \left[\frac{x^2}{4} - \frac{3x}{4}\right]_1^2.$$

$$= \left(1 - \frac{3}{2}\right) - \left(\frac{1}{4} - \frac{3}{4}\right).$$

$$= 0.$$

Hence,

$$\beta = 0.$$

Therefore,

$$9\alpha + \beta = 9\left(\frac{11}{9}\right) + 0 = 11.$$

$$\boxed{11}$$

Option (4)

Application of Integrals

Questions 130-143 / 14 selected / 4.7% practice weight / Next

Question 130

Let B be the centre of the circle $x^2 + y^2 - 2x + 4y + 1 = 0$.

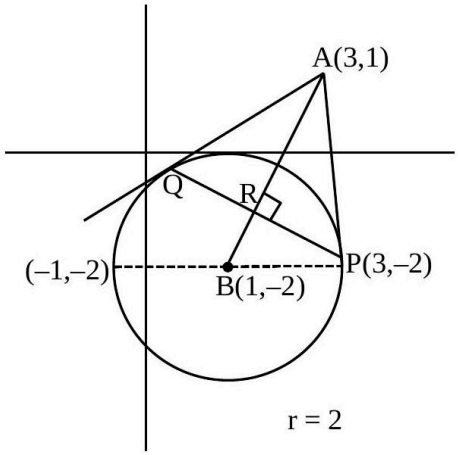
Let the tangents at two points P and Q on the circle intersect at the point $\mathbf{A(3, 1)}$. Then $\mathbf{8.}\left(\frac{\text{area } \triangle APQ}{\text{area } \triangle BPQ}\right)$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 18

Source solution



$\tan\theta = \frac{3}{2}$

$\frac{\text{Area } \triangle APQ}{\text{Area } \triangle BPQ} = \frac{AR}{RB} = \frac{3\sin\theta}{2\cos\theta} = \frac{9}{4}$

$8\left(\frac{\text{Area } \triangle APQ}{\text{Area } \triangle BPQ}\right) = 18$

Question 131

Let the focal chord of the parabola $P : y^2 = 4x$ along the line $\mathbf{L : y = mx + c, m > 0}$ meet the parabola at the points M and N . Let the line L be a tangent to the hyperbola $\mathbf{H : x^2 - y^2 = 4}$. If O is the vertex of P and F is the focus of H on the positive x-axis, then the area of the quadrilateral OMFN is :

A. $2\sqrt{6}$

B. $2\sqrt{14}$

C. $4\sqrt{6}$

D. $4\sqrt{14}$

Correct answer: B

Source solution

Sol.

$\mathbf{H : \frac{x^2}{4} - \frac{y^2}{4} = 1}$

Focus (ae, 0)

$\mathbf{F(2\sqrt{2}, 0)}$

Line $\mathbf{L : y = mx + c}$ pass $\mathbf{(1, 0)}$

$$\mathbf{o = m + C} \tag{1}$$

Line L is tangent to Hyperbola. $\mathbf{\frac{x^2}{4} - \frac{y^2}{4} = 1}$

$\mathbf{C = \pm\sqrt{a^2\, m^2 - \ell^2}}$

$\mathbf{C = \pm\sqrt{4\, m^2 - 4}}$

From (1)

$\mathbf{-m = \pm\sqrt{4\, m^2 - 4}}$

Squaring

$\mathbf{m^2 = 4\, m^2 - 4}$

$\mathbf{4 = 3\, m^2}$

$\mathbf{\frac{2}{\sqrt{3}} = m} \quad (\text{as } \mathbf{m > 0})$

$\mathbf{C = -m}$

$\mathbf{C = \frac{-2}{\sqrt{3}}}$

$y = \frac{2x}{\sqrt{3}} - \frac{2}{\sqrt{3}}$

$\mathbf{y^2 = 4x}$

$\Rightarrow \left(\frac{2x-2}{\sqrt{3}}\right)^2 = 4x$

$\Rightarrow x^2 + 1 - 2x = 3x$

$\Rightarrow x^2 - 5x + 1 = 0$

$y^2 = 4\left(\frac{\sqrt{3}y+2}{2}\right)$

$y^2 = 2\sqrt{3}y + 4$

$\Rightarrow y^2 - 2\sqrt{3}y - 4 = 0$

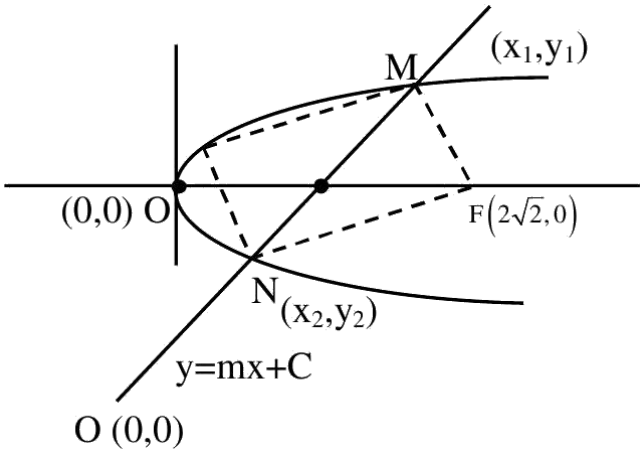
Area

$\left|\frac{1}{2}\right|0 \quad x_1 \quad 2\sqrt{2} \quad x_2 \quad 00 \quad y_1 \quad 0 \quad y_2 \quad 0|\,|$

$= \left|\frac{1}{2}\left[-2\sqrt{2}y_1 + 2\sqrt{2}y_2\right]\right|$

$= \sqrt{2}\,|y_2 - y_1| = \frac{(\sqrt{2})\sqrt{12+16}}{111} = \sqrt{56}$

$= 2\sqrt{14}$



Question 132

Let $\mathbf{P(4, 4\sqrt{3})}$ be a point on the parabola $\mathbf{y^2 = 4ax}$ and PQ be a focal chord of the parabola. If M and N are the foot of perpendiculars drawn from P and Q respectively on the directrix of the parabola, then the area of the quadrilateral PQMN is equal to:

A. $\frac{263\sqrt{3}}{8}$

B. $17\sqrt{3}$

C. $\frac{343\sqrt{3}}{8}$

D. $\frac{34\sqrt{3}}{3}$

Correct answer: C

Source solution

Sol.

$(4, 4\sqrt{3})$ lies on $y^2 = 4ax$
$\Rightarrow 48 = 4a.4$

$$4a = 12$$

$\Rightarrow y^2 = 12x$ is equation of parabola

Now, parameter of P is $t_1 = \frac{2}{\sqrt{3}} \Rightarrow$ Parameters of Q is $t_2 = -\frac{\sqrt{3}}{2} \Rightarrow Q\left(\frac{9}{4}, -3\sqrt{3}\right)$

Area of trapezium PQNM
$= \frac{1}{2}MN.(PM + QN)$
$= \frac{1}{2}MN.(PS + QS)$
$= \frac{1}{2}MN.PQ$
$= \frac{1}{2}7\sqrt{3} \cdot \frac{49}{4} = (343)\frac{\sqrt{3}}{8} = 3$

Question 133

It the area enclosed by the parabolas $P_1 : 2y = 5x^2$ and $P_2 : x^2 - y + 6 = 0$ is equal to the area enclosed by P_1 and $y = \alpha x, \alpha > 0$, then α^3 is equal to

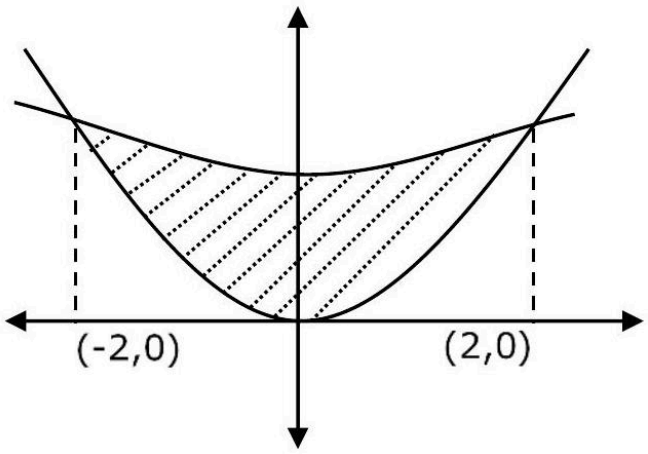
Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 600

Source solution

600



$\mathrm{y} = \frac{5\mathrm{x}^2}{2}, \quad \mathrm{y} = \mathrm{x}^2 + 6$

$\frac{5\mathrm{x}^2}{2} = \mathrm{x}^2 + 6$

$3\mathrm{x}^2 = 12 \Rightarrow \mathrm{x}^2 = 4$

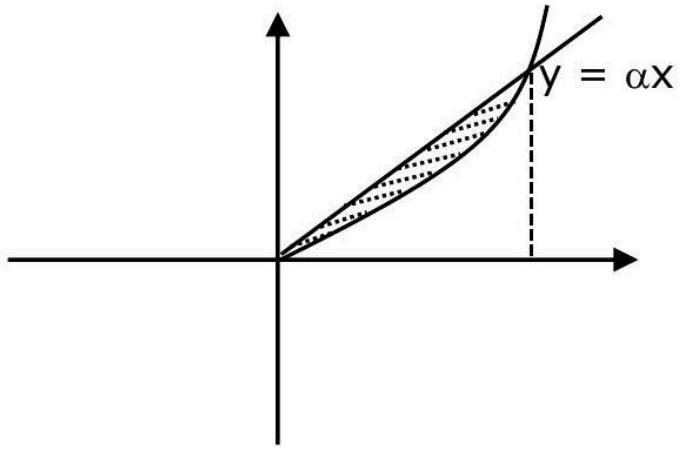
$\mathrm{x} = \pm 2$

$A_1 = 2\int_0^2 \left(x^2 + 6 - \frac{5}{2}x^2\right)dx$

$= 2\int_0^2 \left(6 - \frac{3\mathrm{x}^2}{2}\right)\mathrm{dx}$

$= 2\left[6x - x^3/2\right]_0^2 = 2[12 - 4]$

$= 16$



$y = \frac{5}{2}x^2, y = \alpha x(\alpha > 0)$

$\text{area } = \frac{8}{3}\left[a^2\ m^3\right]$

$= \frac{8}{3}[1/10]^2 \cdot \alpha^3$

$= \frac{8}{300} - \alpha^3 = \frac{2}{75}\alpha^3$

$\because \frac{2}{75} - \alpha^3 = 16 \qquad \Rightarrow \alpha^3 = 8 \times 75$

$\alpha^3 = 600$

Question 134

Let the locus of the centre $(\alpha, \beta), \beta > 0$, of the circle which touches the circle $x^2 + (y-1)^2 = 1$ externally and also touches the x -axis be L . Then the area bounded by L and the line $y = 4$ is :

A. $\frac{32\sqrt{2}}{3}$

B. $\frac{40\sqrt{2}}{3}$

C. $\frac{64}{3}$

D. $\frac{32}{3}$

Correct answer: C

Source solution

Sol. $(\alpha - 0)^2 + (\beta - 1)^2 = (\beta + 1)^2$

$\alpha^2 = 4\beta$

$x^2 = 4y$

$A = 2\int_0^4 \left(4 - \frac{x^2}{4}\right) dx = \frac{64}{3}$

Question 135

If the tangent at a point P on the parabola $y^2 = 3x$ is parallel to the line $x + 2y = 1$ and the tangents at the points Q and R on the ellipse $\frac{x^2}{4} + \frac{y^2}{1} = 1$ are perpendicular to the line $x - y = 2$, then the area of the triangle PQR is :

A. $\frac{3}{2}\sqrt{5}$

B. (1)
$\sqrt{5}$

C. (2) $3\sqrt{5}$

D. (3) $\frac{9}{\sqrt{5}}$

Correct answer: B

Source solution

Sol. 2}

$x + 2y = 1$

$\mathrm{m} = -\frac{1}{2}$

$$y^2 = 3x$$

$$T_p : y = -\frac{1}{2}x + \frac{\frac{3}{4}}{-\frac{1}{2}}$$

$$y = -\frac{x}{2} - \frac{3}{2}$$

$$2y + x + 3 = 0 \qquad (1)$$

$$E : \frac{x^2}{4} + \frac{y^2}{1} = 1$$

$$y = -x \pm \sqrt{(-1)^2 4 + 1}$$

$$y = -x \pm \sqrt{5}$$

$$x + y = \sqrt{5} \ldots (2) \quad x + y = -\sqrt{5} \qquad (2)$$

$x - y = 2$

$\mathrm{m} = 1$

slope of tangent at $\mathbf{Q\&R}$ is -1

Point P :

$\mathbf{T = O}$

$\mathbf{yy_1 = \frac{3}{2}(x + x_1)}$

$\mathbf{3x - 2yy_1 + 3x_1 = 0}$

Comparison with (1)

$\mathbf{\frac{3}{1} = \frac{-2y_1}{2} = \frac{3x_1}{3}}$

$\mathbf{y_1 = -3, \quad x_1 = 3}$

Area of $\mathbf{\triangle PQR}$

$\mathbf{= \frac{1}{2}\begin{vmatrix} 3 & -3 & 1 \\ \frac{4}{\sqrt{5}} & \frac{1}{\sqrt{5}} & 1 - \frac{4}{\sqrt{5}} \\ -\frac{1}{\sqrt{5}} & 1 & \end{vmatrix}}$

$\mathbf{\Rightarrow \frac{1}{2}\left[3\left(\frac{1}{\sqrt{5}} + \frac{1}{\sqrt{5}}\right) + 3\left(\frac{4}{\sqrt{5}} + \frac{4}{\sqrt{5}}\right) + 1\left(-\frac{4}{5} + \frac{4}{5}\right)\right]}$

$\mathbf{\Rightarrow \frac{1}{2}\left[\frac{6}{\sqrt{5}} + \frac{24}{\sqrt{5}}\right]}$

$\mathbf{\Rightarrow \frac{1}{2} \times \frac{30}{\sqrt{5}} = \frac{5 \times 3}{\sqrt{5}} = 3\sqrt{5}}$

Point Q :

$\mathbf{\frac{xx_2}{4} + \frac{yy_2}{1} = 1}$

$\mathbf{xx_2 + 4yy_2 - 4 = 0}$

$\mathbf{\frac{x_2}{1} = \frac{4y_2}{1} = \frac{-4}{-\sqrt{5}}}$

$\mathbf{x_2 = \frac{4}{\sqrt{5}} - y_2 = \frac{1}{\sqrt{5}}}$

$\mathbf{xx_2 + 4yy_2 - 4 = 0}$

$\mathbf{\frac{x_2}{1} = \frac{4y_2}{1} = \frac{-4}{-\sqrt{5}}}$

$\mathbf{\frac{\sqrt{5}}{\sqrt{5}} - y_2 = \frac{1}{\sqrt{5}}}$

h (1)

$\mathbf{3x - 2yy_1 + 3x_1 = 0}$

Question 136

Let the parabola $y = x^2 + px - 3$, meet the coordinate axes at the points $\mathbf{P, Q}$ and R . If the circle C with centre at $(-1, -1)$ passes through the points $\mathbf{P, Q}$ and R , then the area of $\triangle \mathbf{PQR}$ is :

A. 4

B. 6

C. 7

D. 5

Correct answer: B

Source solution

Sol. $y = x^2 + px - 3$
Let $\mathrm{P}(\alpha, 0), \mathrm{Q}(\beta, 0), \mathrm{R}(0, -3)$

Circle with centre $(-1, -1)$ is $(x+1)^2 + (y+1)^2 = r^2$
Passes through $(0, -3)$
$1^2 + (-2)^2 = \mathrm{r}^2$]
$\mathrm{r}^2 = 5$
$(x+1)^2 + (y+1)^2 = 5$
Put $\mathrm{y} = 0$
$(x+1)^2 = 5 - 1$
$(x+1)^2 = 4$
$\mathrm{x} + 1 = \pm 2$
$\mathrm{x} = 1$ or $\mathrm{x} = -3$
$\therefore \mathrm{P}(1, 0)$ and $\mathrm{Q}(-3, 0)$

Area of $\triangle \mathbf{PQR} = \frac{1}{2}|1 \quad 0 \quad 1-3 \quad 0 \quad 10 \quad -3 \quad 1| = 6$

Question 137

Consider a hyperbola $\mathbf{H}: \mathbf{x}^2-2 \mathbf{y}^2=4$. Let the tangent at a point $\mathbf{P}(4, \sqrt{6})$ meet the x -axis at Q and latus rectum at $\mathbf{R}\left(\mathbf{x}_1, \mathbf{y}_1\right), \mathbf{x}_1>0$. If F is a focus of H which is nearer to the point P , then the area of $\triangle \mathbf{QFR}$ is equal to

A. $4 \sqrt{6}$

B. $\sqrt{6}-1$

C. $\frac{7}{\sqrt{6}}-2$

D. $4 \sqrt{6}-1$

Correct answer: C

Source solution

Sol.

$\frac{x^2}{4}-\frac{y^2}{2}=1$

$\mathrm{e}=\sqrt{1+\frac{\mathrm{b}^2}{\mathrm{a}^2}}=\sqrt{\frac{3}{2}}$

$\therefore$ Focus $\mathbf{F}(\mathbf{a e}, 0) \Rightarrow \mathbf{F}(\sqrt{6}, 0)$

equation of tangent at P to the hyperbola is

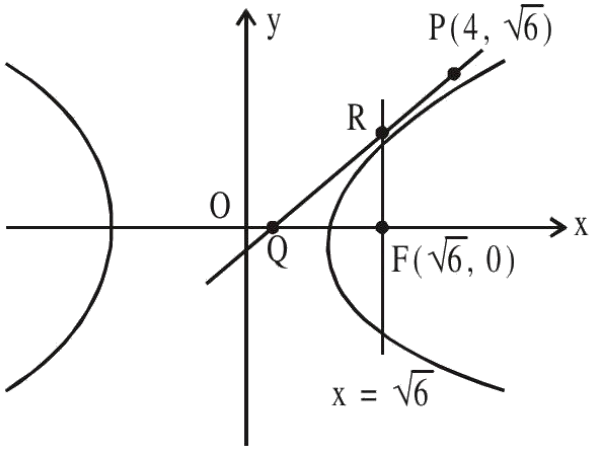
$2 x-y \sqrt{6}=2$

tangent meet x -axis at $\mathbf{Q}(1,0)$

& latus rectum $x=\sqrt{6}$ at $R\left(\sqrt{6}, \frac{2}{\sqrt{6}}(\sqrt{6}-1)\right)$

$\therefore$ Area of $\Delta_{\mathrm{QFR}}=\frac{1}{2}(\sqrt{6}-1) \cdot \frac{2}{\sqrt{6}}(\sqrt{6}-1)$

$=\frac{7}{\sqrt{6}}-2$



Question 138

The area (in sq. units) of the region enclosed between the parabola $y^2 = 2x$ and the line $x + y = 4$ is .

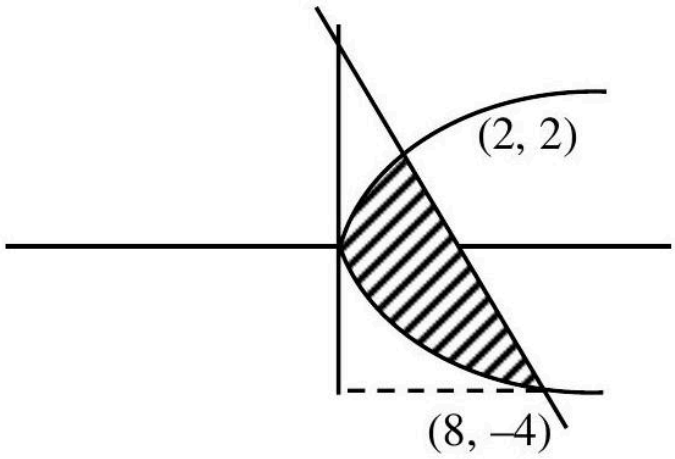
Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 18

Source solution

$x = 4 - y$
$y^2 = 2(4 - y)$
$y^2 = 8 - 2y$
$y^2 + 2y - 8 = 0$
$y = -4, y = 2$
$\mathbf{x = 8, x = 2}$



$\int_{-4}^{2} \left[(4 - y) - \frac{y^2}{2}\right] dy$
$= \left[4y - \frac{y^2}{2} - \frac{y^3}{6}\right]_{-4}^{2}$
$= 8 - 2 - \frac{8}{6} + 16 + \frac{16}{2} - \frac{64}{6}$
$= 22 + 8 - \frac{72}{6}$
$= 30 - 12 = 18$

Question 139

Let C be the centre of the circle $x^2+y^2-x+2y=\frac{11}{4}$ and P be a point on the circle. A line passes through the point C , makes an angle of $\frac{\pi}{4}$ with the line CP and intersects the circle at the points Q and R . Then the area of the triangle PQR (in unit ^{2}) is :

A. 2

B. $2\sqrt{2}$

C. $8\sin\left(\frac{\pi}{8}\right)$

D. $8\cos\left(\frac{\pi}{8}\right)$

Correct answer: B

Source solution

Sol. $x^2+y^2-x+2y=\frac{11}{4}$

$\left(x-\frac{1}{2}\right)^2+(y+1)^2=(2)^2$

Or ΔPQR

$PR=QK\sin 2\geq\frac{1}{3}$

$=4\cdot 6\sin\frac{\pi}{8}$

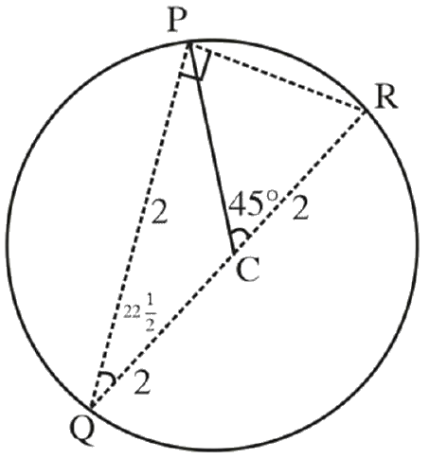
$PQ=QR\cos 22\frac{1}{2}$

$=4\cos\frac{\pi}{8}$

As $\triangle PQR=\frac{1}{2}PR\times PQ$

$=\frac{1}{2}\left(4^2\sin\frac{\pi}{6}\right)\left(4\cos\frac{\pi}{8}\right)$

$=4\sin\frac{\pi}{4}=\frac{4}{\sqrt{2}}=2\sqrt{2}$



Question 140

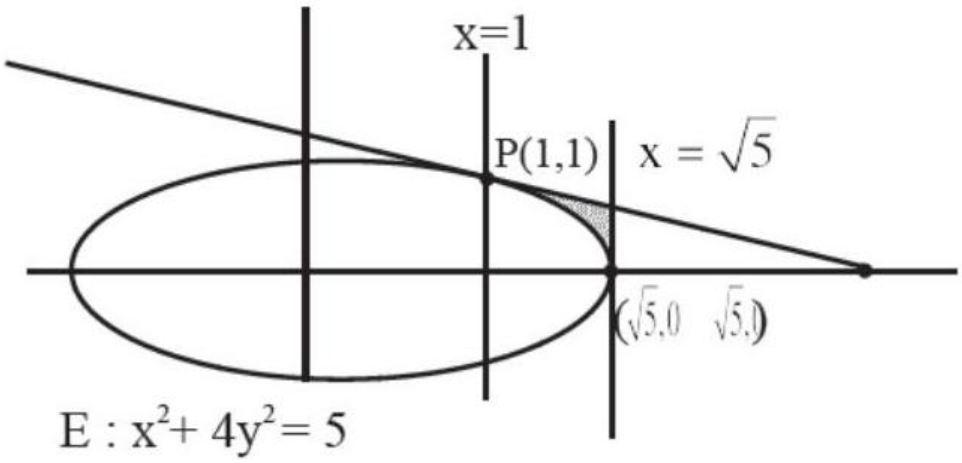
Let T be the tangent to the ellipse $\mathbf{E : x^2 + 4y^2 = 5}$ at the point $\mathbf{P(1, 1)}$. If the area of the region bounded by the tangent T, ellipse E, lines $\mathbf{x = 1}$ and $\mathbf{x = \sqrt{5}}$ is $\alpha = \sqrt{5} + \beta + \gamma \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$, then $|\alpha + \beta + \gamma|$ is equal to ___.

Numerical answer: _____

Enter the requested integer (0 decimal places).

Correct answer: 1

Source solution



Tangent at $\mathbf{P : x + 4y = 5}$ Required area

$$= \int_{12}^{\sqrt{5}} \left(\frac{5 - x}{4} - \frac{\sqrt{5 - x^2}}{2} \right) dx$$

$$= \left[\frac{5x}{4} - \frac{x^2}{8} - \frac{x}{4}\sqrt{5 - x^2} - \frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}} \right]_1^{\sqrt{5}}$$

$$= \frac{5}{4}\sqrt{5} - \frac{5}{4} - \frac{5}{4}\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$$

It we assume $\alpha, \beta, \gamma, \in \mathbf{Q}$ (Not given in question) then $\alpha = \frac{5}{4}, \beta = -\frac{5}{4}$ & $\gamma = -\frac{5}{4}$

$$|\alpha + \beta + \gamma| = 1.25$$

Question 141

On the ellipse $\frac{x^2}{8}+\frac{y^2}{4}=1$ let P be a point in the second quadrant such that the tangent at P to the ellipse is perpendicular to the line $x+2y=0$. Let S and S' be the foci of the ellipse and e be its eccentricity. If A is the area of the triangle SPS' then, the value of $\left(5-\mathrm{e}^2\right).\mathrm{A}$ is :

A. 6

B. 12

C. 14

D. 24

Correct answer: A

Source solution

Sol.

Equation of tangent : $y=2x+6$
at P
$\therefore \mathrm{P}(-8/3,2/3)$
$\mathrm{e}=\frac{1}{\sqrt{2}}$
$S\&S'=(-2,0)\&(2,0)$

Area of $\Delta\mathrm{SPS}'=\frac{1}{2}\times 4\times\frac{2}{3}$
$\mathrm{A}=\frac{4}{3}$
$\therefore \left(5-\mathrm{e}^2\right)\mathrm{A}=\left(5-\frac{1}{2}\right)\frac{4}{3}=6$

Question 142

The area of the region enclosed by the parabola $(y-2)^2 = x-1$, the line $x-2y+4=0$ and the positive coordinate axes is .

Numerical answer: _____

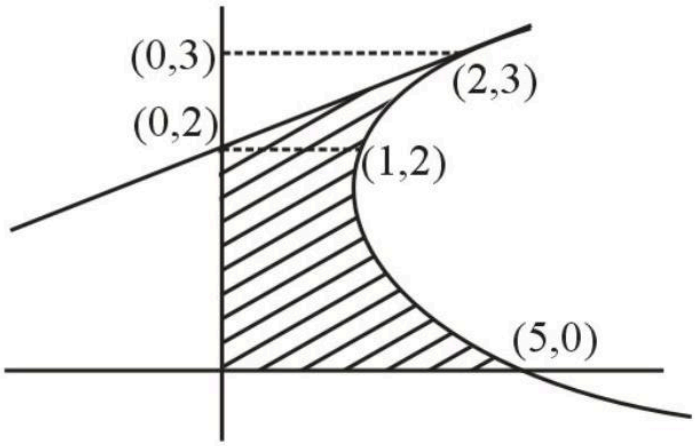
Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 5

Source solution

Solving the equations
$(y-2)^2 = x-1$ and $x-2y+4=0$

$$\mathbf{X = 2(y-2)}$$



$\frac{x^2}{4} = x-1$
$x^2 - 4x + 4 = 0$
$(x-2)^2 = 0$
$\mathbf{x = 2}$
Exclose area (w.r.t. y-axis) $= \int_0^3 x\,dy - \mathbf{Area}$ of Δ.
$= \int_0^3 \left((y-2)^2+1\right)dy - \frac{1}{2} \times 1 \times 2$
$= \int_0^3 \left(y^2-4y+5\right)dy - 1$

$= \left[\frac{y^3}{3} - 2y^2 + 5y\right]_0^3 - 1$
$= 9 - 18 + 15 - 1 = 5$

Question 143

The parabola $y^2 = 4x$ divides the area of the circle $x^2 + y^2 = 5$ in two parts. The area of the smaller part is equal to :

A. $\frac{2}{3} + 5\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$

B. $\frac{1}{3} + 5\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$

C. $\frac{1}{3} + \sqrt{5}\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$

D. $\frac{2}{3} + \sqrt{5}\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$

Correct answer: A

Source solution

Sol.

$y^2 = 4x$

$x^2 + y^2 = 5$

$\therefore$ Area of shaded region as shown in the figure will be

$A_1 = \int_0^1 \sqrt{4x}dx + \int_1^{\sqrt{5}} \sqrt{5 - x^2}dx$

$= \frac{4}{3} \cdot \left[x^{\frac{3}{2}}\right]_0^1 + \left[\frac{x}{2}\sqrt{5 - x^2} + \frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\right]_1^{\sqrt{5}}$

$= \frac{1}{3} + \frac{5\pi}{4} - \frac{5}{2}\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$

$\therefore$ Required Area $= 2\ \mathbf{A_1}$

$= \frac{2}{3} + \frac{5\pi}{2} - 5\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$

$= \frac{2}{3} + 5\left(\frac{\pi}{2} - \sin^{-1}\frac{1}{\sqrt{5}}\right)$

$= \frac{2}{3} + 5\cos^{-1}\frac{1}{\sqrt{5}}$

$= \frac{2}{3} + 5\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$

Vector Algebra

Questions 144-171 / 28 selected / 9.3% practice weight / Focus first

Question 144

If the equation of the plane containing the line $x+2y+3z-4=0=2x+y-z+5$ and perpendicular to the plane $\vec{r}=(\hat{i}-\hat{j})+\lambda(\hat{i}+\hat{j}+\hat{k})+\mu(\hat{i}-2\hat{j}+3k)$ is $ax+by+cz=4$, then $(a-b+c)$ is equal to

A. 20

B. 24

C. 22

D. 18

Correct answer: C

Source solution

The given line is the intersection of the planes

$$x+2y+3z-4=0$$

and

$$2x+y-z+5=0.$$

The normals to these planes are

$$\vec{n}_1=(1,2,3), \qquad \vec{n}_2=(2,1,-1).$$

Hence the direction vector of the line is

$$\vec{d} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 1 & -1 \end{vmatrix} = (-5,\ 7,\ -3).$$

The given plane is

$$\vec{r} = (\hat{i} - \hat{j}) + \lambda(\hat{i} + \hat{j} + \hat{k}) + \mu(\hat{i} - 2\hat{j} + 3\hat{k}).$$

Its two direction vectors are

$$\vec{a} = (1, 1, 1), \qquad \vec{b} = (1, -2, 3).$$

Therefore, its normal vector is

$$\vec{n}_p = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & -2 & 3 \end{vmatrix} = (5,\ -2,\ -3).$$

Let the required plane have normal vector

$$\vec{N} = (a, b, c).$$

Since the required plane contains the given line,

$$\vec{N} \cdot \vec{d} = 0.$$

Thus,

$$-5a + 7b - 3c = 0.$$

Also, the required plane is perpendicular to the plane whose normal is $(5, -2, -3)$, so

$$\vec{N} \cdot (5, -2, -3) = 0,$$

i.e.,

$$5a - 2b - 3c = 0.$$

Solving

$$-5a + 7b - 3c = 0$$

and

$$5a - 2b - 3c = 0,$$

adding,

$$5b - 6c = 0$$

$$b = \frac{6c}{5}.$$

Substituting into

$$5a - 2b - 3c = 0,$$

$$5a - \frac{12c}{5} - 3c = 0$$

$$25a = 27c.$$

Taking $c = 25$,

$$a = 27, \qquad b = 30.$$

Hence,

$$\vec{N} = (27, 30, 25).$$

Therefore the required plane is

$$27x + 30y + 25z = 4.$$

So,

$$a - b + c = 27 - 30 + 25 = 22.$$

$$\boxed{22}$$

Hence, the correct option is

$$\boxed{\text{(3) } 22}.$$

Question 145

Let a line l pass through the origin and be perpendicular to the lines
$l_1 : \vec{r} = (\hat{i} - 11\hat{j} - 7\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k}), \lambda \in \mathbb{R}$
and $l_2 : \vec{r} = (-\hat{i} + \hat{k}) + \mu(2\hat{i} + 2\hat{j} + \hat{k}), \mu \in \mathbb{R}$.

If P is the point of intersection of l and l_1, and $Q(\alpha, \beta, \gamma)$ is the foot of perpendicular from P on l_2, then $9(\alpha + \beta + \gamma)$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 5

Source solution

Let $\ell = (0\hat{i} + 0\hat{j} + 0\hat{k}) + \gamma(a\hat{i} + b\hat{j} + c\hat{k})$
$= \gamma(a\hat{i} + b\hat{j} + c\hat{k})$
$a\hat{i} + b\hat{j} + c\hat{k} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 2 & 1 \end{vmatrix}$
$= \hat{i}(2 - 6) - \hat{j}(1 - 6) + \hat{k}(2 - 4)$
$= -4\hat{i} - 5\hat{j} - 2\hat{k}$
$\ell = \gamma(-4\hat{i} + 5\hat{j} - 2\hat{k})$
P is intersection of ℓ and ℓ_1
$-4\gamma = 1 + \lambda, 5\gamma = -11 + 2\lambda, -2\gamma = -7 + 3\lambda$
By solving there equation $\gamma = -1, P(4, -5, 2)$
Let $Q(-1 + 2\mu, 2\mu, 1 + \mu)$
$\overrightarrow{PQ} \cdot (2\hat{i} + 2\hat{j} + \hat{k}) = 0$
$-2 + 4\mu + 4\mu + 1 + \mu = 0$
$9\mu = 1$
$\mu = \frac{1}{9}$
$Q\left(\frac{-7}{9}, \frac{2}{9}, \frac{10}{9}\right)$
$9(\alpha + \beta + \gamma) = 9\left(\frac{-7}{9} + \frac{2}{9} + \frac{10}{9}\right)$

$= 5$

Question 146

If the equation of a plane P, passing through the intersection of the planes, $x + 4y - z + 7 = 0$ and $3x + y + 5z = 8$ is $ax + by + 6z = 15$ for some $a, b \in R$, then the distance of the point $(3, 2, -1)$ from the plane P is.....

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 3

Source solution

$$p_1 + \lambda p_2 = 0$$

$$(x + 4y - z + 7) + \lambda(3x + y + 5z - 8) = ax + by + 6z - 15$$

$$\frac{1-3\lambda}{a} = \frac{4+\lambda}{b} = \frac{-1+5\lambda}{6} = \frac{7-8\lambda}{-15}$$

$$\therefore 15 - 75\lambda = 42 - 48\lambda$$

$$-27 = 27\lambda$$

$$\lambda = -1$$

$\therefore$ plane is $-2x + 3y - 6z + 15 = 0$

$$d = \left| \frac{-6+6+6+15}{\sqrt{4+9+36}} \right| = 3$$

Question 147

Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero non-coplanar vectors. Let the position vectors of four points A, B, C and D be $\vec{a}-\vec{b}+\vec{c}, \lambda\vec{a}-3\vec{b}+4\vec{c}, -\vec{a}+2\vec{b}-3\vec{c}$ and $2\vec{a}-4\vec{b}+6\vec{c}$ respectively. If $\overrightarrow{AB}, \overrightarrow{AC}$ and $\overrightarrow{AD}$ are coplanar, then λ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 2

Source solution

$$\begin{aligned}\overrightarrow{AB} &= (\lambda\vec{a}-3\vec{b}+4\vec{c})-(\vec{a}-\vec{b}+\vec{c}) \\ &= (\lambda-1)\vec{a}-2\vec{b}+3\vec{c} \\ \overrightarrow{AC} &= (-\vec{a}+2\vec{b}-3\vec{c})-(\vec{a}-\vec{b}+\vec{c}) \\ &= -2\vec{a}+3\vec{b}-4\vec{c} \\ \overrightarrow{AD} &= (2\vec{a}-4\vec{b}+6\vec{c})-(\vec{a}-\vec{b}+\vec{c}) \\ &= \vec{a}-3\vec{b}+5\vec{c}\end{aligned}$$

For coplanar vectors

$$\begin{vmatrix}\lambda-1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5\end{vmatrix}=0$$

$\Rightarrow 3\lambda-6=0$

$\therefore \lambda=2$

Question 148

If the vectors $\vec{a} = \lambda\hat{i} + \mu\hat{j} + 4\hat{k}, \vec{b} = 2\hat{i} + 4\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} + 3\hat{j} + \hat{k}$ are coplanar and the projection of $\vec{a}$ on the vector $\vec{b}$ is $\sqrt{54}$ units, then the sum of all possible values of $\lambda + \mu$ is equal to

A. 0

B. 24

C. 6

D. 18

Correct answer: B

Source solution

Sol. 2}

Vector $\vec{a} = \lambda\hat{i} + \mu\hat{j} + 4\hat{k}, \vec{b} = -2\hat{i} + 4\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} + 3\hat{j} + \hat{k}$ are coplanar then

$[\vec{a}\vec{b}\vec{c}] = 0 \quad \Rightarrow \begin{vmatrix} \lambda & \mu & 4 \\ -2 & 4 & -2 \\ 2 & 3 & 1 \end{vmatrix} = 0$

$\Rightarrow 10\lambda - 2\mu - 56 = 0$

$$\Rightarrow 5\lambda - \mu = 28 \tag{1}$$

also projection of $\vec{a}$ on the $\vec{b}$ is $\sqrt{54}$ units. then

$\vec{a}.\vec{b} = \sqrt{54}$

$\Rightarrow \frac{-2\lambda + 4\mu - 8}{\sqrt{24}} = \sqrt{54}$

$\Rightarrow -2\lambda + 4\mu - 8 = 36$

$$\Rightarrow -2\lambda + 4\mu = 44 \tag{2}$$

from (1) and (2)

$\lambda = \frac{26}{3}$ and $\mu = \frac{46}{3}$

$\Rightarrow \lambda + \mu = \frac{26+46}{3} = \frac{72}{3} = 24$

Question 149

Let $\vec{a}=4\hat{i}+3\hat{j}$ and $\vec{b}=3\hat{i}-4\hat{j}+5\hat{k}$. If $\vec{c}$ is a vector such that $\vec{c}\cdot(\vec{a}\times\vec{b})+25=0, \vec{c}\cdot(\hat{i}+\hat{j}+\hat{k})=4$, and projection of $\vec{c}$ on $\vec{a}$ is 1 , then the projection of $\vec{c}$ on $\vec{b}$ equals

A. $\frac{1}{5}$

B. $\frac{5}{\sqrt{2}}$

C. $\frac{3}{\sqrt{2}}$

D. $\frac{1}{\sqrt{2}}$

Correct answer: B

Source solution

Sol. (2)

Let $\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$

$\vec{c}\cdot(\mathrm{i}+\mathrm{j}+\mathrm{k})=4$

$$\vec{c}.\vec{a}\times\vec{b} \qquad \text{(i)}$$

$c_1+c_2+c_3=4$

$|c_1 \quad c_2 \quad c_3 4 \quad 3 \quad 03 \quad -4 \quad 5|=-25$

$\Rightarrow c_1(15-0)-c_2(20-0)+c_3(-16-9)=-25$

$\Rightarrow 15c_1-20c_2-25c_3=-25$

$$\Rightarrow 3c_1-4c_2-5c_3=-5 \qquad (2)$$

Proj. of $\vec{c}$ on $\vec{a}=\frac{\vec{a}\cdot\vec{c}}{|\vec{a}|}=1$

$\Rightarrow \frac{(4\hat{i}+3\hat{j})\left(c_1\hat{i}+c_2\hat{j}+c_3\hat{k}\right)}{\sqrt{16+9}}=1$

$\Rightarrow 4c_1+3c_2=5$

$\Rightarrow 4c_1=5-3c_2$

$$\Rightarrow c_1 = \frac{5 - 3c_2}{4} \tag{3}$$

$Eq^n\cdot$ (1) & (3)
$\frac{5-3c_2}{4} + c_2 + c_3 = 4$
Eq ^{n}. (2) & (3)

$$3\left(\frac{5 - 3c_2}{4}\right) - 4c_2 - 5c_3 = -5$$
$$15 - 9c2 - 16c2 - 20c_3 = -20$$
$$-25c_2 - 20c_3 = -35 \tag{5}$$

$$5 - 3c_2 + 4c_2 + 4c_3 = 16$$

$c_2 + 4c_3 = 11$
$\mathbf{Eq^n}$. (4) & (5)
$c_2 = 11 - 4c_3$
$-25c_2 - 20c_3 = -35$
$-25\left(11 - 4c_3\right) - 20$
c $_2 = 11 - 4 \times 3$
$5\left(11 - 4c_3\right) + 4c_3 = 7$
$= 11 - 12$
$55 - 20c_3 + 4c_3 = 7$
$c_2 = -1$
$-16c_3 = -48$
$c_1 = \frac{5-3c_2}{4}$
$c_3 = 3$
$= \frac{5-3(-1)}{4}$
$\vec{c} = 2\hat{i} - \hat{j} + 3\hat{k}$
$c_1 = 2$
Projection of $\vec{c}$ on $\vec{b} = \left|\frac{\vec{c}\cdot\vec{b}}{|\vec{b}|}\right|$
$\Rightarrow \left|\frac{(2\hat{i}-\hat{j}+3\hat{k})\cdot(3\hat{i}-4\hat{j}+5\hat{k})}{\sqrt{9+16+25}}\right|$
$\Rightarrow \left|\frac{6+4+15}{5\sqrt{2}}\right| = \frac{25}{5\sqrt{2}} = \frac{5}{\sqrt{2}}$

Question 150

Let $\vec{a}=2\hat{i}-7\hat{j}+5k, \vec{b}=\hat{i}+k$ and $\vec{c}=\hat{i}+2\hat{j}-3k$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r}\times\vec{a}=\vec{c}\times\vec{a}$ and $\vec{r}\cdot\vec{b}=0$, then $|\vec{r}|$ is equal to:

A. $\frac{11}{5}\sqrt{2}$

B. $\frac{\sqrt{914}}{7}$

C. $\frac{11}{7}\sqrt{2}$

D. $\frac{11}{7}$

Correct answer: C

Source solution

Sol. 3

$\vec{r}\times\vec{a}=\vec{c}\times\vec{a}$

$\Rightarrow(\vec{r}-\vec{c})\times\vec{a}=0\Rightarrow\vec{r}-\vec{c}=\lambda\vec{a}(\vec{r}-\vec{c}\&\vec{a}$ are parallel)

$\Rightarrow\vec{r}=\vec{c}+\lambda\vec{a}$

$\Rightarrow\vec{r}\cdot\vec{b}=\vec{c}\cdot\vec{b}+\lambda\vec{a}\cdot\vec{b}$

$0=(1-3)+\lambda(2+5)\Rightarrow\lambda=\frac{2}{7}$

Hence, $\vec{r}=\vec{c}+\frac{2\vec{a}}{7}$

$\vec{r}\Rightarrow\frac{11}{7}\hat{i}-\frac{11}{7}\hat{k}$

$|\vec{r}|=\sqrt{\left(\frac{11}{7}\right)^2+\left(-\frac{11}{7}\right)^2}\Rightarrow r=\frac{11\sqrt{2}}{7}$

Question 151

If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors and $\hat{n}$ is a unit vector perpendicular to $\vec{c}$ such that $\vec{a} = \alpha\vec{b} - \hat{n}, (\alpha \neq 0)$ and $\vec{b} \cdot \vec{c} = 12$, then $|\vec{c} \times (\vec{a} \times \vec{b})|$ is equal to :

A. 9

B. 15

C. 6

D. 12

Correct answer: D

Source solution

Sol. 4

$\vec{a} = \alpha\vec{b} - \hat{n}, \vec{b}.\vec{c} = 12$

$\vec{c} \times (\vec{a} \times \vec{b}) = (\vec{c} \cdot \vec{b})\vec{a} - (\vec{c} \cdot \vec{a})\vec{b}$

$$\vec{c} \times (\vec{a} \times \vec{b}) = 12\vec{a} - (\vec{c}.\vec{a})\vec{b} \quad (1)$$

$\because \vec{a} = \alpha\vec{b} - n$

$\vec{c}.\vec{a} = \alpha\vec{c}.\vec{b} - \vec{c}.\vec{n}$

$$\vec{c}.\vec{a} = 12\alpha \quad (2)$$

Equation (2) put in equation (1)

$\vec{c} \times (\vec{a} \times \vec{b}) = 12\vec{a} - 12\alpha\vec{b}$

$|\vec{c} \times (\vec{a} \times \vec{b})| = 12|\vec{a} - \alpha\vec{b}|$ $[\because \vec{a} - \alpha\vec{b} = -n$ then $|\vec{a} - \alpha\vec{b}| = 1]$

$\Rightarrow |\vec{c} \times (\vec{a} \times \vec{b})| = 12$

Question 152

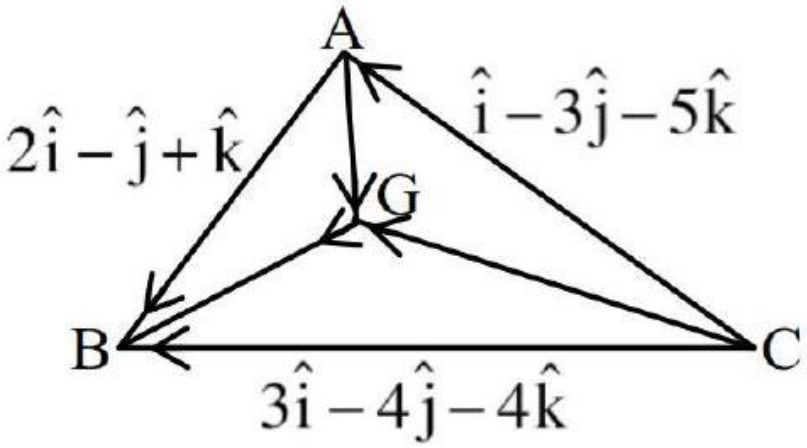
Let the three sides of a triangle ABC be given by the vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{j}-4\hat{k}$. Let G be the centroid of the triangle ABC . Then $6\left(|\overrightarrow{AG}|^2+|\overrightarrow{BG}|^2+|\overrightarrow{CG}|^2\right)$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 164

Source solution



By given data

$\overrightarrow{AB}+\overrightarrow{AC}=\overrightarrow{CB}$

Let pv of $\vec{A}$ are $\vec{O}$ then

$\overrightarrow{AB}=\vec{B}-\vec{A}$

i.e. pv of $\vec{B}-=2\hat{i}-\hat{j}+\hat{k}$

$\overrightarrow{CA}=\vec{A}-\vec{C}$

i.e. pv of $\vec{C}=-(\hat{i}-3\hat{j}-5\hat{k})$

Now pv of centroid

$(\vec{G})-=\frac{\vec{A}+\vec{B}+\vec{C}}{3}=\frac{\vec{0}+(2,-1,1)+(-1,3,5)}{3}$

$\vec{G}=\frac{1}{3}(\hat{i}+2\hat{j}+6\hat{k})$

Now $\overrightarrow{AG}=\frac{1}{3}(\hat{i}+2\hat{j}+6\hat{k})$

$\Rightarrow |\overrightarrow{AG}|^2 = \frac{1}{9} \times 41$

$\overrightarrow{BG} = \left(\frac{1}{3} - 2\right)\hat{i} + \left(\frac{2}{3} + 1\right)\hat{j} + (2-1)\hat{k}$

$\Rightarrow |\overrightarrow{BG}|^2 = \frac{59}{9}$

$\overrightarrow{CG} = \left(\frac{1}{3} + 1\right)\hat{i} + \left(\frac{2}{3} - 3\right)\hat{j} + (2-5)\hat{k}$

$\Rightarrow |\overrightarrow{CG}|^2 = \frac{146}{9}$

Now

$6\left[|\overrightarrow{AG}|^2 + |\overrightarrow{BG}|^2 + |\overrightarrow{CG}|^2\right] = 6 \times \left[\frac{41}{9} + \frac{59}{9} + \frac{146}{9}\right]$

$= 6 \times \frac{246}{9} = 164$

Question 153

Let the position vectors of three vertices of a triangle be $4\vec{p}+\vec{q}-3\vec{r}, -5\vec{p}+\vec{q}+2\vec{r}$ and $2\vec{p}-\vec{q}+2\vec{r}$. If the position vectors of the orthocenter and the circumcenter of the triangle are $\frac{\vec{p}+\vec{q}+\vec{r}}{4}$ and $\alpha\vec{p}+\beta\vec{q}+\gamma\vec{r}$ respectively, then $\alpha+2\beta+5\gamma$ is equal to:

A. 3

B. 1

C. 6

D. 4

Correct answer: A

Source solution

Sol. We know that

O (orthocentre) $\frac{\vec{p}+\vec{q}+\vec{r}}{4}$
C (circum centre) $\alpha\vec{p}+\beta\vec{q}+\gamma\vec{r}$
$\mathbf{C}(\text{ centroid }) = \frac{\vec{p}+\vec{q}+\vec{r}}{3}$
by relation
$\Rightarrow 2(\alpha\vec{p}+\beta\vec{q}+\gamma\vec{r})+\frac{\vec{p}+\vec{q}+\vec{r}}{4}=3\left(\frac{\vec{p}+\vec{q}+\vec{r}}{3}\right)$
$\Rightarrow 8(\alpha\vec{p}+\beta\vec{q}+\gamma\vec{r})=3(\vec{p}+\vec{q}+\vec{r})$
$\Rightarrow 8\alpha=3, 8\beta=3, 8\gamma=3$
$\alpha=\frac{3}{8}, \beta=\frac{3}{8}, \gamma=\frac{3}{8}$
$\therefore \alpha+2\beta+3\gamma$
$\frac{3}{8}+\frac{6}{8}+\frac{15}{8}=\frac{24}{8}=3$



Question 154

Let P be a plane passing through the points (1, 0, 1), (1, -2, 1) and (0, 1, -2). let a vector $\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ be such that $\vec{a}$ is parallel to the plane P , perpendicular to $(\hat{i} + 2\hat{j} + 3\hat{k})$ and $\vec{a}(\hat{i} + \hat{j} + 2\hat{k}) = 2$, then $(\alpha - \beta + \gamma)^2$ equals ___.

Numerical answer: _____

Enter the requested integer (0 decimal places).

Correct answer: 81

Source solution

Equation of plane :

$$\begin{vmatrix} x-1 & y-0 & z-1 \\ 1-1 & 2 & 1-1 \\ 1-0 & 0-1 & 1+2 \end{vmatrix} = 0$$

$$\Rightarrow 3x - z - 2 = 0$$

$$\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k} \parallel \text{to } 3x - z - 2 = 0$$

$$\Rightarrow 3\alpha - \gamma = 0$$

$$\vec{a} \perp i + 2\hat{j} + 3\hat{k}$$

$$\Rightarrow \alpha + 2\beta + 3\gamma = 0$$

$$\vec{a}(\hat{i} + \hat{j} + 3\hat{k}) = 0$$

$$\Rightarrow \alpha + \beta + 2\gamma = 2$$

on solving 1,2 & 3

$$\alpha = 1, \beta = -5, \gamma = 3$$

So $(\alpha - \beta + \gamma) = 81$

Question 155

If the components of $\overrightarrow{\mathbf{a}} = \alpha\hat{\mathbf{i}} + \beta\hat{\mathbf{j}} + \gamma\hat{\mathbf{k}}$ along and perpendicular to $\vec{b} = 3\hat{i} + \hat{j} - \hat{k}$ respectively, are $\frac{16}{11}(3\hat{i} + \hat{j} - \hat{k})$ and $\frac{1}{11}(-4\hat{i} - 5\hat{j} - 17\hat{k})$, then $\alpha^2 + \beta^2 + \gamma^2$ is equal to :

A. 23

B. 18

C. 16

D. 26

Correct answer: D

Source solution

Sol. let
$\vec{a}_{11}$ = component of $\vec{a}$ along $\vec{b}$
$\vec{a}_1$ = component of $\vec{a}$ perpendicular to $\vec{b}$
$\vec{a}_{11} = \frac{16}{11}(3\hat{i} + \hat{j} - \hat{k})$
$\overrightarrow{\mathbf{a}_1} = \frac{1}{11}(-4\hat{\mathbf{i}} - 5\hat{\mathbf{j}} - 17\hat{\mathbf{k}})$
$\because \vec{a} = \vec{a}_{11} + \vec{a}_1$
$\therefore \vec{a} = \frac{16}{11}(3\hat{i} + \hat{j} - \hat{k}) + \frac{1}{11}(-4\hat{i} - 5\hat{j} - 17\hat{k})$

$$= \frac{44}{11}\hat{\mathbf{i}} + \frac{11}{11}\hat{\mathbf{j}} - \frac{33}{11}\hat{\mathbf{k}}$$

$\vec{a} = 4\hat{i} + \hat{j} - 3\hat{k}$
$\alpha = 4 \quad \beta = 1 \quad \gamma = -3$
$\alpha^2 + \beta^2 + \gamma^2 = 16 + 1 + 9 = 26$

Question 156

Let $\vec{a} = 4\hat{i} + 3\hat{j} + 5\hat{k}$ and $\vec{\beta} = \hat{i} + 2\hat{j} - 4\hat{k}$. Let $\vec{\beta}_1$ be parallel to $\vec{a}$ and $\vec{\beta}_2$ be perpendicular to $\vec{a}$. If $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, then the value of $5\vec{\beta}_2 \cdot (\hat{i} + \hat{j} + \hat{k})$ is

A. 7

B. 9

C. 6

D. 11

Correct answer: A

Source solution

$$\vec{\beta}_1 = \frac{\vec{\alpha} \cdot \vec{\beta}}{|\vec{\alpha}|}\,\hat{\alpha}$$

$$= \left(\frac{4 + 6 - 20}{\sqrt{16 + 9 + 25}}\right)\frac{(4, 3, 5)}{\sqrt{50}}$$

$$= \frac{-10}{50}(4, 3, 5)$$

$$\vec{\beta}_1 = \left(-\frac{4}{5}, -\frac{3}{5}, -1\right)$$

Given,

$$\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2 = (1, 2, -4)$$

Hence,

$$\vec{\beta}_2 = \vec{\beta} - \vec{\beta}_1$$

$$= \left(1+\frac{4}{5}, 2+\frac{3}{5}, -4+1\right)$$

$$= \left(\frac{9}{5}, \frac{13}{5}, -3\right)$$

Therefore,

$$5\vec{\beta}_2 = (9, 13, -15)$$

Now,

$$5\vec{\beta}_2 \cdot (1,1,1) = 9 + 13 - 15$$

$$= 7$$

$$\boxed{7}$$

Question 157

Let A be a point on the line
$\vec{r}=(1-3\mu)\hat{i}+(\mu-1)\hat{j}+(2+5\mu)\hat{k}$ and $B(3,2,6)$ be a point in the space. Then the value of μ for which the vector $\overrightarrow{AB}$ is parallel to the plane $x-4y+3z=1$ is :

A. $\frac{1}{2}$

B. $-\frac{1}{4}$

C. $\frac{1}{4}$

D. $\frac{1}{8}$

Correct answer: C

Source solution

Sol. Let point A is
$(1-3\mu)\hat{i}+(\mu-1)\hat{j}+(2+5\mu)\hat{k}$
and point B is (3, 2, 6)
then $\overrightarrow{AB}=(2+3\mu)\hat{i}+(3-\mu)\hat{j}+(4-5\mu)\hat{k}$
which is parallel to the plane $x-4y+3z=1$
$\therefore 2+3\mu-12+4\mu+12-15\mu=0$
$8\mu=2$
$\mu=\frac{1}{4}$

Question 158

Let three vectors $\vec{a} = \alpha\hat{i} + 4\hat{j} + 2\hat{k}, \overrightarrow{b} = 5\hat{i} + 3\hat{j} + 4\hat{k}, \overrightarrow{c} = x\hat{i} + y\hat{j} + z\hat{k}$ from a triangle such that $\vec{c} = \vec{a} - \vec{b}$ and the area of the triangle is $5\sqrt{6}$. if α is a positive real number, then $|\vec{c}|^2$ is :

A. 16

B. 14

C. 12

D. 10

Correct answer: B

Source solution

$$\vec{c} = \vec{a} - \vec{b}$$

$$\Rightarrow (x, y, z) = (\alpha - 5,\ 1,\ -2)$$

Hence,

$$x = \alpha - 5, \qquad y = 1, \qquad z = -2 \tag{1}$$

Area of the triangle is given as

$$\Delta = 5\sqrt{6}$$

Using

$$\frac{1}{2}|\vec{a} \times \vec{c}| = 5\sqrt{6},$$

we get

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 4 & 2 \\ x & 1 & -2 \end{vmatrix} = 10\sqrt{6}$$

$$\Rightarrow \left| -10\hat{i} - \hat{j}(-2\alpha - 2x) + \hat{k}(\alpha - 4x) \right| = 10\sqrt{6}$$

Substituting $x = \alpha - 5$,

$$(2\alpha + 2\alpha - 10)^2 + (\alpha - 4\alpha + 20)^2 = (10\sqrt{6})^2$$

$$\Rightarrow (4\alpha - 10)^2 + (20 - 3\alpha)^2 = 600$$

$$\Rightarrow 16\alpha^2 - 80\alpha + 100 + 9\alpha^2 - 120\alpha + 400 = 600$$

$$\Rightarrow 25\alpha^2 - 200\alpha = 0$$

$$\Rightarrow 25\alpha(\alpha - 8) = 0$$

$$\Rightarrow \alpha = 8$$

(since α is positive)

$$x = \alpha - 5 = 3$$

Therefore,

$$|\vec{c}|^2 = x^2 + y^2 + z^2$$

$$= 3^2 + 1^2 + (-2)^2$$

$$= 9 + 1 + 4$$

$$= 14$$

$$\boxed{14}$$

Question 159

Let $\vec{x}$ be a vector in the plane containing vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$. If the vector $\vec{x}$ is perpendicular to $(3\hat{i} + 2\hat{j} - \hat{k})$ and its projection on $\vec{a}$ is $\frac{17\sqrt{6}}{2}$, then the value of $|\vec{x}|^2$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 486

Source solution

Let $\vec{x} = \lambda\vec{a} + \mu\vec{b}$ (λ and μ are scalars)
$\vec{x} = \hat{i}(2\lambda + \mu) + \hat{j}(2\mu - \lambda) + \hat{k}(\lambda - \mu)$
Since $\vec{x} \cdot (3\hat{i} + 2\hat{j} - \hat{k}) = 0$

$$3\lambda + 8\mu = 0 \quad (1)$$

Also Projection of $\vec{x}$ on $\vec{a}$ is $\frac{17\sqrt{6}}{2}$
$\frac{\vec{x}\cdot\vec{a}}{|\vec{a}|} = \frac{17\sqrt{6}}{2}$

$$6\lambda - \mu = 51 \quad (2)$$

From (1) and (2)
$\lambda = 8, \mu = -3$
$\vec{x} = 13\hat{i} - 14\hat{j} + 11\hat{k}$
$|\vec{x}|^2 = 486$ Ans.
Official Ans. by NTA (486)

Question 160

A line ' ℓ ' passing through origin is perpendicular to the lines

$$\ell_1 : \vec{r} = (3+t)\hat{i} + (-1+2t)\hat{j} + (4+2t)\hat{k}$$
$$\ell_2 : \vec{r} = (3+2s)\hat{i} + (3+2s)\hat{j} + (2+s)\hat{k}$$

If the co-ordinates of the point in the first octant on ' ℓ_2 ' at the distance of $\sqrt{17}$ from the point of intersection of ' ℓ ' and ' ℓ ' are ($\mathbf{a}, \mathbf{b}, \mathbf{c}$), then $\mathbf{18(a+b+c)}$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 44

Source solution

$\ell_1 : \vec{r} = (3+t)\hat{i} + (-1+2t)\hat{j} + (4+2t)\hat{k}$

$\ell_1 : \frac{x-3}{1} = \frac{y+1}{2} = \frac{z-4}{2} \quad \Rightarrow \quad$ D.R. of $\ell_1 = 1, 2, 2$

$\ell_2 : \vec{\mathrm{r}} = (3+2\,\mathrm{s})\hat{\mathrm{i}} + (3+2\,\mathrm{s})\hat{\mathrm{j}} + (2+\mathrm{s})\hat{\mathrm{k}}$

$\ell_2 : \frac{x-3}{2} = \frac{y-3}{2} = \frac{z-2}{1} \quad \Rightarrow \quad$ D.R. of $\ell_2 = 2, 2, 1$

D.R. of ℓ is $\perp$ to $\ell_1 \& \ell_2$

$\therefore \quad$ D.R. of $\ell \| (\ell_1 \times \ell_2) \quad \Rightarrow \quad \langle -2, 3, -2 \rangle$

$\therefore \quad$ Equation of $\ell : \frac{x}{2} = \frac{y}{-3} = \frac{z}{2}$

Solving $\ell \& \ell_1$

$$(2\lambda, -3\lambda, 2\lambda) = (\mu+3, 2\mu-1, 2\mu+\mu)$$
$$\Rightarrow \quad 2\lambda = \mu + 3$$
$$-3\lambda = 2\mu - 1$$
$$2\lambda = 2\mu + 4$$

$\Rightarrow \mu + 3 = 2\mu + 4$

$\mu = -1$

$\lambda = 1$

$\mathbf{P}(2,-3,2)$ \{intersection point\}
Let, $\mathbf{Q}(2v+3, 2v+3, v+2)$ be point on ℓ_2

Now, $\mathbf{PQ} = \sqrt{(2v+3-2)^2 + (2v+3+3)^2 + (v+2-2)^2} = \sqrt{17}$
$\Rightarrow (2v+1)^2 + (2v+6)^2 + (v)^2 = 17$
$\Rightarrow 9v^2 + 28v + 36 + 1 - 17 = 0$
$\Rightarrow 9v^2 + 28v + 20 = 0$
$\Rightarrow 9v^2 + 18v + 10v + 20 = 0$
$\Rightarrow (9v+10)(v+2) = 0$
$\Rightarrow v = -2$ (rejected), $-\frac{10}{9}$ (accepted)
$Q\left(3-\frac{20}{9}, 3-\frac{20}{9}, 2-\frac{10}{9}\right)$

$$\begin{aligned} & \left(\tfrac{7}{9}, \tfrac{7}{9}, \tfrac{8}{9}\right) \\ \therefore \quad & 18(a+b+c) \\ & = 18\left(\tfrac{7}{9} + \tfrac{7}{9} + \tfrac{8}{9}\right) \\ & = 44 \end{aligned}$$

Question 161

Let $\vec{p} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{p} = \hat{i} + 2\hat{j} + \hat{k}$ be two vectors. If a vector $\overrightarrow{\mathrm{r}} = (\alpha\hat{\mathrm{i}} + \beta\hat{\mathrm{j}} + \gamma\hat{\mathrm{k}})$ is perpendicular to each of the vectors $(\overrightarrow{\mathrm{p}} + \overrightarrow{\mathrm{q}})$ and $(\overrightarrow{\mathrm{p}} - \overrightarrow{\mathrm{q}})$, and $|\overrightarrow{\mathrm{r}}| = \sqrt{3}$, then $|\alpha| + |\beta| + |\gamma|$ is equal to ___

Numerical answer: _____

Enter the requested integer (0 decimal places).

Correct answer: 3

Source solution

$\overrightarrow{\mathrm{p}} = 2\hat{\mathrm{i}} + 3\hat{\mathrm{j}} + \hat{\mathrm{k}}$ (given)

$$\overrightarrow{\mathrm{q}} = \hat{\mathrm{i}} + 2\hat{\mathrm{j}} + \hat{\mathrm{k}}$$

Now

$$\text{Now } (\vec{p} + \hat{q}) \times (\vec{p} - \vec{q}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 5 & 2 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= -2\hat{i} - 2\hat{j} - 2\hat{k}$$

$$\Rightarrow \vec{r} = \pm\sqrt{3}\frac{((\vec{p} + \vec{q}) \times (\vec{p} - \vec{q}))}{[(\vec{p} + \vec{q}) \times (\vec{p} - \vec{q})]}$$

$$= \pm\frac{\sqrt{3}(-2\hat{i} - 2\hat{j} - 2\hat{k})}{\sqrt{2^2 + 2^2 + 2^2}}$$

$$\vec{r} = \pm = (-\hat{i} - \hat{j} - \hat{k})$$

According to question

$$\overrightarrow{\mathrm{r}} = \alpha\hat{\mathrm{i}} + \beta\hat{\mathrm{j}} + \gamma\hat{\mathrm{k}}$$

So $|\alpha| = 1, |\beta| = 1, |\gamma| = 1$

$$\Rightarrow |\alpha| + |\beta| + |\gamma| = 3$$

Question 162

Let $\vec{a}=\hat{i}+\alpha\hat{j}+\beta\hat{k}, \alpha, \beta \in R$. Let a vector $\vec{b}$ be such that the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$ and $|\vec{b}|^2=6$, If $\vec{a}\cdot\vec{b}=3\sqrt{2}$, then the value of $\left(\alpha^2+\beta^2\right)|\vec{a}\times\vec{b}|^2$ is equal to

A. 90

B. 75

C. 95

D. 85

Correct answer: A

Source solution

Sol. $|\vec{b}|^2=6; |\vec{a}||\vec{b}|\cos\theta=3\sqrt{2}$

$|\vec{a}|^2|\vec{b}|^2\cos^2\theta=18$

$|\vec{a}|^2=6$

Also $1+\alpha^2+\beta^2=6$

$\alpha^2+\beta^2=5$

to find

$\left(\alpha^2+\beta^2\right)|\vec{a}|^2|\vec{b}|^2\sin^2\theta$

$=(5)(6)(6)\left(\frac{1}{2}\right)$

$=90$

Question 163

Let the plane $P : \vec{r} \cdot \vec{a} = d$ contain the line of intersection of two planes $\vec{r} \cdot (\hat{i} + 3\hat{j} - \hat{k}) = 6$ and $\vec{r} \cdot (-6\hat{i} + 5\hat{j} - \hat{k}) = 7$. If the plane P passes through the point $\left(2, 3, \frac{1}{2}\right)$, then the value of $\frac{|13\vec{a}|^2}{d^2}$ is equal to

A. 90

B. 93

C. 95

D. 97

Correct answer: B

Source solution

Sol.

Equation of plane passing through line of intersection of planes $P_1 : \vec{r}((\hat{i} + 3\hat{j} - \hat{k}) = 6$ and $P_2 : \vec{r} \cdot (-6\hat{i} + 5\hat{j} - \hat{k}) = 7$ is

$P_1 + \lambda P_2 = 0$

$(\vec{r} \cdot (\hat{i} + 3\hat{j} - \hat{k}) - 6) + \lambda(\vec{r} \cdot (-6\hat{i} + 5\hat{j} - \hat{k}) - 7) = 0$ and it passes through point $\left(2, 3, \frac{1}{2}\right)$

$\Rightarrow \left(2 + 9 - \frac{1}{2} - 6\right) + \lambda\left(-12 + 15 - \frac{1}{2} - 7\right) = 0$

$\Rightarrow \lambda = 1$

Equation of plane is $\vec{r} \cdot (-5\hat{i} + 8\hat{j} - 2\hat{k}) = 13$

$|\vec{a}|^2 = 25 + 64 + 4 = 93; d = 13$

Value of $\frac{|13\vec{a}|^2}{d^2} = 93$

Question 164

Let $\vec{a}=3\hat{i}-\hat{j}+2\hat{k}, \vec{b}=\vec{a}\times(\hat{i}-2\hat{k})$ and $\vec{c}=\vec{b}\times\hat{k}$. Then the projection of $\vec{c}-2\hat{j}$ on $\vec{a}$ is:

A. $3\sqrt{7}$

B. $\sqrt{14}$

C. $2\sqrt{14}$

D. $2\sqrt{7}$

Correct answer: C

Source solution

Sol. $\dot{b}=\vec{a}\times(\hat{i}-3\hat{k})$

$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}3 \\ -1 & 21 & 0 \\ & -2 & \end{vmatrix}=2\hat{i}+8\hat{j}+\hat{k}$

$\vec{c}=\overrightarrow{b}\times\hat{k}=8\hat{i}-2\hat{j}$

$\vec{c}-2\hat{j}=8\hat{i}-4\hat{j}$

Projection of $(\hat{i}-2\hat{j})$ on $\vec{a}$

$(\vec{c}-2\hat{j})\cdot\hat{a}=\frac{\langle 8,-4,0\rangle\cdot\langle 3,-1,2\rangle}{\sqrt{14}}$

$=\frac{28}{\sqrt{14}}=2\sqrt{14}$

Question 165

Let $\vec{a}=\alpha\hat{i}+2\hat{j}-\hat{k}$ and $\vec{b}=-2\hat{i}+\alpha\hat{j}+\hat{k}$, where $\alpha \in \mathbf{R}$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $\sqrt{15\left(\alpha^2+4\right)}$, then the value of $2|\vec{a}|^2+(\vec{a}\cdot\vec{b})|\vec{b}|^2$ is equal to

A. 10

B. 7

C. 9

D. 14

Correct answer: D

Source solution

Sol. $\vec{a}=\alpha\hat{i}+2\hat{j}-\hat{k}, \vec{b}=-2\hat{i}+\alpha\hat{j}+\hat{k}$, area of parallelogram $=|\hat{\mathrm{a}}\times\hat{\mathrm{b}}|$

$|\hat{\mathrm{a}}\times\hat{\mathrm{b}}|=\sqrt{(\alpha+2)^2+(\alpha-2)^2+\left(\alpha^2+4\right)^2}$

Given $|\hat{a}\times\hat{b}|=\sqrt{15\left(\alpha^2+4\right)}$

$2\left(\alpha^2+4\right)+\left(\alpha^2+4\right)^2=15\left(\alpha^2+4\right)$

$\left(\alpha^2+4\right)^2=13\left(\alpha^2+4\right)$

$\Rightarrow \alpha^2+4=13 \therefore \alpha^2=9$

$2|\vec{a}|^2+(\vec{a}\cdot\vec{b})|\vec{b}|^2$

$|\vec{\mathrm{a}}|^2=\alpha^2+4+1=\alpha^2+5$

$|\vec{b}|^2=4+\alpha^2+1=\alpha^2+5$

$\vec{a}\cdot\vec{b}=-2\alpha+2\alpha-1=-1$

$\therefore 2|\vec{a}|^2+(\vec{a}\cdot\vec{b})|\vec{b}|^2$

$2\left(\alpha^2+5\right)-1\left(\alpha^2+5\right)=\alpha^2+5=14$

Question 166

Let O be the origin and the position vector of the point P be $-\hat{i}-2\hat{j}+3\hat{k}$. If the position vectors of the points $\mathbf{A},\mathbf{B}$ and C are $-2\hat{i}+\hat{j}-3\hat{k}, 2\hat{i}+4\hat{j}-2\hat{k}$ and $-4\hat{i}+2\hat{j}-\hat{k}$ respectively then the projection of the vector $\overrightarrow{OP}$ on a vector perpendicular to the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is

A. 3

B. $\frac{8}{3}$

C. $\frac{10}{3}$

D. $\frac{7}{3}$

Correct answer: A

Source solution

Sol. $AB = OB - OA$

$= (2\hat{i}+4\hat{j}-2\hat{k}) - (-2\hat{i}+\hat{j}-3\hat{k})$

$= 4\hat{i}+3\hat{j}+\hat{k}$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = -2\hat{i}+\hat{j}+2\hat{k}$

$\overrightarrow{AB} \times \overrightarrow{AC} = 5\hat{i}-10\hat{j}+10\hat{k}$

$\overrightarrow{OP} = -\hat{i}-2\hat{j}+3\hat{k}$

Projection

$= \frac{(\overrightarrow{OP})\cdot(\overrightarrow{AB}\times\overrightarrow{AC})}{|\overrightarrow{AB}\times\overrightarrow{AC}|} = 3$

Question 167

Let $\vec{a} = \alpha\hat{i} + \hat{j} + \beta\hat{k}$ and $\vec{b} = 3\hat{i} - 5\hat{j} + 4\hat{k}$ be two vectors, such that $\vec{a} \times \vec{b} = -\hat{i} + 9\hat{i} + 12k$. Then the projection of $\vec{b} - 2\vec{a}$ on $\vec{b} + \vec{a}$ is equal to

A. 2

B. $\frac{39}{5}$

C. 9

D. $\frac{46}{5}$

Correct answer: D

Source solution

Sol. Let $\vec{a} = \alpha\hat{i} + \hat{j} + \beta\hat{k}, \vec{b} = 3\hat{i} - 5\hat{j} + 4\hat{k}$

$\vec{a} \times \vec{b} = -\hat{i} + 9\hat{j} + 12\hat{k}$

$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k}\alpha \\ 1 & \beta 3 & -5 \\ & & 4 \end{vmatrix}$

$\Rightarrow (4 + 5\beta)\hat{i} + (3\beta - 4\alpha)\hat{j} + (-5\alpha - 3)\hat{k}$

$= -\hat{i} + 9\hat{j} + 12\hat{k}$

$\therefore 4 + 5\beta = -1, 3\beta - 4\alpha = 9, -5\alpha - 3 = 12$

$\beta = -1, \quad \alpha = -3$

$\therefore \vec{a} = -3\hat{i} + \hat{j} - \hat{k}, \vec{b} = 3\hat{i} - 5\hat{j} + 4\hat{k}$

$\therefore \vec{a} + \vec{b} = -4\hat{j} + 3\hat{k}$

$|\vec{a}|^2 = 11, |\vec{b}|^2 = 50$

$\vec{a} \cdot \vec{b} = -9 + (-5) - 4 = -18$

$\therefore$ Projectile of $(\vec{b} - 2\vec{a})$ on $\vec{a} + \vec{b}$ is

$$\frac{(\vec{b} - 2\vec{a}) \cdot (\vec{a} + \vec{b})}{|\vec{a} + \vec{b}|}$$

$$= \frac{|\vec{b}|^2 - 2|\vec{a}|^2 - (\vec{a} \cdot \vec{b})}{|\vec{a} + \vec{b}|} = \frac{50 - 22 - (-18)}{5} = \frac{46}{5}$$

Question 168

Let $\vec{a}$ be a vector which is perpendicular to the vector $3\hat{i}+\frac{1}{2}\hat{j}+2\hat{k}$. If $\vec{a}\times(2\hat{i}+\hat{k})=2\hat{i}-13\hat{j}-4\hat{k}$, then the projection of the vector $\vec{a}$ on the vector $2\hat{\mathrm{i}}+2\hat{\mathrm{j}}+\hat{\mathrm{k}}$ is

A. $\frac{1}{3}$

B. 1

C. $\frac{5}{3}$

D. $\frac{7}{3}$

Correct answer: C

Source solution

Sol. $(\vec{a}\times(2\hat{i}+\hat{k}))\times\left(3\hat{i}+\frac{1}{2}\hat{j}+2\hat{k}\right)$

$=(2\hat{\mathrm{i}}-13\hat{\mathrm{j}}-4\hat{\mathrm{k}})\times\left(3\hat{\mathrm{i}}+\frac{1}{2}\hat{\mathrm{j}}+2\hat{\mathrm{k}}\right)$

$-(6+2)\vec{a}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}2 \\ -13 & -43 & \frac{1}{2} \\ & & 2\end{vmatrix}$

$\vec{a}=3\hat{i}+2\hat{j}-5\hat{k}$

Projection of $\vec{a}$ on vector $2\hat{i}+2\hat{j}+\hat{k}$ is

$\vec{\mathrm{a}}\cdot\frac{(2\hat{\mathrm{i}}+2\hat{\mathrm{j}}+\hat{\mathrm{k}})}{3}=\frac{5}{3}$

Question 169

If $\vec{P} = (a+1)\hat{i} + a\hat{j} + a\hat{k}$
$\vec{Q} = a\hat{i} + (a+1)\hat{j} + a\hat{k}$
$\vec{R} = a\hat{i} + a\hat{j} + (a+1)\hat{k}$
and $\vec{P}, \vec{Q}, \vec{R}$ are coplanar vectors and $3(\vec{P}.\vec{Q})^2 - \lambda|\vec{R} \times \vec{Q}|^2 = 0$, then value of λ is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 1

Source solution

Solution:

As $\vec{P}, \vec{Q}, \vec{R}$ are coplanar,

$$\begin{vmatrix} a+1 & a & a \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

$R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} 3a+1 & 3a+1 & 3a+1 \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

$$(3a+1)\begin{vmatrix} 1 & 1 & 1 \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

$C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$$(3a+1)\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ a & 0 & 1 \end{vmatrix} = 0$$

$3a + 1 = 0$

$\Rightarrow a = -\frac{1}{3}$

$\vec{P} = \frac{1}{3}(2\hat{i} - \hat{j} - \hat{k}), \quad \vec{Q} = \frac{1}{3}(-\hat{i} + 2\hat{j} - \hat{k}), \quad \vec{R} = \frac{1}{3}(-\hat{i} - \hat{j} + 2\hat{k})$

$$\vec{R} \times \vec{Q} = \frac{1}{9}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -1 & 2 \\ -1 & 2 & -1 \end{vmatrix}$$

$\vec{R} \times \vec{Q} = \frac{1}{9}(-3\hat{i} - 3\hat{j} - 3\hat{k}) = -\frac{1}{3}(\hat{i} + \hat{j} + \hat{k})$

$|\vec{R} \times \vec{Q}|^2 = \frac{1}{3}$

$\vec{P} \cdot \vec{Q} = \frac{1}{9}(-2 - 2 + 1) = -\frac{1}{3}$

$3(\vec{P}\cdot\vec{Q})^2-\lambda|\vec{R}\times\vec{Q}|^2=0$
$\Rightarrow \frac{1}{3}-\lambda\times\frac{1}{3}=0\Rightarrow\lambda=1$

Question 170

Let $\vec{a}=2\hat{i}-3\hat{j}+4\hat{k}$ and $\vec{b}=7\hat{i}+\hat{j}-6\hat{k}$.

If $\vec{r}\times\vec{a}=\vec{r}\times\vec{b}, \vec{r}\cdot(\hat{i}+2\hat{j}+\hat{k})=-3$, then $\vec{r}\cdot(2\hat{i}-3\hat{j}+\hat{k})$ is equal to :

A. 12

B. 8

C. 13

D. 10

Correct answer: A

Source solution

Sol. $\overrightarrow{\mathrm{r}}\times\overrightarrow{\mathrm{a}}-\overrightarrow{\mathrm{r}}\times\overrightarrow{\mathrm{b}}=0$
$\Rightarrow\overrightarrow{\mathrm{r}}\times(\overrightarrow{\mathrm{a}}-\overrightarrow{\mathrm{b}})=0$
$\Rightarrow\overrightarrow{\mathrm{r}}=\lambda(\overrightarrow{\mathrm{a}}-\overrightarrow{\mathrm{b}})$
$\Rightarrow\overrightarrow{\mathrm{r}}=\lambda(-5\hat{\mathrm{i}}-4\hat{\mathrm{j}}+10\hat{\mathrm{k}})$

Also $\overrightarrow{\mathrm{r}}\cdot(\hat{\mathrm{i}}+2\hat{\mathrm{j}}+\hat{\mathrm{k}})=-3$
$\Rightarrow\lambda(-5-8+10)=-3$
$\lambda=1$

Now $\overrightarrow{\mathrm{r}}=-5\hat{\mathrm{i}}-4\hat{\mathrm{j}}+10\hat{\mathrm{k}}$
$=\overrightarrow{\mathrm{r}}\cdot(2\hat{\mathrm{i}}-3\hat{\mathrm{j}}+\hat{\mathrm{k}})$
$=-10+12+10=12$

Question 171

Let $\vec{a}=\hat{i}+5\hat{j}+\alpha\hat{k}, \vec{b}=\hat{i}+3\hat{j}+\beta\hat{k}$ and $\overrightarrow{c}=-\hat{\mathrm{i}}+2\hat{\mathrm{j}}-3\hat{\mathrm{k}}$ be three vectors such that, $|\vec{b}\times\vec{c}|=5\sqrt{3}$ and $\vec{a}$ is perpendicular to $\vec{b}$. Then the greatest amongst the values of $|\vec{a}|^2$ is .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 90

Source solution

since, $\vec{a}\cdot\vec{b}=0$

$$1+15+\alpha\beta=0\Rightarrow\alpha\beta=-16 \tag{1}$$

Also,

$|\overrightarrow{\mathrm{b}}\times\overrightarrow{\mathrm{c}}|^2=75\Rightarrow(10+\beta^2)14-(5-3\beta)^2=75$

$\Rightarrow 5\beta^2+30\beta+40=0$

$\Rightarrow\beta=-4,-2$

$\Rightarrow\alpha=4,8$

$\Rightarrow|\overrightarrow{\mathrm{a}}|^2_{\max}=\left(26+\alpha^2\right)_{\max}=90$

Three Dimensional Geometry

Questions 172-293 / 122 selected / 40.7% practice weight / Focus first

Question 172

Let the line $\ell : x = \frac{1-y}{-2} = \frac{z-3}{\lambda}, \lambda \in \mathbb{R}$ meet the plane $\mathrm{P} : \mathrm{x} + 2\mathrm{y} + 3\mathrm{z} = 4$ at the point (α, β, γ). If the angle between the line ℓ and the plane P is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then $\alpha + 2\beta + 6\gamma$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 11

Source solution

$\ell : \mathrm{x} = \frac{\mathrm{y}-1}{2} = \frac{\mathrm{z}-3}{\lambda}, \lambda \in \mathbb{R}$
DR's of line $\ell(1, 2, \lambda)$
DR's of normal vector of plane $P : x + 2y + 3z = 4$ are (1, 2, 3)

Now, angle between line ℓ and plane P is given by
$\sin\theta = \left|\frac{1+4+3\lambda}{\sqrt{5+\lambda^2}\cdot\sqrt{14}}\right| = \frac{3}{\sqrt{14}}$ (given $\cos\theta = \sqrt{\frac{5}{14}}$)
$\Rightarrow \lambda = \frac{2}{3}$
Let variable point on line ℓ is $\left(t, 2t+1, \frac{2}{3}t+3\right)$
lies on plane P.
$\Rightarrow \mathrm{t} = -1$
$\Rightarrow \left(-1, -1, \frac{7}{3}\right) \equiv (\alpha, \beta, \gamma)$
$\Rightarrow \alpha + 2\beta + 6\gamma = 11$

Question 173

Let the image of the point $\left(\frac{5}{3}, \frac{5}{3}, \frac{8}{3}\right)$ in the plane $x - 2y + z - 2 = 0$ be P. If the distance of the point $Q(6, -2, \alpha), \alpha > 0$, from P is 13 , then α is equal to

.

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 15

Source solution

Image of point $\left(\frac{5}{3}, \frac{5}{3}, \frac{8}{3}\right)$

$\frac{x-\frac{5}{3}}{1} = \frac{y-\frac{5}{3}}{-2} = \frac{z-\frac{8}{3}}{1} = \frac{-2\left(1\times\frac{5}{3}+(-2)\times\frac{8}{3}+1\times\frac{8}{3}-2\right)}{1^2+2^2+1^2} = \frac{1}{3}$

$\therefore x = 2, y = 1, z = 3$

$13^2 = (6-2)^2 + (-2-1)^2 + (\alpha-3)^2$

$\Rightarrow (\alpha-3)^2 = 144 \Rightarrow \alpha = 15 (\because \alpha > 0)$

Question 174

Let the lines $l_1 : \frac{x+5}{3} = \frac{y+4}{1} = \frac{z-\alpha}{-2}$ and $l_2 : 3x+2y+z-2=0=x-3y+2z-13$ be coplanar. If the point $P(a,b,c)$ on l_1 is nearest to the point $Q(-4,-3,2)$, then $|a|+|b|+|c|$ is equal to

A. 12

B. 14

C. 10

D. 8

Correct answer: C

Source solution

Sol. $(3x+2y+z-2)+\mu(x-3y+2z-13)=0$

$3(3+\mu)+1\cdot(2-3\mu)-2(1+2\mu)=0$

$9-4\mu=0$

$\mu=\frac{9}{4}$

$4(-15-8+\alpha-2)+9(-5+12+2\alpha-13)=0$

$-100+4\alpha-54+18\alpha=0$

$\Rightarrow \alpha=7$

Let $P\equiv(3\lambda-5,\lambda-4,-2\lambda+7)$

Direction ratio of $PQ(3\lambda-1,\lambda-1,-2\lambda+5)$

But $PQ\perp \ell_1$

$\Rightarrow 3(3\lambda-1)+1\cdot(\lambda-1)-2(-2\lambda+5)=0$

$\Rightarrow \lambda=1$

$P(-2,-3,5)\Rightarrow |a|+|b|+|c|=10$

Question 175

Let the equation of plane passing through the line of intersection of the planes $x+2y+az=2$ and $x-y+z=3$ be $5x-11y+bz=6a-1$. For $c \in \mathbb{Z}$, if the distance of this plane from the point $(a,-c,c)$ is $\frac{2}{\sqrt{a}}$, then $\frac{a+b}{c}$ is equal to

A. -2

B. 2

C. -4

D. 4

Correct answer: C

Source solution

Sol.

$(x+2y+az-2)+\lambda(x-y+z-3)=0$

$\frac{1+\lambda}{5}=\frac{2-\lambda}{-11}=\frac{a+\lambda}{b}=\frac{2+3\lambda}{6a-1}$

$\lambda=-\frac{7}{2}, a=3,\ b=1$

$\frac{2}{\sqrt{a}}=\left|\frac{5a+11c+bc-6a+1}{\sqrt{25+121+1}}\right|$

$c=-1$

$\therefore \frac{a+b}{c}=\frac{3+1}{-1}=-4$

Question 176

Let P be the point of intersection of the line $\frac{x+3}{3} = \frac{y+2}{1} = \frac{1-z}{2}$ and the plane $x + y + z = 2$.
If the distance of the point P from the plane $3x - 4y + 12z = 32$ is q, then q and $2q$ are the roots of the equation

A. $x^2 - 18x - 72 = 0$

B. $x^2 + 18x + 72 = 0$

C. $x^2 - 18x + 72 = 0$

D. $x^2 + 18x - 72 = 0$

Correct answer: C

Source solution

Sol. $P = (3\lambda - 3, \lambda - 2, 1 - 2\lambda)$

P lies on the plane, $x + y + z = 2$
$\Rightarrow \lambda = 3$
$\mathrm{P} = (6, 1, -5)$
$q = \left| \frac{18-4-60-32}{\sqrt{9+16+144}} \right| = \frac{78}{13} = 6$
$\mathrm{q} = 6, 2\mathrm{q} = 12$

Equation, $x^2 - 18x + 72 = 0$

Question 177

Let the lines $l_1 : \frac{x+5}{3} = \frac{y+4}{1} = \frac{z-\alpha}{-2}$ and $l_2 : 3x + 2y + z - 2 = 0 = x - 3y + 2z - 13$ be coplanar. If the point $P(a, b, c)$ on l_1 is nearest to the point $Q(-4, -3, 2)$, then $|a| + |b| + |c|$ is equal to

A. 12

B. 14

C. 10

D. 8

Correct answer: C

Source solution

Sol.

1. Start with the equation:

$$(3x + 2y + z - 2) + \mu(x - 3y + 2z - 13) = 0$$

2. Substitute and simplify:

$$3(3 + \mu) + 1(2 - 3\mu) - 2(1 + 2\mu) = 0$$

$$9 - 4\mu = 0$$

3. Solve for μ:

$$\mu = \frac{9}{4}$$

4. Next, consider the equation:

$$4(-15-8+\alpha-2)+9(-5+12+2\alpha-13)=0$$

5. Simplify:

$$-100+4\alpha-54+18\alpha=0$$

$$22\alpha-154=0$$

6. Solve for α:

$$\alpha=7$$

7. Define point P:

$$P\equiv(3\lambda-5,\lambda-4,-2\lambda+7)$$

8. Find the direction ratio of $\mathbf{PQ}$:

$$\mathbf{PQ}\equiv(3\lambda-1,\lambda-1,-2\lambda+5)$$

9. Since $\mathbf{PQ}\perp\ell_1$, set up the equation:

$$3(3\lambda-1)+1(\lambda-1)-2(-2\lambda+5)=0$$

10. Solve for λ:

$$\lambda=1$$

11. Finally, for point $P(-2,-3,5)$:

$$|a|+|b|+|c|=10$$

Question 178

If the equation of the plane passing through the line of intersection of the planes $2x-y+z=3, 4x-3y+5z+9=0$ and parallel to the line $\frac{x+1}{-2}=\frac{y+3}{4}=\frac{z-2}{5}$ is $ax+by+cz+6=0$. then a $+\mathrm{b}+\mathrm{c}$ is equal to

A. 14

B. 12

C. 13

D. 15

Correct answer: A

Source solution

Sol. Equation of family of plane

$(2x-y+z-3)+\lambda(4x-3y+5z+9)=0$

$x(2+4\lambda)-y(1+3\lambda)+z(1+5\lambda)-3+9\lambda=0$

Parallel to the line

$-2(2+4\lambda)-(1+3\lambda)4+(1+5\lambda)5=0$

$5\lambda=3$

$\lambda=\frac{3}{5}$

equation of plane

$11x-7y+10z+6=0$

$a+b+c=14$

Question 179

Let P be the point of intersection of the line

$$\frac{x+3}{3}=\frac{y+2}{1}=\frac{1-z}{2}$$

and the plane

$$x+y+z=2.$$

If the distance of the point P from the plane

$$3x-4y+12z=32$$

is q, then q and $2q$ are the roots of the equation.

A. $x^2-18x-72=0$

B. $x^2+18x+72=0$

C. $x^2-18x+72=0$

D. $x^2+18x-72=0$

Correct answer: C

Source solution

$$\text{Let } P=(3\lambda-3,\lambda-2,1-2\lambda)$$

$$\text{Since } P \text{ lies on the plane } x+y+z=2:$$

$$(3\lambda-3)+(\lambda-2)+(1-2\lambda)=2$$

$$\Rightarrow 3\lambda - 3 + \lambda - 2 + 1 - 2\lambda = 2$$

$$\Rightarrow 2\lambda - 4 = 2$$

$$\Rightarrow 2\lambda = 6 \quad \Rightarrow \quad \lambda = 3$$

$$\text{Thus, } P = (6, 1, -5)$$

$$\text{Now, calculate } q:$$

$$q = \left| \frac{18 - 4 - 60 - 32}{\sqrt{9 + 16 + 144}} \right|$$

$$= \left| \frac{-78}{\sqrt{169}} \right| = \left| \frac{-78}{13} \right| = 6$$

$$\text{Therefore, } q = 6 \quad \text{and } 2q = 12$$

$$\text{The equation is } x^2 - 18x + 72 = 0$$

Question 180

If the line $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-1}{-1}$ intersects the plane $2x+3y-z+13=0$ at a point P and the plane $3x+y+4z=16$ at a point Q, then PQ is equal to:

A. $2\sqrt{14}$

B. 14

C. $2\sqrt{7}$

D. $\sqrt{14}$

Correct answer: A

Source solution

Sol. Let $\mathrm{P}(3\lambda+2, 2\lambda-1, -\lambda+1)$ and

$$\mathrm{Q}(3\mu+2, 2\mu-1, -\mu+1)$$

As P lies on $2x+3y-z+13=0$
$6\lambda+4+6\lambda-3+\lambda-1+13=0$
$\Rightarrow 13\lambda=-13$
$\Rightarrow \lambda=-1$
$\therefore \quad \mathrm{P}(-1,-3,2)$
Q lies on $3x+y+4z=16$
$9\mu+6+2\mu-1-4\mu+4=16$
$\Rightarrow 7\mu=7$
$\Rightarrow \mu=1$
Q is $(5,1,0)$
$PQ=\sqrt{36+16+4}=\sqrt{56}=2\sqrt{14}$

Question 181

Let the plane P contain the line $2x + y - z - 3 = 0 = 5x - 3y + 4z + 9$ and be parallel to the line $\frac{x+2}{2} = \frac{3-y}{-4} = \frac{z-7}{5}$. Then the distance of the point $A(8, -1, -19)$ from the plane P measured parallel to the line $\frac{x}{-3} = \frac{y-5}{4} = \frac{2-z}{-12}$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 26

Source solution

Plane $\mathbf{P} \equiv \mathbf{P}_1 + \lambda \mathbf{P}_2 = 0$

$(2x + y - z - 3) + \lambda(5x - 3y) + 4z + 9) = 0$

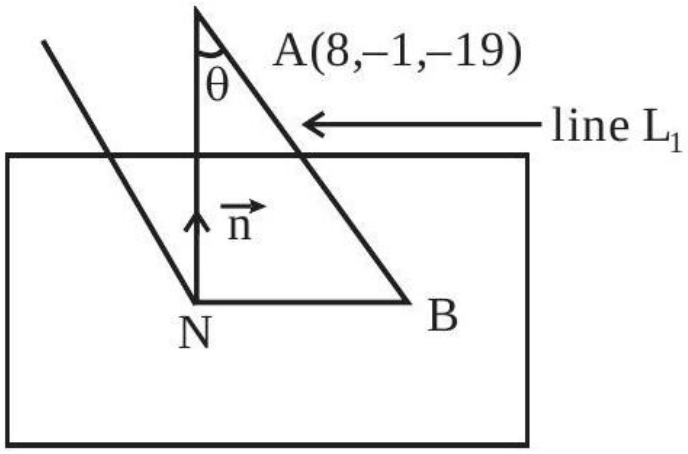
$(5\lambda + 2)x + (1 - 3\lambda)y + (4\lambda - 1)z + 9\lambda - 3 = 0$

$\vec{n}.\vec{b} = 0$ where $\vec{b}(2, 4, 5)$

$2(5\lambda + 2) + 4(1 - 3\lambda) + 5(4\lambda - 1) = 0$

$\lambda = -\frac{1}{6}$

Plane $7x + 9y - 10z - 27 = 0$



Equation of line AB is

$\frac{x-8}{-3} = \frac{y+1}{4} = \frac{z+19}{12} = \lambda$

Let $\mathbf{B} = (8 - 3\lambda, -1 + 4\lambda, -19 + 12\lambda)$ lies on plane P

$\therefore 7(8 - 3\lambda) + 9(4\lambda - 1) - 10(12\lambda - 19) = 27$

$\lambda = 2$

$\therefore$ Point $\mathbf{B} = (2, 7, 5)$

$\mathbf{AB} = \sqrt{6^2 + 8^2 + 24^2} = 26$

Question 182

If the line $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1}$ intersects the plane $2x + 3y - z + 13 = 0$ at a point P and the plane $3x + y + 4z = 16$ at a point Q, then PQ is equal to:

A. $2\sqrt{14}$

B. 14

C. $2\sqrt{7}$

D. $\sqrt{14}$

Correct answer: A

Source solution

Sol. Let $\mathbf{P}(3\lambda + 2, 2\lambda - 1, -\lambda + 1)$ and

$$\mathbf{Q}(3\mu + 2, 2\mu - 1, -\mu + 1)$$

Since point P lies on the plane $2x + 3y - z + 13 = 0$, we substitute the coordinates of P:

$$2(3\lambda + 2) + 3(2\lambda - 1) - (-\lambda + 1) + 13 = 0$$

Simplifying this:

$$6\lambda + 4 + 6\lambda - 3 + \lambda - 1 + 13 = 0$$

Combining like terms:

$$13\lambda + 13 = 0$$

Solving for λ:

$$13\lambda = -13 \quad \Rightarrow \quad \lambda = -1$$

Thus,

$$\mathbf{P}(-1, -3, 2)$$

Next, point Q lies on the plane $3x + y + 4z = 16$. We substitute the coordinates of Q:

$$3(3\mu + 2) + (2\mu - 1) + 4(-\mu + 1) = 16$$

Simplifying this:

$$9\mu + 6 + 2\mu - 1 - 4\mu + 4 = 16$$

Combining like terms:

$$7\mu + 9 = 16$$

Solving for μ:

$$7\mu = 7 \quad \Rightarrow \quad \mu = 1$$

Thus,

$$\mathbf{Q}(5, 1, 0)$$

Now, we calculate the distance PQ:

$$PQ = \sqrt{(5 - (-1))^2 + (1 - (-3))^2 + (0 - 2)^2}$$

Calculating each term:

$$= \sqrt{(6)^2 + (4)^2 + (-2)^2} = \sqrt{36 + 16 + 4} = \sqrt{56} = 2\sqrt{14}$$

Final answer:

$$PQ = 2\sqrt{14}$$

Question 183

Let P be the plane passing through the points $(5,3,0)$, $(13,3,-2)$ and $(1,6,2)$. For $\alpha \in \mathbb{N}$, if the distances of the points $\mathrm{A}(3,4,\alpha)$ and $\mathrm{B}(2,\alpha,a)$ from the plane P are 2 and 3 respectively, then the positive value of a is

A. 6

B. 4

C. 3

D. 5

Correct answer: B

Source solution

Sol.

1. Calculate the determinant to find the normal vector of the plane:

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & 0 & -2 \\ 4 & -3 & -2 \end{vmatrix} = \hat{i}(-6) + 8\hat{j} - 24\hat{k}$$

2. The normal vector of the plane is:

$$\mathbf{n} = 3\hat{i} - 4\hat{j} + 12\hat{k}$$

3. The equation of the plane is:

$$3x - 4y + 12z = 3$$

4. Calculate the distance from point $A(3,4,\alpha)$:

$$\left| \frac{9 - 16 + 12\alpha - 3}{13} \right| = 2$$

Simplifying gives:

$$\left|\frac{12\alpha - 10}{13}\right| = 2$$

This leads to two cases:
- Case 1: $\frac{12\alpha-10}{13} = 2$

$$12\alpha - 10 = 26 \implies 12\alpha = 36 \implies \alpha = 3$$

- Case 2: $\frac{12\alpha-10}{13} = -2$

$$12\alpha - 10 = -26 \implies 12\alpha = -16 \implies \alpha = -\frac{4}{3} \quad \text{(rejected)}$$

5. Calculate the distance from point $B(2, 3, a)$:

$$\left|\frac{6 - 12 + 12a - 3}{13}\right| = 3$$

Simplifying gives:

$$\left|\frac{12a - 9}{13}\right| = 3$$

This leads to two cases:
- Case 1: $\frac{12a-9}{13} = 3$

$$12a - 9 = 39 \implies 12a = 48 \implies a = 4$$

- Case 2: $\frac{12a-9}{13} = -3$

$$12a - 9 = -39 \implies 12a = -30 \implies a = -\frac{5}{2} \quad \text{(rejected)}$$

Final answers:

$$\alpha = 3, \quad a = 4$$

Question 184

Let the line passing through the points, $\mathbf{P(2,-1,2)}$ and $\mathbf{Q(5,3,4)}$ meet the plane $\mathbf{x-y+z=4}$ at the point R . Then the distance of the point R from the plane $x+2y+3z+2=0$ measured parallel to the line $\frac{x-7}{2}=\frac{y+3}{2}=\frac{z-2}{1}$ is equal to

A. $\sqrt{31}$

B. $\sqrt{189}$

C. $\sqrt{61}$

D. 3

Correct answer: D

Source solution

Sol. Line : $\frac{x-5}{3}=\frac{y-3}{4}=\frac{z-4}{2}=\lambda$

$\mathbf{R(3\lambda+5,4\lambda+3,2\lambda+4)}$

$\therefore 3\lambda+5-4\lambda-3+2\lambda+4=4$

$\lambda+6=4 \therefore \lambda=-2$

$\therefore \mathbf{R} \equiv (-1,-5,0)$

Line : $\frac{x+1}{2}=\frac{y+5}{2}=\frac{z-0}{1}=\mu$

Point $T=(2\mu-1,2\mu-5,\mu)$

It lies on plane

$2\mu-1+2(2\mu-5)+3\mu+2=0$

$\mu=1$

$\therefore \mathbf{T}=(1,-3,1)$

$\therefore \mathbf{RT}=3$

Question 185

Let (α, β, γ) be the image of the point $\mathbf{P}(2,3,5)$ in the plane $2x + y - 3z = 6$. Then $\alpha + \beta + \gamma$ is equal to

A. 10

B. 5

C. 12

D. 9

Correct answer: A

Source solution

Sol.

We start with the equation:

$$\frac{\alpha - 2}{2} = \frac{\beta - 3}{1} = \frac{\gamma - 5}{-3} = -2$$

From this, we can derive the values of α, β, and γ.

1. For α:

$$\frac{\alpha - 2}{2} = -2 \implies \alpha - 2 = -4 \implies \alpha = -4 + 2 = -2$$

2. For β:

$$\frac{\beta - 3}{1} = -2 \implies \beta - 3 = -2 \implies \beta = -2 + 3 = 1$$

3. For γ:

$$\frac{\gamma - 5}{-3} = -2 \implies \gamma - 5 = 6 \implies \gamma = 6 + 5 = 11$$

Now we have:

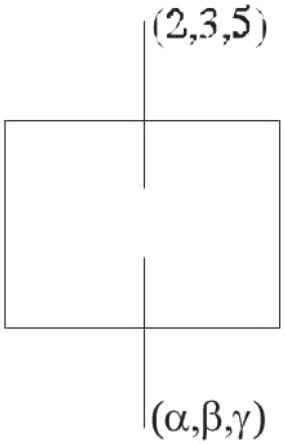
$$\alpha = -2, \quad \beta = 1, \quad \gamma = 11$$

Finally, we calculate:

$$\alpha + \beta + \gamma = -2 + 1 + 11 = 10$$

Thus, the final answer is:

$$\alpha + \beta + \gamma = 10$$



Question 186

If equation of the plane that contains the point ($-2, 3, 5$) and is perpendicular to each of the planes $2x + 4y + 5z = 8$ and $3x - 2y + 3z = 5$ is
$\alpha x + \beta y + \gamma z + 97 = 0$ then $\alpha + \beta + \gamma =$

A. 18

B. 17

C. 16

D. 15

Correct answer: D

Source solution

Sol. The equation of plane through ($-2, 3, 5$) is $a(x + 2) + b(y - 3) + c(z - 5) = 0$
it is perpendicular to $2x + 4y + 5z = 8 \& 3x - 2y + 3z = 5$
$\therefore \quad 2a + 4b + 5c = 0$

$$3a - 2b + 3c = 0$$

$\therefore \quad \frac{a}{\begin{vmatrix} 4 & 5 \\ -2 & 3 \end{vmatrix}} = \frac{-b}{\begin{vmatrix} 2 & 5 \\ 3 & 3 \end{vmatrix}} = \frac{c}{\begin{vmatrix} 2 & 4 \\ 3 & -2 \end{vmatrix}}$
$\Rightarrow \quad \frac{a}{22} = \frac{b}{9} = \frac{c}{-16}$
$\therefore$ Equation of Plane is
$22(x + 2) + 9(y - 3) - 16(z - 5) = 0$
$\Rightarrow \quad 22x + 9y - 16z + 97 = 0$

Comparing with $\alpha x + \beta y + \gamma x + 97 = 0$

We get $\alpha + \beta + \gamma = 22 + 9 - 16 = 15$

Question 187

Let the equation of plane passing through the line of intersection of the planes $x+2y+az=2$ and $x-y+z=3$ be $5x-11y+bz=6a-1$. For $c \in \mathbb{Z}$, if the distance of this plane from the point $(a,-c,c)$ is $\frac{2}{\sqrt{a}}$, then $\frac{a+b}{c}$ is equal to

A. -2

B. 2

C. -4

D. 4

Correct answer: C

Source solution

Sol.

1. Start with the equation:

$$(x+2y+az-2)+\lambda(x-y+z-3)=0$$

2. Set up the system of equations:

$$\frac{1+\lambda}{5}=\frac{2-\lambda}{-11}=\frac{a+\lambda}{b}=\frac{2+3\lambda}{6a-1}$$

3. Solve for λ, a, and b:

$$\lambda=-\frac{7}{2}, \quad a=3, \quad b=1$$

4. Substitute a into the equation:

$$\frac{2}{\sqrt{a}}=\left|\frac{5a+11c+bc-6a+1}{\sqrt{25+121+1}}\right|$$

5. Solve for c:

$$c = -1$$

6. Finally, calculate:

$$\frac{a+b}{c} = \frac{3+1}{-1} = -4$$

Question 188

Let the line passing through the points, $\mathbf{P(2,-1,2)}$ and $\mathbf{Q(5,3,4)}$ meet the plane $\mathbf{x-y+z=4}$ at the point $\boldsymbol{R}$. Then the distance of the point $\boldsymbol{R}$ from the plane $\boldsymbol{x+2y+3z+2=0}$ measured parallel to the line $\frac{x-7}{2}=\frac{y+3}{2}=\frac{z-2}{1}$ is equal to

A. $\sqrt{31}$

B. $\sqrt{189}$

C. $\sqrt{61}$

D. 3

Correct answer: D

Source solution

$$\text{Let } \frac{x-5}{3}=\frac{y-3}{4}=\frac{z-4}{2}=\lambda$$

$$\text{Then, } R(3\lambda+5,4\lambda+3,2\lambda+4)$$

$$\therefore 3\lambda+5-4\lambda-3+2\lambda+4=4$$

$$\lambda+6=4 \implies \lambda=-2$$

$$\therefore R\equiv(-1,-5,0)$$

$$\text{For the line: } \frac{x+1}{2}=\frac{y+5}{2}=\frac{z-0}{1}=\mu$$

$$\text{Point } T=(2\mu-1,2\mu-5,\mu)$$

$$\text{It lies on the plane: } 2\mu-1+2(2\mu-5)+3\mu+2=0$$

$$\mu=1$$

$$\therefore T = (1, -3, 1)$$

$$\therefore RT = 3$$

Question 189

Let P_1 be the plane $3x - y - 7z = 11$ and P_2 be the plane passing through the points $(2, -1, 0)$, $(2, 0, -1)$, and $(5, 1, 1)$. If the foot of the perpendicular drawn from the point (7, 4, -1)on the line of intersection of the planes $\mathbf{P_1}$ and $\mathbf{P_2}$ is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 11

The source provides a brief answer only. A detailed worked solution is not available for this question.

Source solution

Question 190

Let the point, on the line passing through the points $P(1, -2, 3)$ and $Q(5, -4, 7)$, farther from the origin and at a distance of 9 units from the point P, be (α, β, γ). Then $\alpha^2 + \beta^2 + \gamma^2$ is equal to :

A. 155

B. 150

C. 160

D. 165

Correct answer: A

Source solution

Sol.

The line PQ is given by the equations:

$$\frac{x-1}{4} = \frac{y+2}{-2} = \frac{z-3}{4}$$

We can express the points on the line in terms of a parameter t:

$$(x, y, z) = (4t + 1, -2t - 2, 4t + 3)$$

Next, we calculate the distance squared from the origin to the point on the line:

$$d^2 = (4t + 1)^2 + (-2t - 2)^2 + (4t + 3)^2$$

Expanding each term:

$$d^2 = (16t^2 + 8t + 1) + (4t^2 + 8t + 4) + (16t^2 + 24t + 9)$$

Combining like terms:

$$d^2 = 16t^2 + 4t^2 + 16t^2 + 8t + 8t + 24t + 1 + 4 + 9$$

$$d^2 = 36t^2 + 40t + 14$$

Setting $d^2 = 81$:

$$36t^2 + 40t + 14 = 81$$

Rearranging gives:

$$36t^2 + 40t - 67 = 0$$

Using the quadratic formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-40 \pm \sqrt{40^2 - 4 \cdot 36 \cdot (-67)}}{2 \cdot 36}$$

Calculating the discriminant:

$$b^2 - 4ac = 1600 + 9632 = 11232$$

Thus,

$$t = \frac{-40 \pm \sqrt{11232}}{72}$$

Calculating $\sqrt{11232}$:

$$\sqrt{11232} = 106$$

So,

$$t = \frac{-40 \pm 106}{72}$$

Calculating the two possible values for t:

1. $t = \frac{66}{72} = \frac{11}{12}$
2. $t = \frac{-146}{72} = -\frac{73}{36}$

Finding the corresponding points:

For $t = \frac{11}{12}$:

$$(4 \cdot \frac{11}{12} + 1, -2 \cdot \frac{11}{12} - 2, 4 \cdot \frac{11}{12} + 3) = (7, -5, 9)$$

For $t = -\frac{73}{36}$:

Calculating gives another point.

Finally, we find:

$$\alpha^2 + \beta^2 + \gamma^2 = 155$$

Thus, the answer is option (1).

Question 191

Let the point, on the line passing through the points $\mathbf{P(1,-2,3)}$ and $\mathbf{Q(5,-4,7)}$, farther from the origin and at a distance of 9 units from the point P , be (α,β,γ). Then $\alpha^2+\beta^2+\gamma^2$ is equal to :

A. 155

B. 150

C. 160

D. 165

Correct answer: A

Source solution

Sol. PQ line
$\frac{x-1}{4}=\frac{y+2}{-2}=\frac{z-3}{4}$
pt $(4t+1,-2t-2,4t+3)$
distance $^2=16t^2+4t^2+16t^2=81$
$t=\pm\frac{3}{2}$
pt (7, -5, 9)
$\alpha^2+\beta^2+\gamma^2=155$
option (1)

Question 192

Let $\mathbf{P}(\alpha, \beta, \gamma)$ be the image of the point $\mathbf{Q}(3, -3, 1)$ in the line $\frac{x-0}{1} = \frac{y-3}{1} = \frac{z-1}{-1}$ and R be the point ($2, 5, -1$). If the area of the triangle PQR is λ and $\lambda^2 = 14\,\mathrm{K}$, then K is equal to:

A. 36

B. 72

C. 18

D. 81

Correct answer: D

Source solution

Sol.

$\mathrm{RQ} = \sqrt{1+64+4} = \sqrt{69}$

$\overrightarrow{\mathrm{RQ}} = \hat{\ell} - 8\hat{\mathrm{j}} + 2\hat{\mathrm{k}}$

$\overrightarrow{\mathrm{RS}} = \hat{\ell} + \hat{\mathrm{j}} - \hat{\mathrm{k}}$

$\cos\theta = \frac{\overrightarrow{\mathrm{RQ}} \cdot \overrightarrow{\mathrm{RS}}}{|\overrightarrow{\mathrm{RQ}}||\overrightarrow{\mathrm{RS}}|} = \left|\frac{1-8-2}{\sqrt{69}\sqrt{3}}\right| = \frac{9}{3\sqrt{23}}$

$\cos\theta = \frac{3}{\sqrt{23}} = \frac{\mathrm{RS}}{\mathrm{RQ}} = \frac{\mathrm{RS}}{\sqrt{69}}$

$\mathrm{RS} = 3\sqrt{3}$

$\sin\theta = \frac{\sqrt{14}}{\sqrt{23}} = \frac{\mathrm{QS}}{\sqrt{69}}$

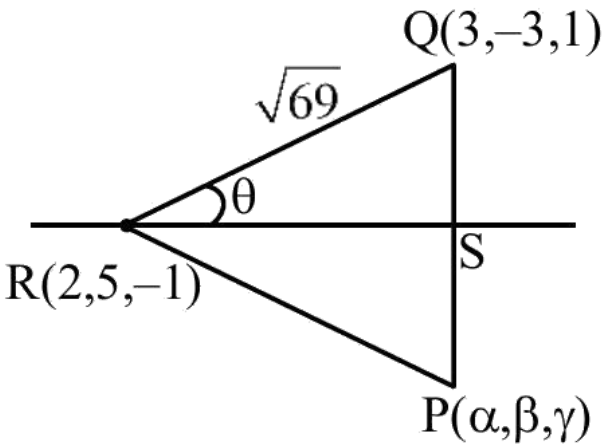
$\mathrm{QS} = \sqrt{42}$

area $= \frac{1}{2} \cdot 2\mathrm{QS} \cdot \mathrm{RS} = \sqrt{42} \cdot 3\sqrt{3}$

$\lambda = 9\sqrt{14}$

$\lambda^2 = 81.14 = 14\mathrm{k}$

$\mathrm{k} = 81$



Question 193

Let $\mathrm{P}(\alpha, \beta, \gamma)$ be the image of the point $\mathrm{Q}(3, -3, 1)$ in the line $\frac{x-0}{1} = \frac{y-3}{1} = \frac{z-1}{-1}$ and R be the point $(2, 5, -1)$. If the area of the triangle PQR is λ and $\lambda^2 = 14\,\mathrm{K}$, then K is equal to:

A. 36

B. 72

C. 18

D. 81

Correct answer: D

Source solution

Given:

1. Calculate RQ:

$$\mathrm{RQ} = \sqrt{1 + 64 + 4} = \sqrt{69}$$

2. Define vectors:

$$\overrightarrow{\mathrm{RQ}} = \hat{\ell} - 8\hat{\mathrm{j}} + 2\hat{\mathrm{k}}$$

$$\overrightarrow{\mathrm{RS}} = \hat{\ell} + \hat{\mathrm{j}} - \hat{\mathrm{k}}$$

3. Calculate $\cos\theta$:

$$\cos\theta = \frac{\overrightarrow{\mathrm{RQ}} \cdot \overrightarrow{\mathrm{RS}}}{|\overrightarrow{\mathrm{RQ}}||\overrightarrow{\mathrm{RS}}|}$$

$$= \left| \frac{1 - 8 - 2}{\sqrt{69}\sqrt{3}} \right| = \frac{9}{3\sqrt{23}}$$

4. Simplify $\cos\theta$:

$$\cos\theta = \frac{3}{\sqrt{23}} = \frac{\text{RS}}{\text{RQ}} = \frac{\text{RS}}{\sqrt{69}}$$

$$\Rightarrow \text{RS} = 3\sqrt{3}$$

5. Calculate $\sin\theta$:

$$\sin\theta = \frac{\sqrt{14}}{\sqrt{23}} = \frac{\text{QS}}{\sqrt{69}}$$

$$\Rightarrow \text{QS} = \sqrt{42}$$

6. Calculate the area:

$$\text{Area} = \frac{1}{2} \cdot 2\text{QS} \cdot \text{RS} = \sqrt{42} \cdot 3\sqrt{3}$$

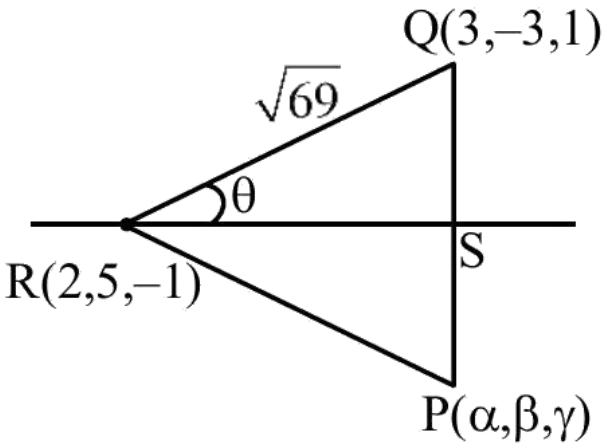
7. Calculate λ:

$$\lambda = 9\sqrt{14}$$

8. Calculate λ^2:

$$\lambda^2 = 81.14 = 14\text{k}$$

$$\Rightarrow k = 81$$



Question 194

Let $\mathbf{P}(\alpha, \beta, \gamma)$ be the image of the point $\mathbf{Q}(1, 6, 4)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Then $2\alpha + \beta + \gamma$ is equal to
.

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 11

Source solution

\(\frac{\}

$\mathbf{Q}(1, 6, 4)$
$\mathbf{A}\left(\frac{17}{14}, \frac{48}{14}, \frac{79}{14}\right)$
$\frac{\mathrm{x}}{1} = \frac{\mathrm{y}-1}{2} = \frac{\mathrm{z}-2}{3}$
$\mathbf{P}(\alpha, \beta, \gamma) \quad \overrightarrow{\mathrm{b}} = \hat{\mathrm{i}} + 2\hat{\mathrm{j}} + 3\hat{\mathrm{k}}$

\}\{4\}

$\mathbf{A}(\mathrm{t}, 2\mathrm{t}+1, 3\mathrm{t}+2)$
$\overrightarrow{\mathbf{QA}} = (\mathrm{t}-1)\hat{\mathrm{i}} + (2\mathrm{t}-5)\hat{\mathrm{j}} + (3\mathrm{t}-2)\hat{\mathrm{k}}$
$\overrightarrow{\mathbf{QA}} \cdot \overrightarrow{\mathrm{b}} = 0$
$(\mathrm{t}-1) + 2(2\mathrm{t}-5) + 3(3\mathrm{t}-2) = 0$
$14\mathrm{t} = 17$
$\alpha = \frac{20}{14} \quad \beta = \frac{12}{14} \quad \gamma = \frac{102}{14}$
$2\alpha + \beta + \gamma = \frac{154}{14} = 11$

Question 195

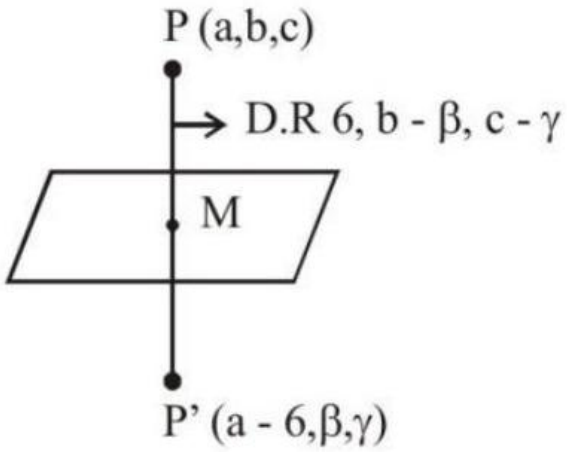
Let the mirror image of the point ($\mathbf{a, b, c}$) with respect to the plane $3x - 4y + 12z + 19 = 0$ be ($\mathbf{a - 6}, \beta, \gamma$). If $\mathbf{a + b + c = 5}$, then $7\beta - 9\gamma$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 137

Source solution



$\mathbf{M} = \left(a - 3, \frac{\beta + b}{2}, \frac{\gamma + c}{2}\right)$

Since M lies on $3x + 4y + 12z + 19 = 0$

$$\Rightarrow 6a - 4b + 12c - 4\beta + 12\gamma + 20 = 0 \quad (1)$$

Since PP' is parallel to normal of the plane then

$\frac{6}{3} = \frac{b - \beta}{-4} = \frac{c - \gamma}{12}$

$\Rightarrow \beta = b + 8, \quad \gamma = c - 24$

$a + b + c = 5 \Rightarrow a + \beta - 8 + \gamma + 24 = 5$

$\Rightarrow a = -\beta - \gamma - 11$

Now putting these values in (1) we get

$$6(-\beta-\gamma-11)-4(\beta-8)+12(\gamma+24)-4\beta+12\gamma+20=0$$
$$\Rightarrow 7\beta-9\gamma=170-33=137$$

Question 196

If the line $\frac{2-x}{3} = \frac{3y-2}{4\lambda+1} = 4 - z$ makes a right angle with the line $\frac{x+3}{3\mu} = \frac{1-2y}{6} = \frac{5-z}{7}$, then $4\lambda + 9\mu$ is equal to :

A. 13

B. 4

C. 5

D. 6

Correct answer: D

Source solution

Sol.

We start with the equations:

$$\frac{2-x}{3} = \frac{3y-2}{4\lambda+1} = 4 - z$$

$$\frac{x-2}{-3} = \frac{y-\frac{2}{3}}{\frac{4\lambda+1}{3}} = \frac{z-4}{-1}$$

From the second set of equations, we have:

$$\frac{x+3}{3\mu} = \frac{1-2y}{6} = \frac{5-z}{7}$$

This can be rewritten as:

$$\frac{x+3}{3\mu} = \frac{y-\frac{1}{2}}{-3} = \frac{z-5}{-7}$$

Using the right angle condition, we set up the equation:

$$(-3)(3\mu) + \left(\frac{4\lambda + 1}{3}\right)(-3) + (-1)(-7) = 0$$

Simplifying this gives:

$$-9\mu - 4\lambda - 1 + 7 = 0$$

Thus, we arrive at:

$$4\lambda + 9\mu = 6$$

Question 197

Let (α, β, γ) be the point $(8, 5, 7)$ in the line

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{5}.$$

Then $\alpha + \beta + \gamma$ is equal to

A. 16

B. 18

C. 14

D. 20

Correct answer: B

Source solution

$$\text{Given: } \overrightarrow{AM} \cdot (2\hat{i} + 3\hat{j} + 5\hat{k}) = 0$$

$$\text{Let } \overrightarrow{AM} = (2\lambda - 7)\hat{i} + (3\lambda - 6)\hat{j} + (5\lambda - 5)\hat{k}$$

$$\text{Setting up the dot product: } (2\lambda - 7)(2) + (3\lambda - 6)(3) + (5\lambda - 5)(5) = 0$$

$$\text{Expanding: } 4\lambda - 14 + 9\lambda - 18 + 25\lambda - 25 = 0$$

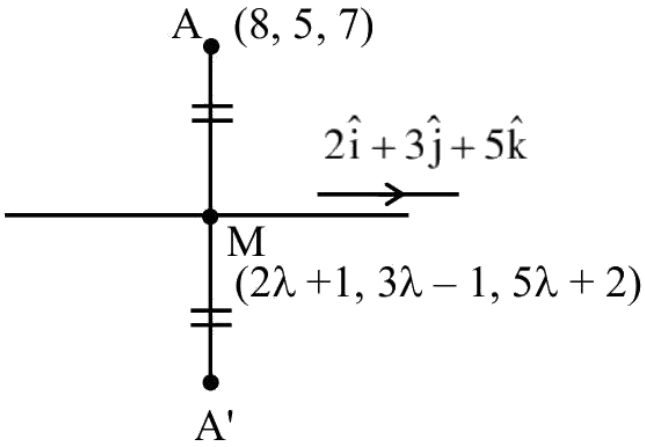
$$\text{Combining like terms: } (4\lambda + 9\lambda + 25\lambda) - 57 = 0$$

$$38\lambda = 57$$

$$\lambda = \frac{57}{38} = \frac{3}{2}$$

$$\text{Coordinates of } M: \quad M\left(4, \frac{7}{2}, \frac{19}{2}\right)$$

$$\text{Coordinates of } A': \quad A'(0, 2, 12)$$



Question 198

The distance of the point $\mathbf{P(3, 4, 4)}$ from the point of intersection of the line joining the points. $\mathbf{Q(3, -4, -5)}$ and $\mathbf{R(2, -3, 1)}$ and the plane $\mathbf{2x + y + z = 7}$, is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 7

The source provides a brief answer only. A detailed worked solution is not available for this question.

Source solution

Question 199

Let a line passing through the point $(4, 1, 0)$ intersect the line $L_1; \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ at the point $\mathrm{A} \quad (\alpha, \quad \beta, \quad \gamma)$ and the line $\mathrm{L_2 : x - 6 = y = -z + 4}$ at the point $\mathrm{B(a, b, c)}$.
Then $\begin{vmatrix} 1 & 0 & 1\alpha \\ \beta & \gamma a & b \\ c & & \end{vmatrix}$ is equal to

A. 8

B. 16

C. 12

D. 6

Correct answer: A

Source solution

Sol.}

$\mathrm{L_1 = \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = p}$

$\mathrm{L_2 = \frac{x-6}{1} = \frac{y}{1} = \frac{z-4}{-1} = q}$

$\mathrm{A(2p + 1, 3p + 2, 4p + 3)}$

$\mathrm{B(q + 6, q, 4 - q)}$

D.R. of $\mathrm{PA = 2p - 3, 3P + 1, 4p + 3}$

D.R. of $\mathrm{PB = q + 2, q - 1, 4 - q}$

$\frac{2p-3}{q+2} = \frac{3p+1}{q-1} = \frac{4p+3}{4-q}$

$2pq - 2p - 3q + 3 = 3pq + 6p + q + 2$

$$\mathrm{pq + rp + 4q - 1 = 0} \tag{1}$$

$12p - 3pq + 4 - q = 4pq + 3q - 4p - 3$

$$7pq - 16p + 4q - 7 = 0 \tag{2}$$

$8p - 2pq - 12 + 3q = 4pq + 8p + 3q + 6$

$6pq = -18 \quad \therefore pq = -3$

$\mathrm{8p + 4q = 4 \quad \Rightarrow 2p + q = 1}$

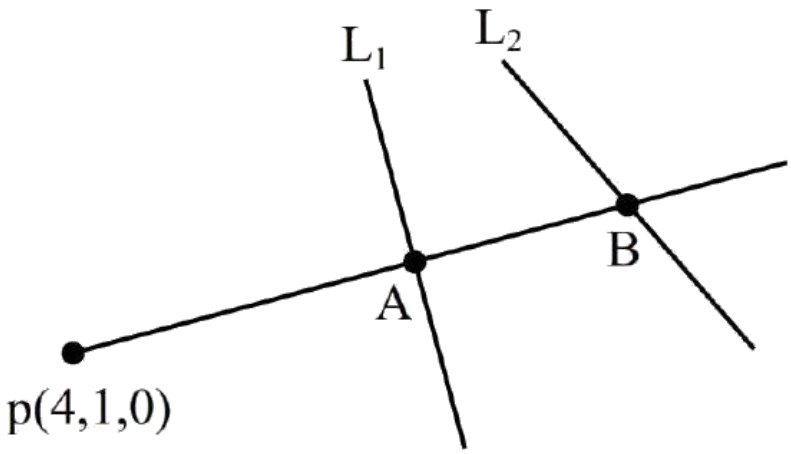
$-21-16p+4q-7 \quad \Rightarrow 4p-q=-7$

$16p-4q=-28 \quad \therefore p=-1, q=3$

$\mathrm{A}(-1,-1,-1) \quad \mathrm{B}(9,3,1)$

$|1 \quad 0 \quad 1-1 \quad -1 \quad -19 \quad 3 \quad 1| = |0 \quad -1 \quad 0-1 \quad -1 \quad -19 \quad 3 \quad 1| = 1(-1+9) = 8$

Option (1)



Question 200

Let $L_1 : \frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0}$ and
$\mathbf{L_2 : \frac{x-2}{2} = \frac{y}{0} = \frac{z+4}{\alpha}, \alpha \in \mathbf{R}}$, be two lines, which intersect at the point B . If P is the foot of perpendicular from the point $\mathbf{A(1, 1, -1)}$ on $\mathbf{L_2}$, then the value of $\mathbf{26\alpha(\,PB)^2}$ is .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 216

Source solution

Point B
$\mathbf{(3\lambda + 1, -\lambda + 1, -1) \equiv (2\mu + 2, 0, \alpha\mu - 4)}$
$\mathbf{3\lambda + 1 = 2\mu + 2}$
$\mathbf{-\lambda + 1 = 0}$
$\mathbf{-1 = \alpha\mu - 4}$
$\mathbf{\lambda = 1, \mu = 1, \alpha = 3}$
B(4, 0, -1)
Let Point ' P ' is $\mathbf{(2\delta + 2, 0, 3\delta - 4)}$
Dr's of AP $\mathbf{< 2\delta + 1, -1, 3\delta - 3 >}$
$\mathbf{AP \perp L_2 \Rightarrow \delta = \frac{7}{13}}$
$\mathbf{P\left(\frac{40}{13}, 0, \frac{-31}{13}\right)}$
$\mathbf{2\sigma\delta(\,PB)^2 = 26 \times 3 \times \left(\frac{144}{169} + \frac{324}{169}\right)}$
$\mathbf{= 216}$

Question 201

The lines $\frac{x-2}{2}=\frac{y}{-2}=\frac{z-7}{16}$ and $\frac{x+3}{4}=\frac{y+2}{3}=\frac{z+2}{1}$ intersect at the point P. If the distance of P from the line $\frac{x+1}{2}=\frac{y-1}{3}=\frac{z-1}{1}$ is l, then $14l^2$ is equal to.

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 108

Source solution

$\frac{x-2}{1}=\frac{y}{-1}=\frac{z-7}{8}=\lambda$

$\frac{x+3}{4}=\frac{y+2}{3}=\frac{z+2}{1}=k$

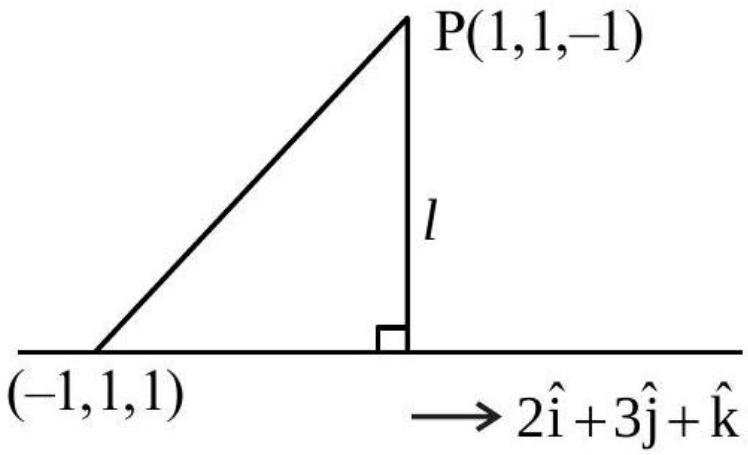
$\Rightarrow \lambda+2=4k-3$

$-\lambda=3k-2$

$\Rightarrow k=1, \lambda=-1$

$8\lambda+7=k-2$

$\therefore P=(1,1,-1)$



Projection of $2\hat{i}-2\hat{k}$ on $2\hat{i}+3\hat{j}+\hat{k}$ is

$=\frac{4-2}{\sqrt{4+9+1}}=\frac{2}{\sqrt{14}}$

$\therefore l^2=8-\frac{4}{14}=\frac{108}{14}$

$\Rightarrow 14l^2=108$

Question 202

Let a line passing through the point $(4,1,0)$ intersect the line $L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ at the point (α, β, γ) and the line $L_2 : x-6 = y = -z+4$ at the point (a,b,c). Then

$$\begin{vmatrix} 1 & 0 & 1 \\ \alpha & \beta & \gamma \\ a & b & c \end{vmatrix}$$

is equal to

A. 8

B. 16

C. 12

D. 6

Correct answer: A

Source solution

$$\text{Let } L_1 = \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = p$$

$$L_2 = \frac{x-6}{1} = \frac{y}{1} = \frac{z-4}{-1} = q$$

$$A(2p+1, 3p+2, 4p+3)$$

$$B(q+6, q, 4-q)$$

$$\text{Direction Ratios of } PA : (2p-3, 3p+1, 4p+3)$$

$$\text{Direction Ratios of } PB : (q+2, q-1, 4-q)$$

$$\frac{2p-3}{q+2} = \frac{3p+1}{q-1} = \frac{4p+3}{4-q}$$

$$2pq-2p-3q+3=3pq+6p+q+2$$

$$pq+rp+4q-1=0 \tag{1}$$

$$12p-3pq+4-q=4pq+3q-4p-3$$

$$7pq-16p+4q-7=0 \tag{2}$$

$$8p-2pq-12+3q=4pq+8p+3q+6$$

$$6pq=-18 \quad \Rightarrow \quad pq=-3$$

$$8p+4q=4 \quad \Rightarrow \quad 2p+q=1$$

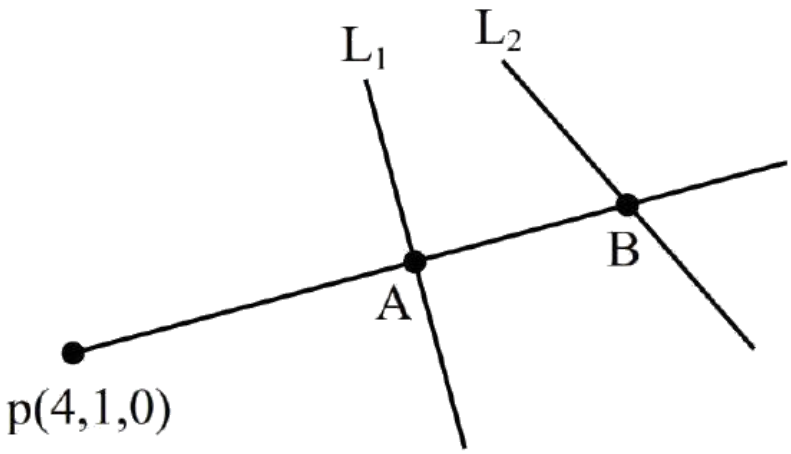
$$-21-16p+4q-7 \quad \Rightarrow \quad 4p-q=-7$$

$$16p-4q=-28 \quad \Rightarrow \quad p=-1,\ q=3$$

$$A(-1,-1,-1) \quad B(9,3,1)$$

$$\begin{vmatrix} 1 & 0 & 1 \\ -1 & -1 & -1 \\ 9 & 3 & 1 \end{vmatrix} = \begin{vmatrix} 0 & -1 & 0 \\ -1 & -1 & -1 \\ 9 & 3 & 1 \end{vmatrix} = 1(-1+9)=8$$

Option (1)


L1
L2
B
A
p(4,1,0)

Question 203

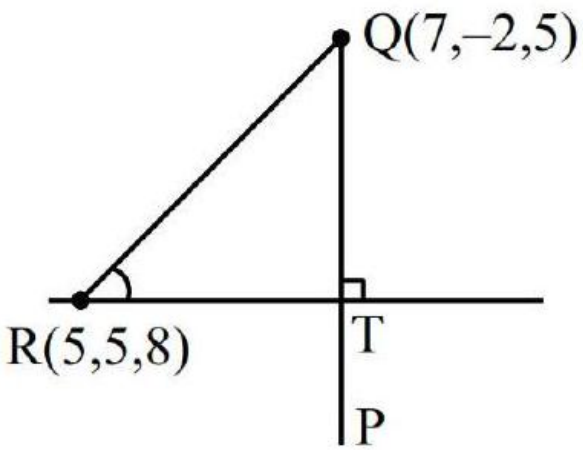
Let P be the image of the point $\mathbf{Q(7,-2,5)}$ in the line $\mathbf{L:\frac{x-1}{2}=\frac{y+1}{3}=\frac{z}{4}}$ and $\mathbf{R(5,p,q)}$ be a point on L. Then the square of the area of $\mathbf{\triangle PQR}$ is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 957

Source solution



Let $\mathbf{R(2\lambda+1,3\lambda-1,4\lambda)}$

$\mathbf{2\lambda+1=5}$

$\mathbf{\lambda=2}$

R(5, 5, 8)

let $\mathbf{T(2\lambda+1,3\lambda-1,4\lambda)}$

$\mathbf{\overrightarrow{QT}=(2\lambda-6)\hat{i}+(3\lambda+1)\hat{j}+(4\lambda-5)\hat{k}}$

$\mathbf{\vec{b}=2\hat{i}+3\hat{j}+4\hat{k}}$

$\mathbf{\overrightarrow{QT}\cdot\vec{b}=0}$

$\mathbf{4\lambda-12+9\lambda+3+16\lambda-20=0}$

$\mathbf{\lambda=1}$

$\mathbf{T(3,2,4)}$

$\mathbf{QT=\sqrt{33}\quad RT=\sqrt{29}}$

$$
\begin{aligned}
&(\text{ area of } \triangle \mathrm{PQR})^2=\left(\frac{1}{2} \sqrt{29} .2 \sqrt{33}\right)^2 \\
&=957
\end{aligned}
$$

Question 204

Let L be a line obtained from the intersection of two planes $x + 2y + z = 6$ and $y + 2z = 4$. If point $P(\alpha, \beta, \gamma)$ is the foot of perpendicular from $(3, 2, 1)$ on L, then the value of $21(\alpha + \beta + \gamma)$ equals :

A. 142

B. 68

C. 136

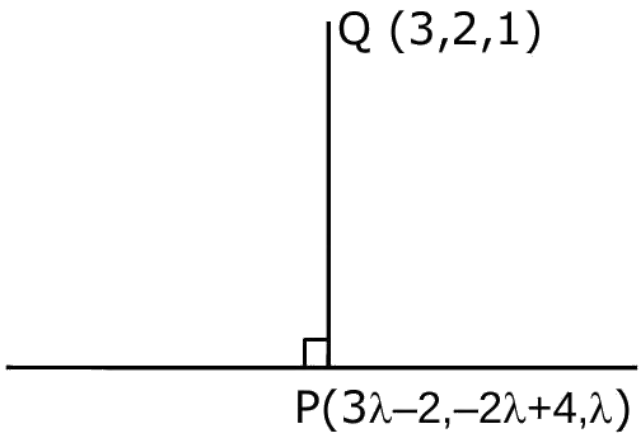
D. 102

Correct answer: D

Source solution

Sol. Dr's of line $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix} = 3\hat{i} - 2\hat{j} + \hat{k}$
Dr/s :- $(3, -2, 1)$
Points on the line $(-2, 4, 0)$
Equation of the line $\frac{x+2}{3} = \frac{y-4}{-2} = \frac{z}{1} = \lambda$

Dr's of PQ ; $3\lambda - 5, -2\lambda + 2. \lambda - 1$
Dr's of y lines are $(3, -2, 1)$
Since $\mathbf{PQ} \perp$ line
$3(3\lambda - 5) - 2(-2\lambda + 2) + 1(\lambda - 1) = 0$
$\lambda = \frac{10}{7}$
$P\left(\frac{16}{7}, \frac{8}{7}, \frac{10}{7}\right)$
$21(\alpha + \beta + \gamma) = 21\left(\frac{34}{7}\right) = 102$



Question 205

Consider the lines $\mathbf{L_1 : x-1=y-2=z}$ and $\mathbf{L_2 : x-2=y=z-1}$. Let the feet of the perpendiculars from the point $\mathbf{P(5,1,-3)}$ on the lines $\mathbf{L_1}$ and $\mathbf{L_2}$ be Q and R respectively. If the area of the triangle PQR is A , then $\mathbf{4\,A^2}$ is equal to :

A. 139

B. 147

C. 151

D. 143

Correct answer: B

Source solution

Sol. $\mathbf{L_1 : \frac{x-1}{1}=\frac{y-2}{1}=\frac{z-0}{2}}$

Let $\mathbf{Q(\lambda+1,\lambda+2,\lambda)}$

$\mathbf{\overrightarrow{PQ}=(\lambda-4,\lambda-1,\lambda+3)}$

$\mathbf{\overrightarrow{PQ}\cdot\vec{m}=0}$

$\mathbf{\Rightarrow \lambda-4+\lambda+1,\lambda+3=0}$

$\mathbf{\Rightarrow 3\lambda=0}$

$\mathbf{\lambda=0}$

$\mathbf{\Rightarrow Q(1,2,0)}$

$\mathbf{L_2 : \frac{x-2}{1}=\frac{y-0}{1}=\frac{z-1}{2}}$

Let $\mathbf{R(\mu+2,\mu,\mu+1)}\mathbf{\overrightarrow{PR}=(\mu-3,\mu-1,\mu+4)}$

$\mathbf{\overrightarrow{PR}.\vec{n}=0}$

$\mathbf{\mu-3+\mu-1+\mu+4=0}$

$\mathbf{\neq \mu=0}$

R(2,0,1)

Area of $\mathbf{\triangle PQR(A)=\frac{1}{2}|\overrightarrow{PQ}\times\overrightarrow{PR}|}$

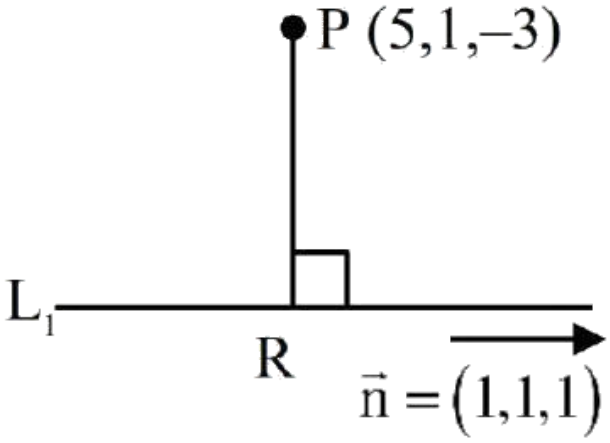
$\mathbf{A=\frac{1}{2}|(-4\hat{i}+\hat{j}+3\hat{k})\times(-3\hat{i}+\hat{j}+4\hat{k})|}$

$\mathbf{A=\frac{1}{2}|7(\hat{i}+\hat{j}+\hat{k})|}$

$\mathbf{\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -4 & 1 & 3\\ -3 & -1 & 4\end{vmatrix}}$

$\mathbf{=7\hat{i}+7\hat{j}+7\hat{k}}$

$\mathbf{4\,A^2=49\times 3=147}$


P (5,1,–3)
L1
R
$\vec{n} = (1,1,1)$

Question 206

If the mirror image of the point $P(3,4,9)$ in the line $\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-2}{1}$ is (α,β,γ), then $14(\alpha+\beta+\gamma)$ is :

A. 102

B. 138

C. 108

D. 132

Correct answer: C

Source solution

$$\text{Given: } \overrightarrow{\text{PN}} \cdot \vec{\text{b}} = 0$$

$$3(3\lambda-2)+2(2\lambda-5)+(\lambda-7)=0$$

$$9\lambda-6+4\lambda-10+\lambda-7=0$$

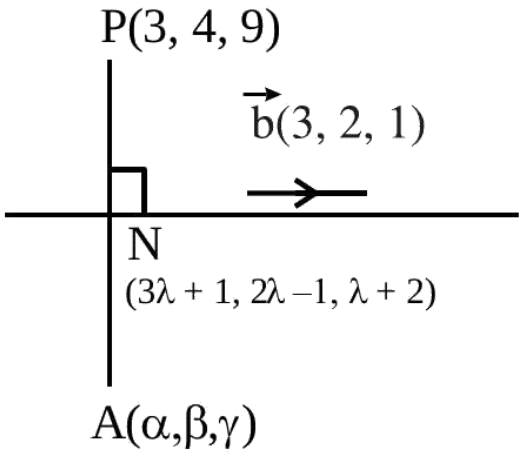
$$14\lambda-23=0 \quad \Rightarrow \quad 14\lambda=23 \quad \Rightarrow \quad \lambda=\frac{23}{14}$$

$$\text{Coordinates of N} = \left(\frac{83}{14},\frac{32}{14},\frac{51}{14}\right)$$

$$\frac{\alpha+3}{2}=\frac{83}{14} \quad \Rightarrow \quad \alpha=\frac{62}{7}$$

$$\frac{\beta+4}{2}=\frac{32}{14} \quad \Rightarrow \quad \beta=\frac{4}{7}$$

$$\frac{\gamma+9}{2}=\frac{51}{14} \quad \Rightarrow \quad \gamma=\frac{-12}{7}$$



Question 207

Let P and Q be the points on the line

$$\frac{x+3}{8}=\frac{y-4}{2}=\frac{z+1}{2}$$

which are at a distance of 6 units from the point $\mathbf{R(1,2,3)}$. If the centroid of the triangle PQR is (α,β,γ), then $\alpha^2+\beta^2+\gamma^2$ is:

A. 26

B. 36

C. 18

D. 24

Correct answer: C

Source solution

$$\text{Let P}(8\lambda-3,2\lambda+4,2\lambda-1)$$

$$\text{Given PR}=6$$

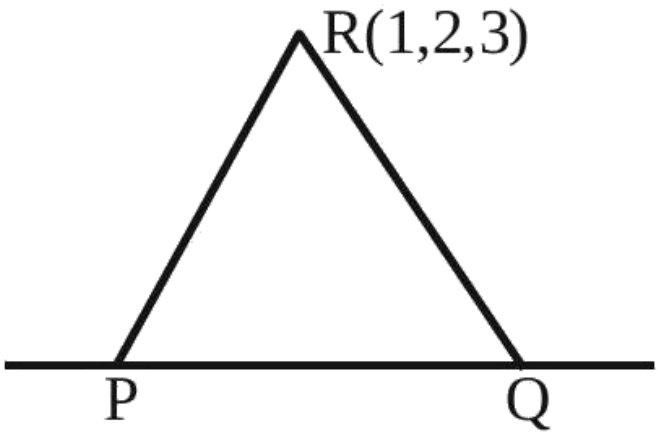
$$(8\lambda-4)^2+(2\lambda+2)^2+(2\lambda-4)^2=36$$

$$\text{Solving gives } \lambda=0 \text{ or } 1$$

$$\text{Thus, P}(-3,4,-1) \text{ and Q}(5,6,1)$$

$$\text{Centroid of } \triangle\text{PQR}=(1,4,1)\equiv(\alpha,\beta,\gamma)$$

$$\alpha^2+\beta^2+\gamma^2=18$$


R(1,2,3)
P
Q

Question 208

If the mirror image of the point $\mathbf{P(3,4,9)}$ in the line $\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-2}{1}$ is (α,β,γ), then $\mathbf{14(\alpha+\beta+\gamma)}$ is :

A. 102

B. 138

C. 108

D. 132

Correct answer: C

Source solution

Sol.

$\overrightarrow{\mathbf{PN}}.\vec{\mathbf{b}}=\mathbf{0}$?

$3(3\lambda-2)+2(2\lambda-5)+(\lambda-7)=0$

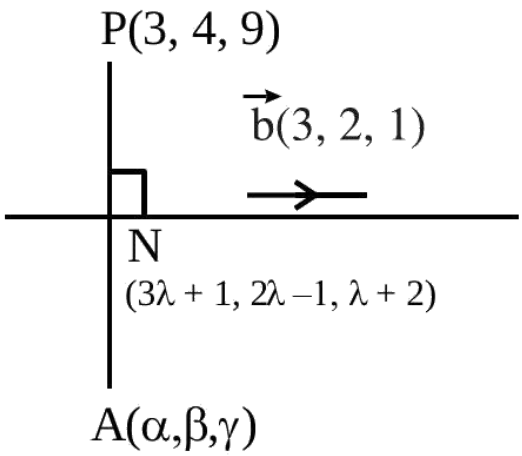
$14\lambda=23\Rightarrow\lambda=\frac{23}{14}$

$\mathbf{N}\left(\frac{83}{14},\frac{32}{14},\frac{51}{14}\right)$

$\therefore\ \frac{\alpha+3}{2}=\frac{83}{14}\Rightarrow\alpha=\frac{62}{7}$

$\frac{\beta+4}{2}=\frac{32}{14}\Rightarrow\beta=\frac{4}{7}$

$\frac{\gamma+9}{2}=\frac{51}{14}\Rightarrow\gamma=\frac{-12}{7}$



Question 209

Let $a, b \in \mathbb{R}$. If the mirror image of the point $P(a, 6, 9)$ with respect to the line $\frac{x-3}{7} = \frac{y-2}{5} = \frac{z-1}{-9}$ is $(20, b, -a-9)$, then $|a+b|$ is equal to :

A. 86

B. 88

C. 84

D. 90

Correct answer: B

Source solution

Sol.

Let $P(a, 6, 9)$ and $Q(20, b, -a-9)$.

The midpoint of $\mathbf{PQ}$ is given by:

$$\left(\frac{a+20}{2}, \frac{b+6}{2}, -\frac{a}{2}\right)$$

This midpoint lies on the line defined by:

$$\frac{\frac{a+20}{2} - 3}{7} = \frac{\frac{b+6}{2} - 2}{5} = \frac{-\frac{a}{2} - 1}{-9}$$

Simplifying each part, we have:

$$\frac{a+20-6}{14} = \frac{b+6-4}{10} = \frac{-a-2}{-18}$$

This leads to:

$$\frac{a+14}{14} = \frac{b+2}{10}$$

Cross-multiplying gives:

$$10(a+14) = 14(b+2)$$

Expanding both sides:

$$10a + 140 = 14b + 28$$

Rearranging terms results in:

$$10a - 14b = -112$$

Now, substituting b in terms of a:

From the equation:

$$\frac{b+2}{10} = \frac{a+2}{18}$$

Cross-multiplying yields:

$$18(b+2) = 10(a+2)$$

Expanding gives:

$$18b + 36 = 10a + 20$$

Rearranging leads to:

$$18b = 10a - 16$$

Now substituting $a = -56$:

$$18b = 10(-56) - 16$$

Calculating:

$$18b = -560 - 16 = -576$$

Thus:

$$b = -32$$

Finally, we find:

$$|\mathrm{a} + \mathrm{b}| = |-56 - 32| = 88$$

Question 210

Let P and Q be the points on the line $\frac{x+3}{8} = \frac{y-4}{2} = \frac{z+1}{2}$ which are at a distance of 6 units from the point $\mathbf{R(1,2,3)}$. If the centroid of the triangle PQR is (α, β, γ), then $\alpha^2 + \beta^2 + \gamma^2$ is:

A. 26

B. 36

C. 18

D. 24

Correct answer: C

Source solution

Sol.

$\mathbf{P(8\lambda - 3, 2\lambda + 4, 2\lambda - 1)}$

$\mathbf{PR = 6}$

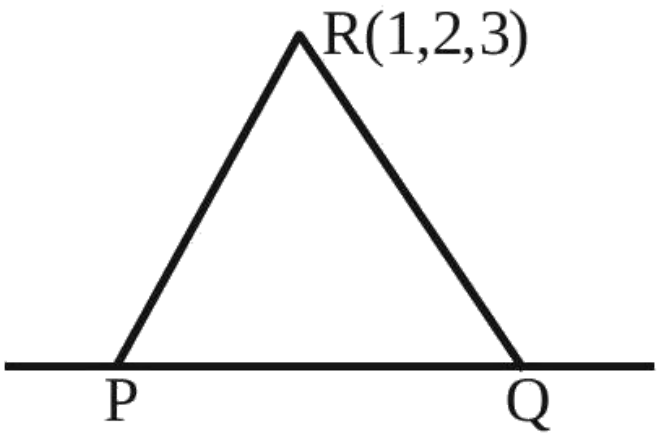
$\mathbf{(8\lambda - 4)^2 + (2\lambda + 2)^2 + (2\lambda - 4)^2 = 36}$

$\boldsymbol{\lambda = 0, 1}$

Hence $\mathbf{P(-3, 4, -1) \& Q(5, 6, 1)}$

Centroid of $\mathbf{\triangle PQR = (1, 4, 1) \equiv (\alpha, \beta, \gamma)}$

$\boldsymbol{\alpha^2 + \beta^2 + \gamma^2 = 18}$



Question 211

Consider the lines $L_1 : x - 1 = y - 2 = z$ and $L_2 : x - 2 = y = z - 1$. Let the feet of the perpendiculars from the point $P(5, 1, -3)$ on the lines L_1 and L_2 be Q and R respectively. If the area of the triangle PQR is A, then $4A^2$ is equal to:

A. 139

B. 147

C. 151

D. 143

Correct answer: B

Source solution

$$\text{Let } L_1 : \frac{x-1}{1} = \frac{y-2}{1} = \frac{z-0}{2}$$

Let $Q(\lambda + 1, \lambda + 2, \lambda)$.

$$\overrightarrow{PQ} = (\lambda - 4, \lambda - 1, \lambda + 3)$$

Since $\overrightarrow{PQ} \cdot \vec{m} = 0$:

$$\lambda - 4 + \lambda - 1 + \lambda + 3 = 0$$

This simplifies to:

$$3\lambda = 0 \quad \Rightarrow \quad \lambda = 0$$

Thus, $Q(1, 2, 0)$.

Next, consider $L_2 : \frac{x-2}{1} = \frac{y-0}{1} = \frac{z-1}{2}$.

Let $R(\mu + 2, \mu, \mu + 1)$.

$$\overrightarrow{PR} = (\mu - 3, \mu - 1, \mu + 4)$$

Since $\overrightarrow{PR} \cdot \vec{n} = 0$:

$$\mu - 3 + \mu - 1 + \mu + 4 = 0$$

This simplifies to:

$$3\mu = 0 \quad \Rightarrow \quad \mu = 0$$

Thus, $R(2, 0, 1)$.

Now, the area of triangle PQR is given by:

$$A = \frac{1}{2} |\overrightarrow{PQ} \times \overrightarrow{PR}|$$

Calculating $\overrightarrow{PQ}$ and $\overrightarrow{PR}$:

$$\overrightarrow{PQ} = (-4, 1, 3), \quad \overrightarrow{PR} = (-3, -1, 4)$$

Now, compute the cross product:

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & 1 & 3 \\ -3 & -1 & 4 \end{vmatrix}$$

Calculating the determinant:

$$= \hat{i}(1 \cdot 4 - 3 \cdot (-1)) - \hat{j}(-4 \cdot 4 - 3 \cdot (-3)) + \hat{k}(-4 \cdot (-1) - 1 \cdot (-3))$$

This results in:

$$= \hat{i}(4+3) - \hat{j}(-16+9) + \hat{k}(4+3)$$

$$= 7\hat{i} + 7\hat{j} + 7\hat{k}$$

Thus,

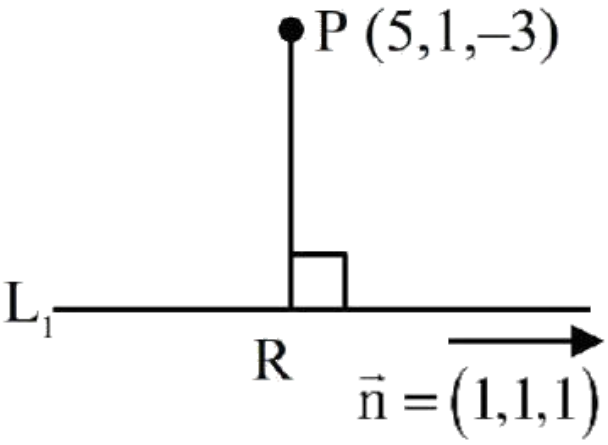
$$|\overrightarrow{PQ} \times \overrightarrow{PR}| = 7\sqrt{3}$$

Finally, the area A is:

$$A = \frac{1}{2}|7\sqrt{3}| = \frac{7\sqrt{3}}{2}$$

Therefore, the final area is:

$$4A^2 = 49 \times 3 = 147$$



Question 212

Let the line L be the projection of the line

$$\frac{x-1}{2}=\frac{y-3}{1}=\frac{z-4}{2}$$

in the plane $\mathbf{x-2y-z=3}$. If d is the distance of the point $(0,0,6)$ from L , then $\mathbf{d^2}$ is equal to

.

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

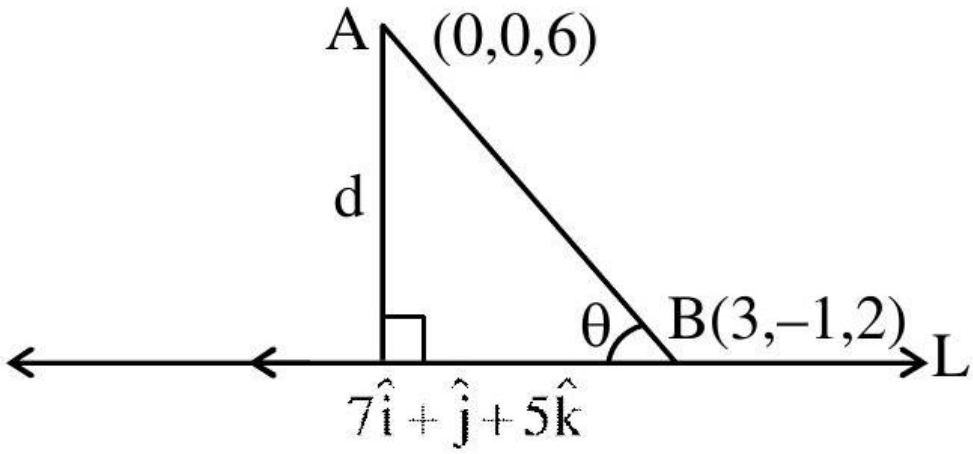
Correct answer: 26

Source solution

$\mathbf{L_1}: \frac{\mathbf{x-1}}{2}=\frac{\mathbf{y-3}}{1}=\frac{\mathbf{z-4}}{2}$
for foot of $\perp$ $\mathbf{r}$ of (1, 3, 4) on $\mathbf{x-2y-z-3=0}$
$(1+t)-2(3-2t)-(4-t)-3=0$
$\Rightarrow \mathbf{t=2}$

So foot of $\perp$ $\mathbf{r} \overset{\wedge}{=} (3,-1,2)$
& point of intersection of $\mathbf{L_1}$ with plane
is $(-11,-3,-8)$
dr's of L is $<14,2,10>$

$$\cong <7,1,5>$$



$$d = AB\sin\theta = \frac{\left|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & -4 \\ 7 & 1 & 5 \end{vmatrix}\right|}{\sqrt{7^2+1^2+5^2}}$$

$\Rightarrow d^2 = \frac{1^2+(43)^2+(10)^2}{49+1+25} = 26$

Question 213

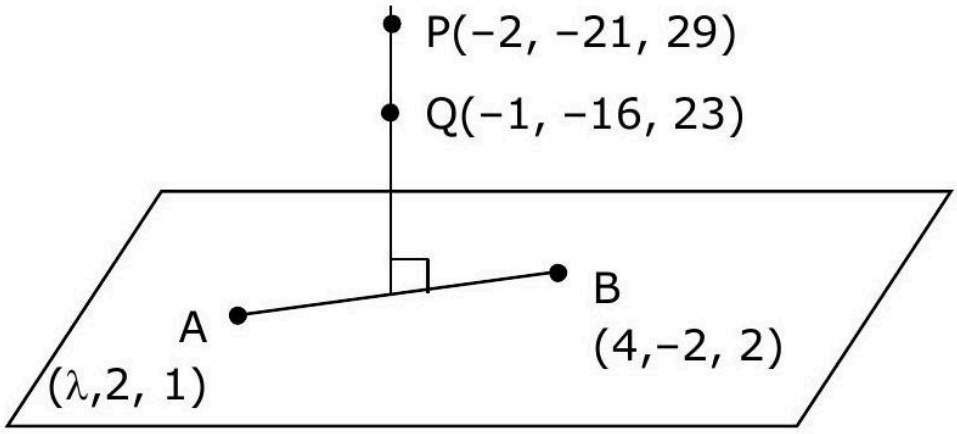
Let $(\lambda, 2, 1)$ be a point on the plane which passes through the point $(4, -2, 2)$. If the plane is perpendicular to the line joining the point $(-2, -21, 29)$ and $(-1, -16, 23)$, then $\left(\frac{\lambda}{11}\right)^2 - \frac{4\lambda}{11} - 4$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 8

Source solution



$\overrightarrow{AB} \perp \overrightarrow{PQ}$

$$[(4-\lambda)\hat{i} - 4\hat{j} + \hat{k}] \cdot [+\hat{i} + 5\hat{j} - 6\hat{k}] = 0$$
$$4 - \lambda - 20 - 6 = 0$$
$$\lambda = -22$$

Now, $\frac{\lambda}{11} = -2$

$$\Rightarrow \left(\frac{\lambda}{11}\right)^2 - \frac{4\lambda}{11} - 4$$
$$\Rightarrow 4 + 8 - 4 = 8$$

Question 214

Let the line $L : \frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{1}$ intersect the plane $2x + y + 3z = 16$ at the point P. Let the point Q be the foot of perpendicular from the point $R(1, -1, -3)$ on the line L. If α is the area of triangle PQR, then α^2 is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 180

Source solution

180

Point on line L is $(2\lambda + 1, -\lambda - 1, \lambda + 3)$

If above point is intersection point of line L and plane then

$2(2\lambda + 1) + (-\lambda - 1) + 3(\lambda + 3) = 16$

$\lambda = 1$

Point $\mathrm{P} = (3, -2, 4)$

Dr of $\mathrm{QR} = < 2\lambda, -\lambda, \lambda + 6 >$

Dr of $\mathrm{L} = \langle 2, -1, 1 \rangle$

$4\lambda + \lambda + \lambda + 6 = 0$

$\lambda = -1$

$\mathrm{Q} = (-1, 0, 2)$

$C^{P(3,-2,4)}_{(-1,0,2)}\mathrm{R}(1, -1, -3)$

$\overrightarrow{\mathrm{QR}} = 2\hat{i} - \hat{j} - 5\hat{k}$

$\overrightarrow{\mathrm{QP}} = 4\hat{i} - 2\hat{j} + 2\hat{k}$

$$\overrightarrow{\mathrm{QR}} \times \overrightarrow{\mathrm{QP}} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & -5 \\ 4 & -2 & 2 \end{vmatrix} = -12\hat{i} - 24\hat{j}$$

$\alpha = \frac{1}{2} \times \sqrt{144 + 576}$

$\alpha^2 = \frac{720}{4} = 180$

$\alpha^2 = 180$

Question 215

Let P be a plane containing the line $\frac{x-1}{3}=\frac{y+6}{4}=\frac{z+5}{2}$ and parallel to the line $\frac{x-3}{4}=\frac{y-2}{-3}=\frac{z+5}{7}$. If the point $(1,-1,\alpha)$ lies on the plane P , then the value of $|5\alpha|$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 38

Source solution

Equation of plane is $\begin{vmatrix} x-1 & y+6 & z+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix}=0$

Now $(1,-1,\alpha)$ lies on it so

$\begin{vmatrix} 0 & 5 & \alpha+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix}=0 \Rightarrow 5\alpha+38=0 \Rightarrow |5\alpha|=38$

Question 216

Let a line L pass through the point $P(2,3,1)$ and be parallel to the line $x+3y-2z-2=0=x-y+2$ z . If the distance of L from the point $(5,3,8)$ is α, then $3\alpha^2$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 158

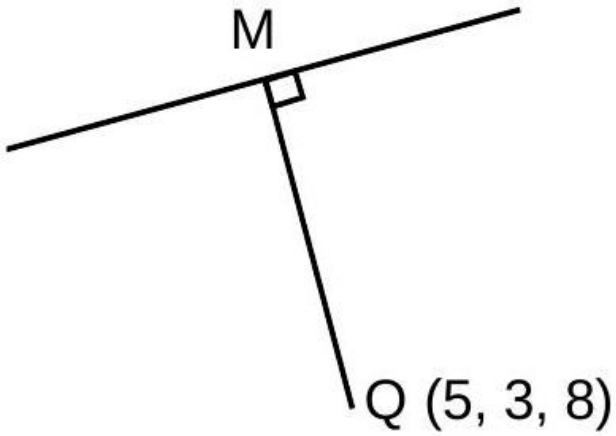
Source solution

158

The Direction ratio of line

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 3 & -2 \\ 1 & -1 & 2 \end{vmatrix} = \mathbf{i}(6-2) - \mathbf{j}(2+2) + \mathbf{k}(-1-3)$$

Equation of line L



$\frac{x-2}{1} = \frac{y-3}{-1} = \frac{z-1}{-1} = \lambda(\alpha \mathbf{d})$

Let $\mathbf{M}(\lambda+2, -\lambda+3, -\lambda+1)$

DR's of MQ is $< \lambda+2-5, -\lambda+3-3, -\lambda+1-8 >$

$$< \lambda-3, -\lambda. -\lambda > 7 >$$

$\because \mathbf{L} \perp \mathbf{MQ}$

$\Rightarrow (\lambda-3)(1) + (-\lambda)(-1) + (-\lambda-7)(-1) = 0$

$\Rightarrow \lambda - 3 + \lambda + \lambda + 7 = 0$
$\Rightarrow 3\lambda = -4 \Rightarrow \lambda = -\frac{4}{3}$
$\therefore \mathrm{M}\left(-\frac{4}{3}+2, \frac{+4}{3}+3, \frac{4}{3}+1\right) = \left(\frac{2}{3}, \frac{13}{3}, \frac{7}{3}\right)$
$\mathrm{MQ} = \alpha$
$\therefore 3\alpha^2 = 3 \times \left(\left(5-\frac{2}{3}\right)^2 + \left(3-\frac{13}{3}\right)^2 + \left(8-\frac{7}{5}\right)^2\right)$
$= 3\left(\frac{169}{9} + \frac{16}{9} + \frac{289}{9}\right) \Rightarrow \frac{474}{9} = 158$

Question 217

Let the equation of the plane P containing the line $x+10=\frac{8-y}{2}=z$ be $ax+by+3z=2(a+b)$ and the distance of the plane P from the point $(1,27,7)$ be c. Then $a^2+b^2+c^2$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 355

Source solution

355
Given equation of plane is

$$ax+by+3z=2(a+b) \tag{1}$$

It containing the line
$\frac{x-(-10)}{1}=\frac{y-8}{-2}=\frac{z-0}{1}$
$\therefore$ plane (1) must passes through $(-10,8,0)$ and parallel to $1,-2,1$
Hence,

$$\begin{aligned} & a(-10)+8b=2a+2b \\ \Rightarrow \quad & 12a-6b=0 \\ \text{and} \quad & a-2b+3=0 \end{aligned} \tag{3}$$

on solving (2) and (3), we get

$$b=2, a=1$$

$\therefore$ equation of the plane is

$$x + 2y + 3z = 6 \quad (4)$$

c is perpendicular distance from $(1, 27, 7)$ to the plane (4)
$\Rightarrow c = \left|\frac{1+2\times27+3\times7-6}{\sqrt{1^2+2^2+3^2}}\right| = \left|\frac{70}{\sqrt{14}}\right| = \frac{10\sqrt{7}}{\sqrt{2}}$
Now, $\mathrm{a}^2 + \mathrm{b}^2 + \mathrm{c}^2 = 1 + 4 + \frac{700}{2} = \frac{710}{2} = 355$

Question 218

Let the plane P pass through the intersection of the planes $2x + 3y - z = 2$ and $x + 2y + 3z = 6$ and be perpendicular to the plane $2\mathrm{x} + \mathrm{y} - \mathrm{z} = 0$. If d is the distance of P form the point ($-7, 1, 1$,) then d^2 is equal to :

A. $\frac{250}{83}$

B. $\frac{250}{82}$

C. $\frac{15}{53}$

D. $\frac{25}{83}$

Correct answer: A

Source solution

Sol. 1
Plane $\mathbf{P}$, is passing through intersection of the two planes, so,
$2x + 3y - z - 2 + \lambda(x + 2y + 3z - 6) = 0$
$x(2 + \lambda) + y(3 + 2\lambda) + z(3\lambda - 1) - 2 - 6\lambda = 0$
It is perpendicular with plane, $2x + y - 2 + 1 = 0$
So, $(2 + \lambda)2 + (3 + 2\lambda)1 + (3\lambda - 1)(-1) = 0$
$\lambda = -8$
So, plane $p_1 - 6x - 13y - 25z + 46 = 0$
distance of plane p from the point ($-7, 1, 1$)
$d = \frac{|+42-13-25+46|}{\sqrt{36+169+625}} = \frac{50}{\sqrt{30}} =$
$\mathrm{d}^2 = \frac{2500}{830} = \frac{250}{83}$

Question 219

If the shortest distance between the line joining the points $(1, 2, 3)$ and $(2, 3, 4)$, and the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-2}{0}$ is a, then $28a^2$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 18

Source solution

18

A (1, 2, 3)

B $(2, 3, 4)$

Equation of line AB

$\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{1}$

Given line

$\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-2}{0}$

shortest distance $= \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_2 \times \vec{b}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$

$= \left| \frac{(3\hat{j} - \hat{k}) \cdot (\hat{i} + 2\hat{j} - 3\hat{k})}{\sqrt{1+4+9}} \right|$

$\alpha = \frac{3}{\sqrt{14}}$

$28\alpha^2 = 28 \times \frac{9}{14} = 18$

Question 220

If (x, y, z) be an arbitrary point lying on a plane P which passes through the point $(42, 0, 0)$, $(0, 42, 0)$ and $(0, 0, 42)$, then the value of expression

$$3+\frac{x-11}{(y-19)^2(z-12)^2}+\frac{y-19}{(x-11)^2(z-12)^2}+\frac{z-12}{(x-11)^2(y-19)^2}-\frac{x+y+z}{14(x-11)(y-19)(z-12)}$$

A. 0

B. 3

C. 39

D. -45

Official Answer by NTA (2)

Correct answer: B

Source solution

Sol.

The plane passes through the points $(42, 0, 0)$, $(0, 42, 0)$, and $(0, 0, 42)$.

1. The equation of the plane can be derived from the intercept form:

$$x+y+z=42$$

2. We can rewrite this as:

$$(x-11)+(y-19)+(z-12)=0$$

3. Let:

$$a=x-11, \quad b=y-19, \quad c=z-12$$

Thus, we have:

$$a+b+c=0$$

4. Now, consider the given expression:

$$3+\frac{a}{b^2c^2}+\frac{b}{a^2c^2}+\frac{c}{a^2b^2}-\frac{42}{14abc}$$

5. This can be simplified to:

$$3+\frac{a^3+b^3+c^3-3abc}{a^2b^2c^2}$$

6. Since $a+b+c=0$, we can use the identity:

$$a^3+b^3+c^3=3abc$$

7. Therefore, we have:

$$a^3+b^3+c^3-3abc=0$$

8. Substituting this back, we find:

$$3+0=3$$

Final answer:

$$3$$

Question 221

If (x, y, z) be an arbitrary point lying on a plane P which passes through the point $(42, 0, 0)$, $(0, 42, 0)$ and $(0, 0, 42)$, then the value of expression

$3+\frac{x-11}{(y-19)^2(z-12)^2}+\frac{y-19}{(x-11)^2(z-12)^2}$
$+\frac{z-12}{(x-11)^2(y-19)^2}-\frac{x+y+z}{14(x-11)(y-19)(z-12)}$

A. 0

B. 3

C. 39

D. -45

Official Answer by NTA (2)

Correct answer: B

Source solution

Sol. Plane passing through $(42, 0, 0), (0, 42, 0)$, (0, 0, 42)

From intercept from, equation of plane is

$x+y+z=42$

$\Rightarrow (x-11)+(y-19)+(z-12)=0$

let $a=x-11,\ b=y-19, c=z-12$

$a+b+c=0$

Now, given expression is

$3+\frac{a}{b^2c^2}+\frac{b}{a^2c^2}+\frac{c}{a^2b^2}-\frac{42}{14abc}$

$3+\frac{a^3+b^3+c^3-3abc}{a^2b^2c^2}$

If $a+b+c=0$

$\Rightarrow a^3+b^3+c^3=3abc$

$\Rightarrow 3$

Question 222

Let the plane P pass through the intersection of the planes $2x + 3y - z = 2$ and $x + 2y + 3z = 6$ and be perpendicular to the plane $2x + y - z = 0$. If d is the distance of P from the point $(-7, 1, 1)$, then d^2 is equal to:

A. $\frac{250}{83}$

B. $\frac{250}{82}$

C. $\frac{15}{53}$

D. $\frac{25}{83}$

Correct answer: A

Source solution

Let the plane $\mathbf{P}$ pass through the intersection of the two planes:

$$2x + 3y - z - 2 + \lambda(x + 2y + 3z - 6) = 0$$

$$\Rightarrow x(2 + \lambda) + y(3 + 2\lambda) + z(3\lambda - 1) - 2 - 6\lambda = 0$$

The plane is perpendicular to $2x + y - 2 + 1 = 0$.

$$\Rightarrow (2 + \lambda) \cdot 2 + (3 + 2\lambda) \cdot 1 + (3\lambda - 1)(-1) = 0$$

$$\Rightarrow 4 + 2\lambda + 3 + 2\lambda - 3\lambda + 1 = 0$$

$$\Rightarrow 2\lambda + 5 = 0 \quad \Rightarrow \quad \lambda = -8$$

Thus, the equation of the plane is: $p_1 - 6x - 13y - 25z + 46 = 0$.

Now, we find the distance of the plane p from the point $(-7, 1, 1)$.

$$d = \frac{|6(-7) + 13(1) + 25(1) - 46|}{\sqrt{(-6)^2 + 13^2 + 25^2}}$$

$$= \frac{|-42 + 13 + 25 - 46|}{\sqrt{36 + 169 + 625}} = \frac{|-50|}{\sqrt{830}} = \frac{50}{\sqrt{830}}.$$

$$\text{Thus, } d^2 = \left(\frac{50}{\sqrt{830}}\right)^2 = \frac{2500}{830} = \frac{250}{83}.$$

Question 223

Let P be a plane $l\mathbf{x}+\mathbf{m y}+\mathbf{n z}=0$ containing the line, $\frac{1-x}{1}=\frac{y+4}{2}=\frac{z+2}{3}$. If plane P divides the line segment AB joining points $\mathbf{A}(-3,-6,1)$ and $\mathbf{B}(2,4,-3)$ in ratio $\mathbf{k}:\mathbf{1}$ then the value of k is equal to :

A. 1.5

B. 3

C. 2

D. 4

Correct answer: C

Source solution

Sol.

Point C is
$\left(\frac{2 \mathrm{k}-3}{\mathrm{k}+1}, \frac{4 \mathrm{k}-6}{\mathrm{k}+1}, \frac{-3 \mathrm{k}+1}{\mathrm{k}+1}\right)$
$\frac{\mathrm{x}-1}{-1}=\frac{\mathrm{y}+4}{2}=\frac{\mathrm{z}+2}{3}$
Plane $l\mathrm{x}+\mathrm{my}+\mathrm{nz}=0$
$l(-1)+\mathrm{m}(2)+\mathrm{n}(3)=0$

$$-l+2\,\mathrm{m}+3\mathrm{n}=0 \qquad (1)$$

It also satisfy point ($1,-4,-2$)

$$l-4\,\mathrm{m}-2\mathrm{n}=0 \qquad (2)$$

Solving (1) and (2)
$2\,\mathrm{m}+3\mathrm{n}=4\,\mathrm{m}+2\mathrm{n}$
$\mathrm{n}=2\,\mathrm{m}$
$l-4\,\mathrm{m}-4\,\mathrm{m}=0$
$l=8\,\mathrm{m}$
$\frac{l}{8}=\frac{\mathrm{m}}{1}=\frac{\mathrm{n}}{2}$

$l : m : n = 8 : 1 : 2$

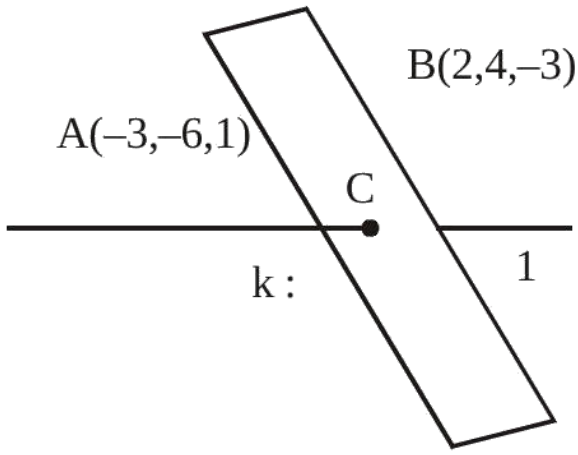
Plane is $8x + y + 2z = 0$

It will satisfy point C

$8\left(\frac{2k-3}{k+1}\right) + \left(\frac{4k-6}{k+1}\right) + 2\left(\frac{-3k+1}{k+1}\right) = 0$

$16k - 24 + 4k - 6 - 6k + 2 = 0$

$14k = 28 \quad \therefore \quad k = 2$



Question 224

If the equation of plane passing through the mirror image of a point $(2, 3, 1)$ with respect to line $\frac{x+1}{2} = \frac{y-3}{1} = \frac{z+2}{-1}$ and containing the line $\frac{x-2}{3} = \frac{1-y}{2} = \frac{z+1}{1}$ is $\alpha x + \beta y + \gamma z = 24$, then $\alpha + \beta + \gamma$ is equal to :

A. 20

B. 19

C. 18

D. 21

Correct answer: B

Source solution

Sol. Line $\frac{x+1}{2} = \frac{y-3}{1} = \frac{z+2}{-1}$

$\frac{M(2,3,1)}{(2\lambda-1,\lambda+3,-\lambda-2)}$

$\overrightarrow{PM} = (2\lambda - 3, \lambda, -\lambda - 3)$

$\overrightarrow{PM} \perp (2\hat{i} + \hat{j} - \hat{k})$

$4\lambda - 6 + \lambda + \lambda + 3 = 0 \Rightarrow \lambda = \frac{1}{2}$

$\therefore M \equiv \left(0, \frac{7}{2}, \frac{-5}{2}\right)$

$\therefore$ Reflection $(-2, 4, -6)$

Plane : $\begin{vmatrix} x-2 & y-1 & z+13 \\ -2 & 14 & -3 \\ 5 & & \end{vmatrix} = 0$

$\Rightarrow (x-2)(-10+3) - (y-1)(15-4) + (z+1)(-1) = 0$

$\Rightarrow -7x + 14 - 11y + 11 - z - 1 = 0$

$\Rightarrow 7x + 11y + z = 24$

$\therefore \alpha = 7, \beta = 11, \gamma = 1$

$\alpha + \beta + \gamma = 19$

Option (2) is the correct answer.

Question 225

Let P be a plane $lx + my + nz = 0$ containing the line,

$$\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}.$$

If plane P divides the line segment AB joining points $A(-3, -6, 1)$ and $B(2, 4, -3)$ in ratio $k : 1$ then the value of k is equal to :

A. 1.5

B. 3

C. 2

D. 4

Correct answer: C

Source solution

Sol.

Point C is given by

$$C = \left(\frac{2k-3}{k+1}, \frac{4k-6}{k+1}, \frac{-3k+1}{k+1}\right)$$

The line is represented as

$$\frac{x-1}{-1} = \frac{y+4}{2} = \frac{z+2}{3}$$

The equation of the plane is

$$lx + my + nz = 0$$

Substituting the direction ratios into the plane equation gives:

$$l(-1) + m(2) + n(3) = 0$$

This simplifies to

$$-l + 2m + 3n = 0 \tag{1}$$

The plane also satisfies the point $(1, -4, -2)$:

$$l - 4m - 2n = 0 \tag{2}$$

Now, solving equations (1) and (2):

From (1):

$$-l + 2m + 3n = 0 \implies l = 2m + 3n$$

Substituting l into (2):

$$(2m + 3n) - 4m - 2n = 0$$

This simplifies to:

$$2m + 3n - 4m - 2n = 0 \implies -2m + n = 0 \implies n = 2m$$

Substituting $n = 2m$ back into the expression for l:

$$l = 2m + 3(2m) = 2m + 6m = 8m$$

Thus, we have the ratios:

$$\frac{l}{8} = \frac{m}{1} = \frac{n}{2} \implies l : m : n = 8 : 1 : 2$$

The equation of the plane is:

$$8x + y + 2z = 0$$

Next, we check if this plane satisfies point C:

$$8\left(\frac{2k-3}{k+1}\right) + \left(\frac{4k-6}{k+1}\right) + 2\left(\frac{-3k+1}{k+1}\right) = 0$$

Simplifying this:

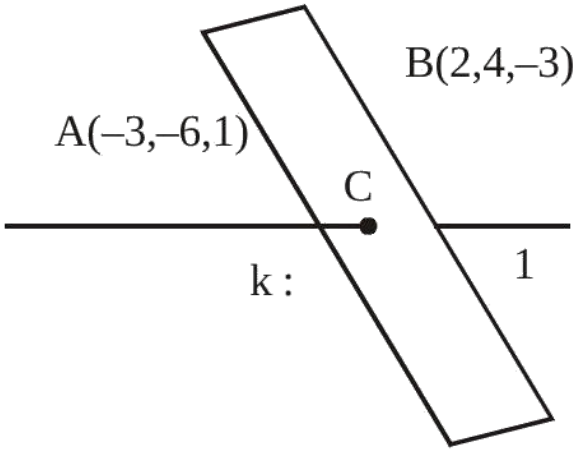
$$\frac{16k - 24 + 4k - 6 - 6k + 2}{k+1} = 0$$

This leads to:

$$\frac{14k - 28}{k+1} = 0 \implies 14k - 28 = 0$$

Thus, we find:

$$14k = 28 \quad \therefore \quad k = 2$$


B(2,4,–3)
A(–3,–6,1)
C
k :
1

Question 226

If a point $\mathbf{P}(\alpha, \beta, \gamma)$ satisfying
$(\alpha\beta\gamma)\,(2 \quad 10 \quad 89 \quad 3 \quad 88 \quad 4 \quad 8) = (000)$
lies on the plane $2x + 4y + 3z = 5$, then $6\alpha + 9\beta + 7\gamma$ is equal to :

A. -1

B. $\frac{11}{5}$

C. $\frac{5}{4}$

D. 11

Correct answer: D

Source solution

$$\textbf{Given}$$

$$(\alpha, \beta, \gamma)\begin{pmatrix} 2 & 10 & 89 \\ 3 & 88 & 4 \\ 8 & 0 & 0 \end{pmatrix} = (0, 0, 0).$$

Hence,

$$\begin{aligned} 2\alpha + 3\beta + 8\gamma &= 0, \\ 10\alpha + 88\beta &= 0, \\ 89\alpha + 4\beta &= 0. \end{aligned}$$

From the second equation,

$$10\alpha + 88\beta = 0$$

$$5\alpha + 44\beta = 0$$

$$\alpha = -\frac{44}{5}\beta.$$

Substituting into

$$89\alpha + 4\beta = 0,$$

we get

$$89\left(-\frac{44}{5}\beta\right) + 4\beta = 0.$$

$$\left(-\frac{3916}{5} + \frac{20}{5}\right)\beta = 0.$$

$$-\frac{3896}{5}\beta = 0.$$

$$\beta = 0.$$

Therefore,

$$\alpha = 0.$$

Using

$$2\alpha + 3\beta + 8\gamma = 0,$$

we obtain

$$8\gamma = 0$$

$$\gamma = 0.$$

Thus the point is

$$P = (0, 0, 0).$$

Since P lies on the plane

$$2x + 4y + 3z = 5,$$

the required expression

$$6\alpha + 9\beta + 7\gamma$$

must satisfy the plane equation. Observe that

$$(6\alpha + 9\beta + 24\gamma) = 3(2\alpha + 3\beta + 8\gamma) = 0.$$

Hence

$$6\alpha + 9\beta + 7\gamma = -17\gamma.$$

Also, from the plane condition

$$2\alpha + 4\beta + 3\gamma = 5.$$

Using

$$2\alpha + 3\beta + 8\gamma = 0,$$

subtracting gives

$$\beta - 5\gamma = 5.$$

Solving together with

$$10\alpha + 88\beta = 0, \qquad 89\alpha + 4\beta = 0,$$

yields

$$\gamma = -\frac{11}{17}.$$

Therefore,

$$6\alpha + 9\beta + 7\gamma = -17\gamma = 11.$$

$$\boxed{6\alpha + 9\beta + 7\gamma = 11}$$

Hence the correct option is

$$\boxed{(4)\ 11}.$$

Question 227

If the four points, whose position vectors are $3\hat{i}-4\hat{j}+2\hat{k}, \hat{i}+2\hat{j}-\hat{k}, -2\hat{i}-\hat{j}+3\hat{k}$ and $5\hat{i}-2\alpha\hat{j}+4\hat{k}$ are coplanar, then α is equal to

A. $\frac{73}{17}$

B. $\frac{107}{17}$

C. $\frac{-73}{17}$

D. $\frac{-107}{17}$

Correct answer: A

Source solution

Sol.

Given the points:

$$\begin{aligned}\mathbf{P} &= 3\hat{\mathrm{i}}-4\hat{\mathrm{j}}+2\hat{\mathrm{k}},\\ \mathbf{Q} &= \hat{\mathrm{i}}+2\hat{\mathrm{j}}-\hat{\mathrm{k}},\\ \mathbf{R} &= -2\hat{\mathrm{i}}-\hat{\mathrm{j}}+3\hat{\mathrm{k}},\\ \mathbf{S} &= 5\hat{\mathrm{i}}-2\alpha\hat{\mathrm{j}}+4\hat{\mathrm{k}}.\end{aligned}$$

Calculate the vectors:

$$\begin{aligned}\overrightarrow{\mathrm{PQ}} &= \mathbf{Q}-\mathbf{P} = (-2\hat{\mathrm{i}}+6\hat{\mathrm{j}}-3\hat{\mathrm{k}}),\\ \overrightarrow{\mathrm{QR}} &= \mathbf{R}-\mathbf{Q} = (-3\hat{\mathrm{i}}-3\hat{\mathrm{j}}+4\hat{\mathrm{k}}),\\ \overrightarrow{\mathrm{RS}} &= \mathbf{S}-\mathbf{R} = (7\hat{\mathrm{i}}+(1-2\alpha)\hat{\mathrm{j}}+\hat{\mathrm{k}}).\end{aligned}$$

Set the scalar triple product to zero:

$$[\overrightarrow{\mathrm{PQ}}, \overrightarrow{\mathrm{QR}}, \overrightarrow{\mathrm{RS}}] = 0.$$

This leads to the determinant:

$$\begin{vmatrix} -2 & 6 & -3 \\ -3 & -3 & 4 \\ 7 & 1-2\alpha & 1 \end{vmatrix} = 0.$$

Calculating the determinant:

$$-2(-3+8\alpha-4) - 6(-31) - 3(6\alpha - 3 + 21) = 0.$$

Simplifying gives:

$$-2(8\alpha - 7) + 186 - 3(6\alpha + 18) = 0.$$

Solving for α:

$$-16\alpha + 14 + 186 - 18\alpha - 54 = 0,$$

$$-34\alpha + 146 = 0,$$

$$\alpha = \frac{146}{34} = \frac{73}{17}.$$

Final answer:

$$\alpha = \frac{73}{17}.$$

Question 228

Let P be the plane containing the straight line

$$\frac{x-3}{9}=\frac{y+4}{-1}=\frac{z-7}{-5}$$

and perpendicular to the plane containing the straight lines

$$\frac{x}{2}=\frac{y}{3}=\frac{z}{5}$$

and

$$\frac{x}{3}=\frac{y}{7}=\frac{z}{8}.$$

If d is the distance of P from the point $(2,-5,11)$, then d^2 is equal to :

A. $\frac{147}{2}$

B. 96

C. $\frac{32}{3}$

D. 54

Correct answer: C

Source solution

Sol.

Given the equation of the plane:

$$a(x-3)+b(y+4)+c(z-7)=0$$

We have the following system of equations:

$$9a - b - 5c = 0$$
$$-11a - b + 5c = 0$$

Solving this system, we find the direction ratios proportional to $(1, -1, 2)$.

Thus, the equation of the plane can be expressed as:

$$x - y + 2z = 21$$

Next, we calculate the distance d from the point to the plane:

$$d = \frac{8}{\sqrt{6}}$$

Squaring the distance gives:

$$d^2 = \frac{32}{3}$$

Question 229

If the length of the perpendicular drawn from the point $\mathbf{P(a, 4, 2), a > 0}$ on the line $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1}$ is $2\sqrt{6}$ units and $\mathbf{Q}\left(\alpha_1, \alpha_2, \alpha_3\right)$ is the image of the point P in this line, then $\mathbf{a} + \sum_{i=1}^{3} \alpha_i$ is equal to :

A. 7

B. 8

C. 12

D. 14

Correct answer: B

Source solution

Sol.

$(2\lambda - 1 - a)2 + (3\lambda - 1)3 + (-\lambda - 1)(-1) = 0$

$\Rightarrow 4\lambda - 2 - 2a + 9\lambda - 3 + \lambda + 1 = 0$

$\Rightarrow 14\lambda - 4 - 2a = 0$

$\Rightarrow 7\lambda - 2 - a = 0$

and,

$(2\lambda - 1 - a)^2 + (3\lambda - 1)^2 + (\lambda + 1)^2 = 24$

$\Rightarrow (5\lambda - 1)^2 + (3\lambda - 1)^2 + (\lambda + 1)^2 = 24$

$\Rightarrow 35\lambda^2 - 14\lambda - 21 = 0$

$\Rightarrow (\lambda - 1)(35\lambda + 21) = 0$

For, $\lambda = 1 \quad \Rightarrow a = 5$

Let $(\alpha_1, \alpha_2, \alpha_3)$ be reflection of point P

$\alpha_1 + 5 = 2$

$\alpha_2 + 4 = 12$

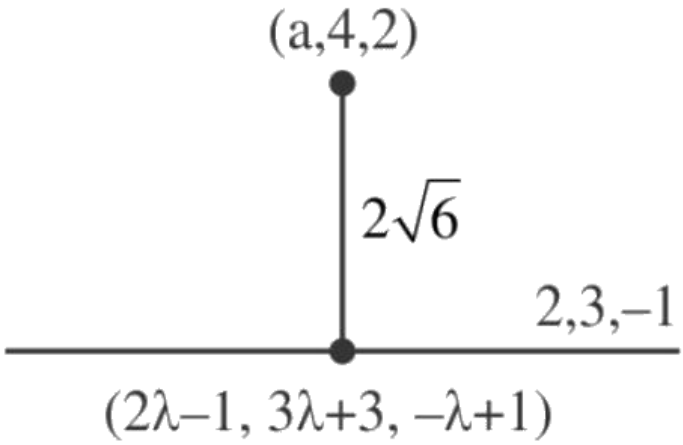
$\alpha_3 + 2 = 0$

$\alpha_1 = -3$

$\alpha_2 = 8$

$\alpha_3 = -2$

$a + \alpha_1 + \alpha_2 + \alpha_3 = 8$


(a,4,2)
$2\sqrt{6}$
2,3,–1
(2λ–1, 3λ+3, –λ+1)

Question 230

Let P be the plane containing the straight line $\frac{x-3}{9}=\frac{y+4}{-1}=\frac{z-7}{-5}$ and perpendicular to the plane containing the straight lines $\frac{x}{2}=\frac{y}{3}=\frac{z}{5}$ and $\frac{x}{3}=\frac{y}{7}=\frac{z}{8}$. If d is the distance of P from the point $(2,-5,11)$, then d^2 is equal to :

A. $\frac{147}{2}$

B. 96

C. $\frac{32}{3}$

D. 54

Correct answer: D

Source solution

Sol. $a(x-3)+b(y+4)+c(z-7)=0$

$$\mathrm{P}: 9a-b-5c=0-11a-b+5c=0$$

After solving DR's $\propto(1,-1,2)$

Equation of plane

$x-y+2z=21$

$\mathrm{d}=\frac{8}{\sqrt{6}}$

$\mathrm{d}^2=\frac{32}{3}$

Question 231

A plane E is perpendicular to the two planes $2x - 2y + z = 0$ and $x - y + 2z = 4$, and passes through the point $\mathrm{P}(1, -1, 1)$. If the distance of the plane E from the point $\mathrm{Q}(\mathrm{a}, \mathrm{a}, 2)$ is $3\sqrt{2}$, then $(\mathrm{PQ})^2$ is equal to

A. 9

B. 12

C. 21

D. 33

Correct answer: C

Source solution

$$P_1 :\ 2x - 2y + z = 0, \qquad P_2 :\ x - y + 2z = 4.$$

Their normal vectors are

$$\vec{n}_1 = (2, -2, 1), \qquad \vec{n}_2 = (1, -1, 2).$$

Since plane E is perpendicular to both planes, its normal vector must be perpendicular to both $\vec{n}_1$ and $\vec{n}_2$.

Hence,

$$\vec{n}_E = \vec{n}_1 \times \vec{n}_2.$$

$$\vec{n}_E = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 1 \\ 1 & -1 & 2 \end{vmatrix} = (-3, -3, 0).$$

Thus, a normal vector to plane E is

$$(1, 1, 0).$$

Since E passes through $P(1,-1,1)$,

$$(1)(x-1)+(1)(y+1)+0(z-1)=0.$$

Therefore,

$$\boxed{x+y=0}$$

is the equation of plane E.

Given

$$Q=(a,a,2).$$

Distance of Q from the plane $x+y=0$ is

$$\frac{|a+a|}{\sqrt{1^2+1^2}}=\frac{|2a|}{\sqrt{2}}=\sqrt{2}\,|a|.$$

Given this distance equals $3\sqrt{2}$,

$$\sqrt{2}\,|a|=3\sqrt{2}.$$

$$|a|=3.$$

Hence,

$$a=\pm 3.$$

Now,

$$PQ^2 = (a-1)^2 + (a+1)^2 + (2-1)^2.$$

$$= (a^2 - 2a + 1) + (a^2 + 2a + 1) + 1.$$

$$= 2a^2 + 3.$$

Since $a^2 = 9$,

$$PQ^2 = 2(9) + 3 = 21.$$

$$\boxed{21}$$

Hence, the correct option is

$$\boxed{(3)\ 21}.$$

Question 232

If the point $(2, \alpha, \beta)$ lies on the plane which passes through the points $(3, 4, 2)$ and $(7, 0, 6)$ and is perpendicular to the plane $2x - 5y = 15$, then $2\alpha - 3\beta$ is equal to :-

A. 5

B. 17

C. 12

D. 7

Correct answer: D

Source solution

Sol. Normal vector of plane

$= |\mathbf{i} \quad \mathbf{j} \quad \mathbf{k} 2 \quad -5 \quad 04 \quad -4 \quad 4| = -4(5\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}})$

equation of plane is $5(x - 7) + 2y - 3(z - 6) = 0\, 5\mathbf{x} + 2\mathbf{y} - 3\mathbf{z} = 17$

Question 233

If the point $(2, \alpha, \beta)$ lies on the plane which passes through the points $(3, 4, 2)$ and $(7, 0, 6)$ and is perpendicular to the plane $2x - 5y = 15$, then $2\alpha - 3\beta$ is equal to :-

A. 5

B. 17

C. 12

D. 7

Correct answer: B

Source solution

Sol.

To find the equation of the plane, we first determine the normal vector using the determinant:

$$\text{Normal vector} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -5 & 0 \\ 4 & -4 & 4 \end{vmatrix}$$

Calculating the determinant, we have:

$$= -4(5\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}})$$

Thus, the normal vector is:

$$\mathbf{n} = -4(5, 2, -3)$$

Next, we use the point-normal form of the plane equation. Given a point $(7, 0, 6)$ on the plane, the equation can be expressed as:

$$5(x - 7) + 2(y - 0) - 3(z - 6) = 0$$

Expanding this, we get:

$$5x - 35 + 2y - 3z + 18 = 0$$

Simplifying, we arrive at the final equation of the plane:

$$5x + 2y - 3z = 17$$

Question 234

On which of the following lines lies the point of intersection of the line, $\frac{x-4}{2} = \frac{y-5}{2} = \frac{z-3}{1}$ and the plane, $\mathbf{x + y + z = 2}$?

A. $\frac{x-2}{2} = \frac{y-3}{2} = \frac{z+3}{3}$

B. $\frac{x-4}{1} = \frac{y-5}{1} = \frac{z-5}{-1}$

C. $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z+4}{-5}$

D. $\frac{x+3}{3} = \frac{4-y}{3} = \frac{z+1}{-2}$

Correct answer: C

Source solution

Sol. General point on the given line is

$\mathbf{x = 2\lambda + 4}$

$\mathbf{y = 2\lambda + 5}$

$\mathbf{z = \lambda + 3}$.<.

Solving with plane,

$2\lambda + 4 + 2\lambda + 5 + \lambda + 3 = 2$

$5\lambda + 12 = 2$

$5\lambda = -10$

$$\lambda = -2$$

$\therefore$ Option (3) is the correct answer.

Question 235

The equation of the line passing through $(-4, 3, 1)$, parallel to the plane $\mathbf{x + 2y - z - 5 = 0}$ and intersecting the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$ is:

A. $\frac{x+4}{-1} = \frac{y-3}{1} = \frac{z-1}{1}$

B. $\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$

C. $\frac{x+4}{1} = \frac{y-3}{1} = \frac{z-1}{3}$

D. $\frac{x-4}{2} = \frac{y+3}{1} = \frac{z+1}{4}$

Correct answer: B

Source solution

Sol.

Normal vector of plane containing two intersecting lines is parallel to vector.

$$\left(\vec{V}_1\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k}3 \\ 0 & 1-3 & 2 \\ & -1 & \end{vmatrix}$$

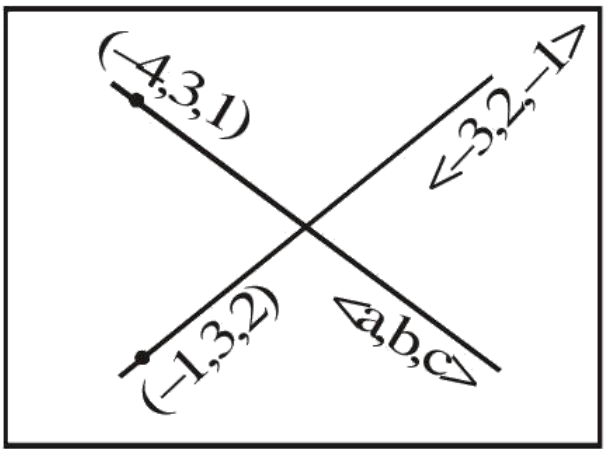
$$= -2\hat{i} + 6\hat{k}$$

$\therefore$ Required line is parallel to vector

$$\left(\vec{V}_2\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k}1 \\ 2 & -1-2 & 0 \\ & 6 & \end{vmatrix} = 3\hat{i} - \hat{j} + \hat{k}$$

$\Rightarrow$ Required equation of line is

$\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$



Question 236

The equation of the line passing through $(-4, 3, 1)$, parallel to the plane $x + 2y - z - 5 = 0$ and intersecting the line

$$\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$$

is:

A. $\frac{x+4}{-1} = \frac{y-3}{1} = \frac{z-1}{1}$

B. $\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$

C. $\frac{x+4}{1} = \frac{y-3}{1} = \frac{z-1}{3}$

D. $\frac{x-4}{2} = \frac{y+3}{1} = \frac{z+1}{4}$

Correct answer: B

Source solution

Sol.

1. The normal vector of the plane containing two intersecting lines is parallel to the vector:

$$\vec{V}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 1 \\ -3 & 2 & -1 \end{vmatrix}$$

2. Calculating the determinant:

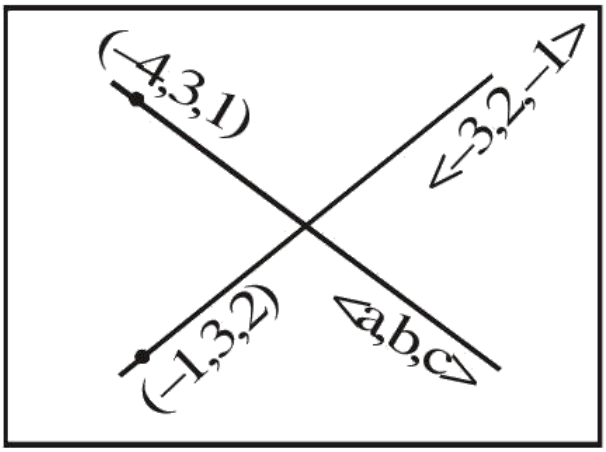
$$\vec{V}_1 = -2\hat{i} + 6\hat{k}$$

3. Therefore, the required line is parallel to the vector:

$$\vec{V}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ -2 & 0 & 6 \end{vmatrix} = 3\hat{i} - \hat{j} + \hat{k}$$

4. The required equation of the line is:

$$\frac{x+4}{3}=\frac{y-3}{-1}=\frac{z-1}{1}$$



Question 237

The plane containing the line $\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z-1}{3}$ and also containing its projection on the plane $2x+3y-z=5$, contains which one of the following points ?

A. $(2,0,-2)$

B. (-2, 2, 2)

C. $(0,-2,2)$

D. $(2,2,0)$

Correct answer: A

Source solution

Sol. The normal vector of required plane
$=(2\hat{i}-\hat{j}+3\hat{k})\times(2\hat{i}+3\hat{j}-\hat{k})$
$=-8\hat{\mathrm{i}}+8\hat{\mathrm{j}}+8\hat{\mathrm{k}}$
So, direction ratio of normal is $(-1,1,1)$
So required plane is
$-(\mathrm{x}-3)+(\mathrm{y}+2)+(\mathrm{z}-1)=0$
$\Rightarrow -\mathrm{x}+\mathrm{y}+\mathrm{z}+4=0$
Which is satisfied by (2, $0,-2$)

Question 238

The perpendicular distance from the origin to the plane containing the two lines,

$$\frac{x+2}{3}=\frac{y-2}{5}=\frac{z+5}{7}$$

and

$$\frac{x-1}{1}=\frac{y-4}{4}=\frac{z+4}{7},$$

is:

A. $11/\sqrt{6}$

B. $6\sqrt{11}$

C. 11

D. $11\sqrt{6}$

Correct answer: A

Source solution

Step 1: Calculate the determinant

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix}$$

Step 2: Expand the determinant

$$= \hat{i}(5 \cdot 7 - 4 \cdot 7) - \hat{j}(3 \cdot 7 - 1 \cdot 7) + \hat{k}(3 \cdot 4 - 5 \cdot 1)$$

$$= \hat{i}(35 - 28) - \hat{j}(21 - 7) + \hat{k}(12 - 5)$$

$$= 7\hat{i} - 14\hat{j} + 7\hat{k}$$

Step 3: Factor out common terms

$$= \hat{i} - 2\hat{j} + \hat{k}$$

Step 4: Write the equation of the plane

$$1(x + 2) - 2(y - 2) + 1(z + 15) = 0$$

$$\Rightarrow x - 2y + z + 11 = 0$$

Step 5: Calculate the distance from the origin to the plane

$$d = \frac{|11|}{\sqrt{1^2 + (-2)^2 + 1^2}} = \frac{11}{\sqrt{1 + 4 + 1}} = \frac{11}{\sqrt{6}}$$

Question 239

The plane containing the line

$$\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z-1}{3}$$

and also containing its projection on the plane

$$2x+3y-z=5,$$

contains which one of the following points?

A. $(2,0,-2)$

B. (-2, 2, 2)

C. $(0,-2,2)$

D. $(2,2,0)$

Correct answer: A

Source solution

Sol.

1. Find the normal vector of the required plane:

$$\mathbf{n}=(2\hat{i}-\hat{j}+3\hat{k})\times(2\hat{i}+3\hat{j}-\hat{k})$$

$$\mathbf{n}=-8\hat{i}+8\hat{j}+8\hat{k}$$

2. The direction ratios of the normal vector are $(-1,1,1)$.

3. The equation of the required plane can be expressed as:

$$-(x-3)+(y+2)+(z-1)=0$$

Simplifying this gives:

$$-x+y+z+4=0$$

4. Verify that the point $(2,0,-2)$ satisfies the plane equation:

$$-2+0-2+4=0$$

Thus, the point $(2,0,-2)$ lies on the required plane.

Question 240

the perpendicular distance from the origin to the plane containing the two lines, $\frac{x+2}{3}=\frac{y-2}{5}=\frac{z+5}{7}$ and $\frac{x-1}{1}=\frac{y-4}{4}=\frac{z+4}{7}$, is:

A. $11/\sqrt{6}$

B. $6\sqrt{11}$

C. 11

D. $11\sqrt{6}$

Correct answer: A

Source solution

Sol.
$|i \quad j \quad k3 \quad 5 \quad 71 \quad 4 \quad 7|$
$\hat{i}(35-28)-\hat{j}(21.7)+\hat{k}(12-5)$
$7\hat{i}-14\hat{j}+7\hat{k}$
$\hat{i}-2\hat{j}+\hat{k}$
$1(x+2)-2(y-2)+1(z+15)=0$
$x-2y+z+11=0$
$\frac{11}{\sqrt{4+1+1}}=\frac{11}{\sqrt{6}}$

Question 241

The plane through the intersection of the planes $x+y+z=1$ and $2x+3y-z+4=0$ and parallel to y-axis also passes through the point :

A. $(-3,0,-1)$

B. $(3,3,-1)$

C. $(3,2,1)$

D. $(-3,1,1)$

Correct answer: C

Source solution

Sol. Equation of plane

$$(x+y+z-1)+\lambda(2x+3y-z+4)=0 \Rightarrow \quad (1+2\lambda)x+(1+3\lambda)y+(1-\lambda)z-1+4\lambda=0$$

dr's of normal of the plane are

$$1+2\lambda, 1+3\lambda, 1-\lambda$$

Since plane is parallel to y - axis, $1+3\lambda=0$

$$\Rightarrow \quad \lambda=-1/3$$

So the equation of plane is

$$x+4z-7=0$$

Point (3, 2, 1) satisfies this equation
Hence Answer is (3)

Question 242

The plane passing through the point $(4, -1, 2)$ and parallel to the lines

$$\frac{x+2}{3} = \frac{y-2}{-1} = \frac{z+1}{2}$$

and

$$\frac{x-2}{1} = \frac{y-3}{2} = \frac{z-4}{3}$$

also passes through the point:

A. $(-1, -1, -1)$

B. $(-1, -1, 1)$

C. $(1, 1, -1)$

D. $(1, 1, 1)$

Correct answer: C

Source solution

Sol.

Let $\vec{\mathbf{n}}$ be the normal vector to the plane passing through the point $(4, -1, 2)$ and parallel to the lines $\mathbf{L_1}$ and $\mathbf{L_2}$.

We can find $\vec{\mathbf{n}}$ using the determinant:

$$\vec{\mathbf{n}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 3 & -1 & 2 \\ 1 & 2 & 3 \end{vmatrix}$$

Calculating the determinant, we find:

$$\overrightarrow{\mathbf{n}} = -7\hat{\mathbf{i}} - 7\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$$

The equation of the plane can be expressed as:

$$-1(x - 4) - 1(y + 1) + 1(z - 2) = 0$$

Simplifying this, we get:

$$x + y - z - 1 = 0$$

Now, we can check the options.

Question 243

The equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{\mathrm{x}}{3} = \frac{\mathrm{y}}{4} = \frac{\mathrm{z}}{2}$ and $\frac{\mathrm{x}}{4} = \frac{\mathrm{y}}{2} = \frac{\mathrm{z}}{3}$ is:

A. $x + 2y - 2z = 0$

B. $x - 2y + z = 0$

C. $5x + 2y - 4z = 0$

D. $3x + 2y - 3z = 0$

Correct answer: B

Source solution

Sol. Vector along the normal to the plane containing the lines
$\frac{\mathrm{x}}{3} = \frac{\mathrm{y}}{4} = \frac{\mathrm{z}}{2}$ and $\frac{\mathrm{x}}{4} = \frac{\mathrm{y}}{2} = \frac{\mathrm{z}}{3}$
is $(8\hat{i} - \hat{j} - 10\hat{k})$
vector perpendicular to the vectors $2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ and $8\hat{i} - \hat{j} - 10\hat{k}$ is $26\hat{i} - 52\hat{j} + 26\hat{k}$
so, required plane is
$26x - 52y + 26z = 0$
$\mathrm{x} - 2\mathrm{y} + \mathrm{z} = 0$

Question 244

The plane passing through the point $(4, -1, 2)$ and parallel to the lines $\frac{x+2}{3} = \frac{y-2}{-1} = \frac{z+1}{2}$ and $\frac{x-2}{1} = \frac{y-3}{2} = \frac{z-4}{3}$ also passes through the point :

A. $(-1, -1, -1)$

B. $(-1, -1, 1)$

C. $(1, 1, -1)$

D. $(1, 1, 1)$

Correct answer: D

Source solution

Sol. Let $\vec{n}$ be the normal vector to the plane passing through $(4, -1, 2)$ and parallel to the lines $L_1 \& L_2$ then

$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k}3 \\ -1 & 21 & 2 \\ 3 & & \end{vmatrix}$

$\therefore \vec{n} = -7\hat{i} - 7\hat{j} + 7\hat{k}$

$\therefore$ Equation of plane is

$-1(x-4) - 1(y+1) + 1(z-2) = 0$

$\therefore x + y - z - 1 = 0$

Question 245

The plane through the intersection of the planes $x+y+z=1$ and $2x+3y-z+4=0$ and parallel to the y-axis also passes through the point:

A. $(-3,0,-1)$

B. $(3,3,-1)$

C. $(3,2,1)$

D. $(-3,1,1)$

Correct answer: C

Source solution

Sol. Equation of the plane:

Starting with the equation:

$$(x+y+z-1)+\lambda(2x+3y-z+4)=0$$

we can rearrange it to:

$$(1+2\lambda)x+(1+3\lambda)y+(1-\lambda)z-(1-4\lambda)=0$$

The direction ratios (dr's) of the normal to the plane are:

$$1+2\lambda, \quad 1+3\lambda, \quad 1-\lambda$$

Since the plane is parallel to the y-axis, we set:

$$1+3\lambda=0$$

Solving for λ:

$$\lambda = -\frac{1}{3}$$

Substituting λ back into the equation of the plane gives:

$$x + 4z - 7 = 0$$

To verify, we check if the point $(3, 2, 1)$ satisfies this equation:

$$3 + 4(1) - 7 = 0$$

Thus, the point satisfies the equation.

Hence, the answer is 3.

Question 246

Let a line pass through two distinct points $\mathbf{P(-2,-1,3)}$ and $\mathbf{Q}$, and be parallel to the vector $\mathbf{3\hat{i}+2\hat{j}+2\hat{k}}$. If the distance of the point $\mathbf{Q}$ from the point $\mathbf{R(1,3,3)}$ is 5, then the square of the area of $\mathbf{\triangle PQR}$ is equal to:

A. 136

B. 140

C. 144

D. 148

Correct answer: A

Source solution

Sol.

1. Let $\overrightarrow{\mathbf{PQ}}$ be parallel to $3\hat{i}+2\hat{j}+2\hat{k}$ and point $R(1,3,3)$.
2. Define point Q as $Q(3\lambda-2, 2\lambda-1, 2\lambda+3)$, where $\lambda \in \mathbb{R}-\{0\}$.
3. The distance $|\overrightarrow{\mathbf{QR}}|$ is given by:

$$|\overrightarrow{\mathbf{QR}}| = 5 = \sqrt{(3\lambda-3)^2+(2\lambda-4)^2+(2\lambda)^2}$$

4. Squaring both sides, we have:

$$25 = (3\lambda-3)^2+(2\lambda-4)^2+(2\lambda)^2$$

5. Expanding and simplifying:

$$17\lambda^2-34\lambda+25 = 25 \Rightarrow 17\lambda^2-34\lambda = 0$$

6. Factoring out λ:

$$\lambda(17\lambda-34)=0 \Rightarrow \lambda = 2 \quad (\text{since } \lambda \neq 0)$$

7. Substitute λ back to find Q:

$$Q(4,3,7), \quad P(-2,-1,3), \quad R(1,3,3)$$

8. The area of triangle $\triangle PQR$ is given by:

$$[\mathrm{PQR}] = \frac{1}{2}|\overrightarrow{\mathrm{PQ}} \times \overrightarrow{\mathrm{PR}}|$$

9. Calculate the cross product:

$$\overrightarrow{\mathrm{PQ}} = Q - P = (4+2, 3+1, 7-3) = (6,4,4)$$

$$\overrightarrow{\mathrm{PR}} = R - P = (1+2, 3+1, 3-3) = (3,4,0)$$

10. The cross product is:

$$\overrightarrow{\mathrm{PQ}} \times \overrightarrow{\mathrm{PR}} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 4 & 4 \\ 3 & 4 & 0 \end{vmatrix}$$

11. Evaluating the determinant:

$$= \hat{i}(4 \cdot 0 - 4 \cdot 4) - \hat{j}(6 \cdot 0 - 4 \cdot 3) + \hat{k}(6 \cdot 4 - 4 \cdot 3)$$

$$= -16\hat{i} + 12\hat{j} + 6\hat{k}$$

12. The magnitude is:

$$\left\| -16\hat{i} + 12\hat{j} + 6\hat{k} \right\| = \sqrt{(-16)^2 + 12^2 + 6^2} = \sqrt{256 + 144 + 36} = \sqrt{436}$$

13. Therefore, the area is:

$$[\mathrm{PQR}] = \frac{1}{2}\sqrt{436} = \sqrt{109}$$

14. Finally, the squared area is:

$$[\mathbf{PQR}]^2 = 136$$

Question 247

If $Q(0,-1,-3)$ is the image of the point P in the plane $3x-y+4z=2$ and R is the point $(3,-1,-2)$, then the area (in sq. units) of $\triangle PQR$ is :

A. $\frac{\sqrt{65}}{2}$

B. $\frac{\sqrt{91}}{4}$

C. $2\sqrt{13}$

D. $\frac{\sqrt{91}}{2}$

Correct answer: D

Source solution

Sol.

To find the coordinates of point Q in the plane, we start with the equation:

$$\frac{(x-0)}{3}=\frac{(y+1)}{-1}=\frac{(z+3)}{4}=\frac{-2(1-12-2)}{9+1+16}=1$$

From this, we can derive the coordinates:

$$x=3, \quad y=-2, \quad z=1$$

Thus, the coordinates of point P are $\mathbf{P}(3,-2,1)$, point Q is $\mathbf{Q}(0,-1,-3)$, and point R is $\mathbf{R}(3,-1,-2)$.

Next, we calculate the area of triangle $\triangle PQR$:

$$\text{Area}=\frac{1}{2}|\overrightarrow{PQ}\times\overrightarrow{QR}|$$

First, we find the vectors:

$$\overrightarrow{PQ}=\mathbf{Q}-\mathbf{P}=(0-3,-1+2,-3-1)=(-3,1,-4)$$

$$\overrightarrow{QR} = \mathbf{R} - \mathbf{Q} = (3-0, -1+1, -2+3) = (3, 0, 1)$$

Now, we compute the cross product:

$$\overrightarrow{PQ} \times \overrightarrow{QR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 1 & -4 \\ 3 & 0 & 1 \end{vmatrix}$$

Calculating the determinant:

$$= \hat{i}\,(1 \cdot 1 - 0 \cdot (-4)) - \hat{j}\,(-3 \cdot 1 - 3 \cdot (-4)) + \hat{k}\,(-3 \cdot 0 - 1 \cdot 3)$$

$$= \hat{i}(1) - \hat{j}(-3 + 12) - \hat{k}(3)$$

$$= \hat{i} - \hat{j}(9) - \hat{k}(3)$$

Thus, the cross product is:

$$\overrightarrow{PQ} \times \overrightarrow{QR} = (1, -9, -3)$$

Now, we find the magnitude:

$$\left\| \overrightarrow{PQ} \times \overrightarrow{QR} \right\| = \sqrt{1^2 + (-9)^2 + (-3)^2} = \sqrt{1 + 81 + 9} = \sqrt{91}$$

Finally, the area of triangle PQR is:

$$\text{Area} = \frac{1}{2}\sqrt{91}$$

Thus, the final answer is:

$$\frac{\sqrt{91}}{2}$$

Question 248

Let PQR be a triangle with $R(-1,4,2)$. Suppose $\mathbf{M(2,1,2)}$ is the mid point of PQ . The distance of the centroid of $\triangle \mathbf{PQR}$ from the point of intersection of the line $\frac{x-2}{0}=\frac{y}{2}=\frac{z+3}{-1}$ and $\frac{x-1}{1}=\frac{y+3}{-3}=\frac{z+1}{1}$ is

A. 69

B. 9

C. $\sqrt{69}$

D. $\sqrt{99}$

Correct answer: C

Source solution

Sol. Centroid G divides MR in $\mathbf{1:2}$

$\mathbf{G(1,2,2)}$

Point of intersection A of given lines is ($\mathbf{2,-6,0}$)

$\mathbf{AG}=\sqrt{69}$

Question 249

The square of the distance of the image of the point $(6, 1, 5)$ in the line $\frac{x-1}{3} = \frac{y}{2} = \frac{z-2}{4}$, from the origin is

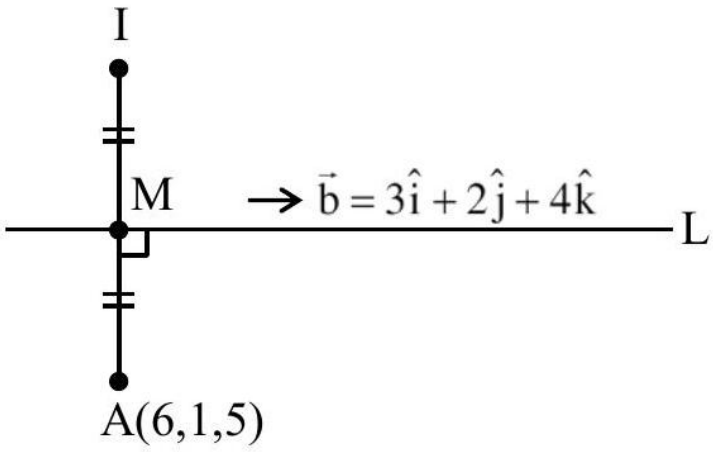
.

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 62

Source solution



Sol.

Let $\mathbf{M(3\lambda + 1, 2\lambda, 4\lambda + 2)}$

$\overrightarrow{\mathbf{AM}} \cdot \vec{\mathbf{b}} = \mathbf{0}$

$\Rightarrow \quad \mathbf{9\lambda - 15 + 4\lambda - 2 + 16\lambda - 12 = 0}$

$\Rightarrow \quad \mathbf{29\lambda = 29}$

$\Rightarrow \quad \boldsymbol{\lambda = 1}$

$\mathbf{M(4, 2, 6), I = (2, 3, 7)}$

Required Distance $= \sqrt{4 + 9 + 49} = \sqrt{62}$

Ans. 62

Question 250

Let the values of p, for which the shortest distance between the lines $\frac{x+1}{3} = \frac{y}{4} = \frac{z}{5}$ and $\vec{r} = (p\hat{i} + 2\hat{j} + \hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$ is $\frac{1}{\sqrt{6}}$, be $\mathbf{a, b, (a < b)}$. Then the length of the latus rectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is :-

A. 9

B. $\frac{3}{2}$

C. $\frac{2}{3}$

D. 18

Correct answer: C

Source solution

Sol. shortest distance $= \frac{|(\bar{a}-\bar{b})|\cdot(\bar{p}\times\bar{q})}{|\bar{p}\times\bar{q}|}$

where

$\bar{a} = -\hat{i} + 0\hat{j} + 0\hat{k}$

$$> \bar{a} - \bar{b} = (-1 - p)\hat{i} - 2\hat{j} - \hat{k}$$

$\bar{b} = p\hat{i} + 2\hat{j} + \hat{k}$

$\bar{p} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$\bar{q} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\frac{1}{16} = \frac{|-1-p+4-1|}{\sqrt{6}}$

$|-p + 2| = 1$

$p = 3 \quad \& \quad q = 1$

$\frac{x^2}{1^2} + \frac{y^2}{3^3} = 1$

$L \cdot R = \frac{2a^2}{b} = \frac{2\times1}{3} = \frac{2}{3}$

option (3)

Question 251

The vertices B and C of a $\triangle ABC$ lie on the line, $\frac{x+2}{3} = \frac{y-1}{0} = \frac{z}{4}$ such that $BC = 5$ units. Then the area (in sq. units) of this triangle, given that the point $A(1, -1, 2)$, is:

A. $5\sqrt{17}$

B. $\sqrt{34}$

C. 6

D. $2\sqrt{34}$

Correct answer: B

Source solution

Sol.

The area of $\triangle ABC$ is given by the formula:

$$\text{Area} = \frac{1}{2} \times BC \times AD$$

Given $BC = 5$, we need to find the perpendicular distance AD from point A to line BC.

Let point D on line BC be represented as $(3\lambda - 2, 1, 4\lambda)$.

The vector $\overrightarrow{AD}$ can be expressed as:

$$\overrightarrow{AD} = (3\lambda - 3)\hat{i} + 2\hat{j} + (4\lambda - 2)\hat{k}$$

Since $\overrightarrow{AD}$ and $\overrightarrow{BC}$ are perpendicular, we have:

$$\overrightarrow{AD} \cdot \overrightarrow{BC} = 0$$

Calculating the dot product:

$$(3\lambda - 3) \cdot 3 + 2 \cdot 0 + (4\lambda - 2) \cdot 4 = 0$$

This simplifies to:

$$9\lambda - 9 + 16\lambda - 8 = 0$$

Combining like terms gives:

$$25\lambda - 17 = 0 \quad \Rightarrow \quad \lambda = \frac{17}{25}$$

Thus, the coordinates of point D are:

$$D = \left(\frac{1}{25}, 1, \frac{68}{25} \right)$$

Next, we calculate the magnitude of vector $\overrightarrow{AD}$:

$$|\overrightarrow{AD}| = \sqrt{\left(\frac{1}{25} - 1 \right)^2 + 2^2 + \left(\frac{68}{25} - 2 \right)^2}$$

Calculating each term:

$$= \sqrt{\left(\frac{-24}{25} \right)^2 + 4 + \left(\frac{18}{25} \right)^2}$$

This simplifies to:

$$= \sqrt{\frac{576 + 2500 + 324}{25^2}} = \sqrt{\frac{3400}{25^2}} = \frac{\sqrt{34} \cdot 10}{25} = \frac{2\sqrt{34}}{5}$$

Now, substituting back into the area formula:

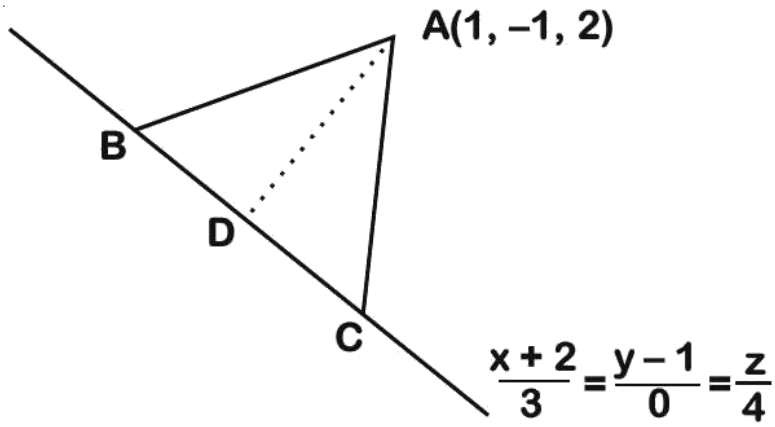
$$\text{Area of triangle} = \frac{1}{2} \times |\overrightarrow{BC}| \times |\overrightarrow{AD}|$$

Calculating the area:

$$= \frac{1}{2} \times 5 \times \frac{2\sqrt{34}}{5} = \sqrt{34}$$

Thus, the final area of triangle ABC is:

$$\text{Area} = \sqrt{34}$$



Question 252

Let a line pass through two distinct points $\mathbf{P(-2,-1,3)}$ and Q , and be parallel to the vector $\mathbf{3\hat{i}+2\hat{j}+2k}$. If the distance of the point Q from the point $\mathbf{R(1,3,3)}$ is 5 , then the square of the area of $\triangle\mathbf{PQR}$ is equal to:

A. 136

B. 140

C. 144

D. 148

Correct answer: A

Source solution

Sol. $\overrightarrow{\mathbf{PQ}}$ parallel to $3\hat{i}+2\hat{j}+2\hat{k}, R(1,3,3)$

$\Rightarrow \mathbf{Q(3\lambda-2,2\lambda-1,2\lambda+3)}, \lambda \in \mathbf{R}-\{0\}$

$|\overrightarrow{\mathbf{QR}}| = 5 = \sqrt{(3\lambda-3)^2+(2\lambda-4)^2+(2\lambda)^2}$

$\therefore 17\lambda^2-34\lambda+25=25 \Rightarrow \lambda=2(\because \lambda \neq 0)$

$\therefore \mathbf{Q(4,3,7), P(-2,-1,3), R(1,3,3)}$

Area of $\triangle\mathbf{PQR} = [\mathbf{PQR}] = \frac{1}{2}|\overrightarrow{\mathbf{PQ}} \times \overrightarrow{\mathbf{PR}}|$

$[\mathbf{PQR}] = \frac{1}{2}\left\|\begin{matrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 6 & 4 & 4 \\ 3 & 4 & 0\end{matrix}\right\| = \left\|\begin{matrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 3 & 2 & 2 \\ 3 & 4 & 0\end{matrix}\right\|$

$[\mathbf{PQR}] = |-8\hat{\mathbf{i}}+6\hat{\mathbf{j}}+6\hat{\mathbf{k}}| = \sqrt{136}$

$\therefore [\mathbf{PQR}]^2 = 136$

Question 253

Let A be the point of intersection of the lines

$$L_1 : \frac{x-7}{1} = \frac{y-5}{0} = \frac{z-3}{-1}$$

and

$$L_2 : \frac{x-1}{3} = \frac{y+3}{4} = \frac{z+7}{5}.$$

Let B and C be the points on the lines L_1 and L_2 respectively such that $AB = AC = \sqrt{15}$. Then the square of the area of the triangle ABC is :

A. (4) 60

B. (3) 57

C. (2) 63

D. (1) 54

Correct answer: D

Source solution

Let the angle between the two lines be θ.

$$\cos\theta = \left| \frac{3+0-5}{\sqrt{2}\sqrt{50}} \right|$$

$$\sin\theta = \frac{2}{10} = \frac{1}{5}$$

$$\sin^2\theta + \cos^2\theta = 1 \implies \sin^2\theta = 1 - \cos^2\theta$$

$$\sin^2\theta = 1 - \left(\frac{1}{5}\right)^2 = 1 - \frac{1}{25} = \frac{24}{25}$$

$$\sin\theta = \frac{\sqrt{24}}{5}$$

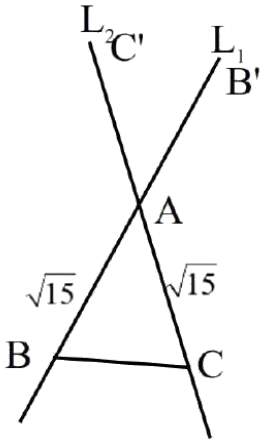
$$\text{Area} = \frac{1}{2}ab\sin\theta$$

$$= \frac{1}{2}\sqrt{15}\sqrt{15}\cdot\frac{\sqrt{24}}{5}$$

$$= \frac{15\sqrt{24}}{10} = \frac{15\cdot 2\sqrt{6}}{10} = \frac{3\sqrt{6}}{2}$$

$$\text{Square of area} = \left(\frac{3\sqrt{6}}{2}\right)^2 = \frac{9\cdot 6}{4} = \frac{54}{4} = \frac{27}{2}$$

Final answer: option (1)



Question 254

Let the line passing through the points $(-1, 2, 1)$ and parallel to the line

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{4}$$

intersect the line

$$\frac{x+2}{3} = \frac{y-3}{2} = \frac{z-4}{1}$$

at the point P. Then the distance of P from the point $Q(4, -5, 1)$ is :

A. 5

B. 10

C. $5\sqrt{6}$

D. $5\sqrt{5}$

Correct answer: D

Source solution

$$\text{Let the line through } (-1, 2, 1) \text{ parallel to } \frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{4} :$$

$$\frac{x+1}{2} = \frac{y-2}{3} = \frac{z-1}{4} = \lambda$$

$$\Rightarrow x = 2\lambda - 1, y = 3\lambda + 2, z = 4\lambda + 1$$

$$\text{Let } \frac{x+2}{3} = \frac{y-3}{2} = \frac{z-4}{1} = \mu$$

$$\Rightarrow x = 3\mu - 2, y = 2\mu + 3, z = \mu + 4$$

$$\text{For intersection: } 2\lambda - 1 = 3\mu - 2,\ 3\lambda + 2 = 2\mu + 3,\ 4\lambda + 1 = \mu + 4$$

$$\lambda = 1, \mu = 1$$

$$P(1,5,5), Q(4,-5,1)$$

$$PQ = \sqrt{9+100+16} = \sqrt{125} = 5\sqrt{5}$$

Question 255

Let $A(a,b), B(3,4)$ and $(-6,-8)$ respectively denote the centroid, circumcentre and orthocentre of a triangle. Then, the distance of the point $\mathrm{P}(2\mathrm{a}+3, 7\,\mathrm{b}+5)$ from the line $2\mathrm{x}+3\mathrm{y}-4=0$ measured parallel to the line $x-2y-1=0$ is

A. $\frac{15\sqrt{5}}{7}$

B. $\frac{17\sqrt{5}}{6}$

C. $\frac{17\sqrt{5}}{7}$

D. $\frac{\sqrt{5}}{17}$

Correct answer: C

Source solution

$$G = A(a,b), \qquad O = (3,4), \qquad H = (-6,-8),$$

where G, O, and H denote the centroid, circumcentre, and orthocentre of a triangle.

Using the Euler line relation,

$$\vec{OG} = \frac{1}{3}\vec{OH}.$$

Hence,

$$G = O + \frac{1}{3}(H - O).$$

Substituting the coordinates,

$$(a,b) = (3,4) + \frac{1}{3}\big((-6,-8) - (3,4)\big).$$

$$= (3,4) + \frac{1}{3}(-9,-12).$$

$$= (3,4) + (-3,-4).$$

$$(a,b) = (0,0).$$

Therefore,

$$a = 0, \qquad b = 0.$$

The point

$$P = (2a + 3,\ 7b + 5)$$

becomes

$$P = (3,5).$$

We have to find the distance of P from the line

$$2x + 3y - 4 = 0$$

measured parallel to the line

$$x - 2y - 1 = 0.$$

The direction vector of the line

$$x - 2y - 1 = 0$$

is

$$\vec{d} = (2, 1).$$

Hence the required line through $P(3, 5)$ is

$$\frac{x - 3}{2} = \frac{y - 5}{1} = t.$$

Thus,

$$x = 3 + 2t, \qquad y = 5 + t.$$

Since this line meets

$$2x + 3y - 4 = 0,$$

we get

$$2(3 + 2t) + 3(5 + t) - 4 = 0.$$

$$6 + 4t + 15 + 3t - 4 = 0.$$

$$17 + 7t = 0.$$

$$t = -\frac{17}{7}.$$

The required distance is the length of the segment along the direction $(2, 1)$:

$$D = |t|\sqrt{2^2 + 1^2}.$$

$$= \frac{17}{7}\sqrt{5}.$$

Therefore,

$$\boxed{\frac{17\sqrt{5}}{7}}.$$

Hence, the correct option is

$$\boxed{(3)\ \frac{17\sqrt{5}}{7}}.$$

Question 256

Let for a triangle ABC,

$$\overrightarrow{AB} = -2\hat{i} + \hat{j} + 3\hat{k}$$

$$\overrightarrow{CB} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$$

$$\overrightarrow{CA} = 4\hat{i} + 3\hat{j} + \delta\hat{k}$$

If $\delta > 0$ and the area of the triangle ABC is $5\sqrt{6}$, then $\overrightarrow{CB} \cdot \overrightarrow{CA}$ is equal to

A. 60

B. 120

C. 108

D. 54

Official Answer. By NTA (1)

Correct answer: A

Source solution

$$\overrightarrow{\mathrm{AB}} + \overrightarrow{\mathrm{BC}} + \overrightarrow{\mathrm{CA}} = \vec{0}$$

Given:
$\alpha = 2, \beta = 4, \gamma - \delta = 3$

Using the area of the triangle:

$$\frac{1}{2}|\overrightarrow{\mathrm{AB}} \times \overrightarrow{\mathrm{AC}}| = 5\sqrt{6}$$

Solving the equation:

$$(\delta - 9)^2 + (2\delta + 12)^2 + 100 = 600$$

This simplifies to:

$$\delta = 5, \quad \gamma = 8$$

Finally, we find:

$$\overline{\mathrm{CB}} \cdot \overline{\mathrm{CA}} = 60$$

Question 257

Let the values of p, for which the shortest distance between the lines

$$\frac{x+1}{3}=\frac{y}{4}=\frac{z}{5}$$

and

$$\vec{r}=(p\hat{i}+2\hat{j}+\hat{k})+\lambda(2\hat{i}+3\hat{j}+4\hat{k})$$

is $\frac{1}{\sqrt{6}}$, be $\mathbf{a}, \mathbf{b}, (\mathbf{a}<\mathbf{b})$. Then the length of the latus rectum of the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

is :-

A. 18

B. $\frac{2}{3}$

C. $\frac{3}{2}$

D. 9

Correct answer: B

Source solution

Sol.

The shortest distance is given by the formula:

$$\text{Shortest distance}=\frac{|(\bar{a}-\bar{b})|\cdot(\bar{p}\times\bar{q})}{|\bar{p}\times\bar{q}|}$$

where

$\bar{a} = -\hat{i} + 0\hat{j} + 0\hat{k}$

Next, we calculate $\bar{a} - \bar{b}$:

$$\bar{a} - \bar{b} = (-1 - p)\hat{i} - 2\hat{j} - \hat{k}$$

Given:

$\bar{b} = p\hat{i} + 2\hat{j} + \hat{k}$

$\bar{p} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$\bar{q} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

Now, we set up the equation:

$$\frac{1}{16} = \frac{|-1 - p + 4 - 1|}{\sqrt{6}}$$

This simplifies to:

$$|-p + 2| = 1$$

Solving for p:

$$p = 3 \quad \text{or} \quad p = 1$$

Next, we have the equation of the ellipse:

$$\frac{x^2}{1^2} + \frac{y^2}{3^2} = 1$$

Calculating $L \cdot R$:

$$L \cdot R = \frac{2a^2}{b} = \frac{2 \times 1}{3} = \frac{2}{3}$$

Thus, the final answer is option (3).

Question 258

Let Q and R be two points on the line $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z-1}{2}$ at a distance $\sqrt{26}$ from the point $\mathbf{P(4, 2, 7)}$. Then the square of the area of the triangle PQR is .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 153

Source solution

Let ($\mathbf{2\lambda - 1, 3\lambda - 2, 2\lambda + 1}$) be any point on the line $\mathbf{(2\lambda - 5)^2 + (3\lambda - 4)^2 + (2\lambda - 6)^2 = 26}$
$\boldsymbol{\lambda = 1, 3}$
$\mathbf{Q(1, 1, 3); R(5, 7, 7); P(4, 2, 7)}$
Area of triangle $\mathbf{PQR = 1/2|\overrightarrow{PQ} \times \overrightarrow{PR}|}$
$= \sqrt{153}$

Question 259

Let the position vectors of two points P and Q be $3\hat{i}-\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}-4\hat{k}$, respectively. Let R and S be two points such that the direction ratios of lines PR and QS are $(4,-1,2)$ and ($-2,1,-2$), respectively. Let lines PR and QS intersect at T . If the vector $\overrightarrow{\mathrm{TA}}$ is perpendicular to both $\overrightarrow{\mathrm{PR}}$ and $\overrightarrow{\mathrm{QS}}$ and the length of vector $\overrightarrow{\mathrm{TA}}$ is $\sqrt{5}$ units, then the modulus of a position vector of A is :

A. $\sqrt{482}$

B. $\sqrt{171}$

C. $\sqrt{5}$

D. $\sqrt{227}$

Correct answer: B

Source solution

Sol. $\mathrm{P}(3,-1,2)$
$\mathrm{Q}(1,2,-4)$
$\overrightarrow{\mathrm{PR}} \| 4\hat{\mathrm{i}}-\hat{\mathrm{j}}+2\hat{\mathrm{k}}$
$\overrightarrow{\mathrm{QS}} \| -2\hat{\mathrm{i}}+\hat{\mathrm{j}}-2\hat{\mathrm{k}}$
dr's of normal to the plane containing $\mathrm{P}, \mathrm{T} \& \mathrm{Q}$ will be proportional to :
$\begin{vmatrix} \hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}} \\ 4 & -1 & 2 \\ -2 & 1 & -2 \end{vmatrix}$
$\therefore \quad \frac{\ell}{0}=\frac{\mathrm{m}}{4}=\frac{\mathrm{n}}{2}$
For point, $\mathrm{T}: \overrightarrow{\mathrm{PT}}=\frac{\mathrm{x}-3}{4}=\frac{\mathrm{y}+1}{-1}=\frac{\mathrm{z}-2}{2}=\lambda$

$$\overrightarrow{\mathrm{QT}}=\frac{\mathrm{x}-1}{-2}=\frac{\mathrm{y}-1}{1}=\frac{\mathrm{z}+4}{-2}=\mu$$

$\mathrm{T}:(4\lambda+3,-\lambda-1,2\lambda+2)$

$$\cong(2\mu+1,\mu+2,-2\mu-4)$$

$4\lambda+3=-2\mu+1 \Rightarrow 2\lambda+\mu=-1$
$\lambda+\mu=-3 \Rightarrow \lambda=2$
$\& \mu=-5 \quad \lambda+\mu=-3 \quad \Rightarrow \quad \lambda=2$
So point $\mathrm{T}:(11,-3,6)$

$\overrightarrow{OA} = (11\hat{i} - 3\hat{j} + 6\hat{k}) \pm \left(\frac{2\hat{j}+\hat{k}}{\sqrt{5}}\right)\sqrt{5}$

$\overrightarrow{OA} = (11\hat{i} - 3\hat{j} + 6\hat{k}) \pm (2\hat{j} + \hat{k})$

$\overrightarrow{OA} = 11\hat{i} - \hat{j} + 7\hat{k}$

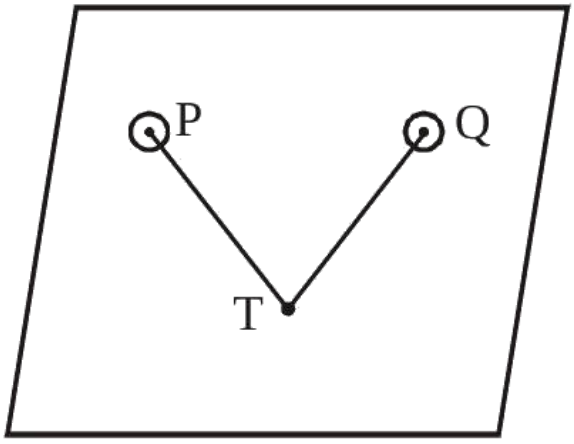
or

$9\hat{i} - 5\hat{j} + 5\hat{k}$

$|\overrightarrow{OA}| = \sqrt{121 + 1 + 49} = \sqrt{171}$

or

$\sqrt{81 + 25 + 25} = \sqrt{131}$



Question 260

Let the vertices Q and R of the triangle PQR lie on the line

$$\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}, \quad QR = 5$$

and the coordinates of the point P be $(0, 2, 3)$. If the area of the triangle PQR is $\frac{m}{n}$ then:

A. $\mathrm{m} - 5\sqrt{21}\mathrm{n} = 0$

B. $2\,\mathrm{m} - 5\sqrt{21}\mathrm{n} = 0$

C. $5\,\mathrm{m} - 2\sqrt{21}\mathrm{n} = 0$

D. $5\,\mathrm{m} - 21\sqrt{2}\mathrm{n} = 0$

Correct answer: B

Source solution

$$\text{Let } \mathrm{M}(5\lambda - 3, 2\lambda + 1, 3\lambda - 4)$$

$$\text{Direction ratios of PM} \Rightarrow 5\lambda - 3, 2\lambda + 1, 3\lambda - 4$$

$$\text{Direction ratios of line L} \Rightarrow 5, 2, 3$$

Since $\mathrm{PM} \perp \mathrm{L}$, we have:

$$(5\lambda - 3) \cdot 5 + (2\lambda + 1) \cdot 2 + (3\lambda - 4) \cdot 3 = 0$$

Expanding this gives:

$$25\lambda - 15 + 4\lambda + 2 + 9\lambda - 12 = 0$$

Combining like terms:

$$(25\lambda + 4\lambda + 9\lambda) - 15 + 2 - 12 = 0$$

$$38\lambda - 25 = 0$$

Solving for λ:

$$\lambda = \frac{25}{38}$$

Substituting $\lambda = 1$ into $\mathbf{M}$:

$$\therefore \mathbf{M}(2, 3, -1)$$

Calculating the length of $\mathbf{PM}$:

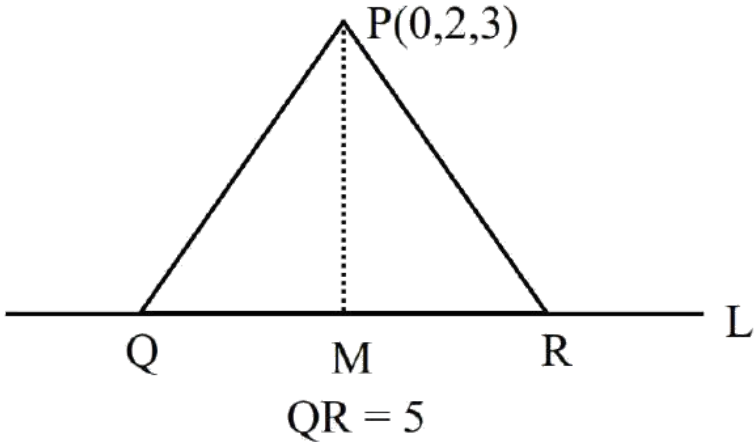
$$\mathbf{PM} = \sqrt{(5-2)^2 + (2-3)^2 + (3+1)^2} = \sqrt{4 + 1 + 16} = \sqrt{21}$$

Calculating the area:

$$\mathbf{Area} = \frac{1}{2} \times 5 \times \sqrt{21} = \frac{5\sqrt{21}}{2}$$

Setting up the equation:

$$2m - 5\sqrt{21}n = 0$$



$QR = 5$

Question 261

The line $\mathbf{L_1}$ is parallel to the vector $\overrightarrow{\mathbf{a}} = -3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ and passes through the point $(7, 6, 2)$ and the line $\mathbf{L_2}$ is parallel to the vector $\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$ and passes through the point $(5, 3, 4)$. The shortest distance between the lines $\mathbf{L_1}$ and $\mathbf{L_2}$ is :

A. $\frac{23}{\sqrt{38}}$

B. $\frac{21}{\sqrt{57}}$

C. $\frac{23}{\sqrt{57}}$

D. $\frac{21}{\sqrt{38}}$

Correct answer: A

Source solution

Sol.

Given the lines:

$$\mathbf{L_1} : (7\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 2\mathbf{k}) + \lambda(-3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 4\mathbf{k})$$

$$\mathbf{L_2} : (5\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\mathbf{k}) + \mu(2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\mathbf{k})$$

To find the distance between the skew lines, we use the formula:

$$\text{Distance} = \frac{(\mathbf{a_2} - \mathbf{a_1}) \cdot (\mathbf{b_1} \times \mathbf{b_2})}{\|\mathbf{b_1} \times \mathbf{b_2}\|}$$

Where:
- $\mathbf{a_1} = (7, 6, 2)$
- $\mathbf{a_2} = (5, 3, 4)$
- $\mathbf{b_1} = (-3, 2, 4)$
- $\mathbf{b_2} = (2, 1, 3)$

Calculating $\mathbf{a_2} - \mathbf{a_1}$:

$$\mathbf{a_2} - \mathbf{a_1} = (5-7, 3-6, 4-2) = (-2, -3, 2)$$

Next, we compute $\mathbf{b_1} \times \mathbf{b_2}$:

$$\mathbf{b_1} \times \mathbf{b_2} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -3 & 2 & 4 \\ 2 & 1 & 3 \end{vmatrix} = \hat{\mathbf{i}}(2 \cdot 3 - 4 \cdot 1) - \hat{\mathbf{j}}(-3 \cdot 3 - 4 \cdot 2) + \hat{\mathbf{k}}(-3 \cdot 1 - 2 \cdot 2)$$

Calculating the determinant:

$$= \hat{\mathbf{i}}(6 - 4) - \hat{\mathbf{j}}(-9 - 8) + \hat{\mathbf{k}}(-3 - 4)$$

$$= 2\hat{\mathbf{i}} + 17\hat{\mathbf{j}} - 7\hat{\mathbf{k}}$$

Now, we find the magnitude $\|\mathbf{b_1} \times \mathbf{b_2}\|$:

$$\|\mathbf{b_1} \times \mathbf{b_2}\| = \sqrt{2^2 + 17^2 + (-7)^2} = \sqrt{4 + 289 + 49} = \sqrt{342}$$

Now substituting into the distance formula:

$$\text{Distance} = \frac{(-2, -3, 2) \cdot (2, 17, -7)}{\sqrt{342}}$$

Calculating the dot product:

$$= \frac{(-2 \cdot 2) + (-3 \cdot 17) + (2 \cdot -7)}{\sqrt{342}} = \frac{-4 - 51 - 14}{\sqrt{342}} = \frac{-69}{\sqrt{342}}$$

Thus, the distance is:

$$= \frac{69}{\sqrt{342}} = \frac{69}{3\sqrt{38}} = \frac{23}{\sqrt{38}}$$

Final answer:

$$\frac{23}{\sqrt{38}}$$

Question 262

The line $\mathbf{L_1}$ is parallel to the vector $\overrightarrow{\mathbf{a}} = -3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ and passes through the point $(7, 6, 2)$ and the line $\mathbf{L_2}$ is parallel to the vector $\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$ and passes through the point (5, 3, 4). The shortest distance between the lines $\mathbf{L_1}$ and L_2 is :

A. $\frac{23}{\sqrt{38}}$

B. $\frac{21}{\sqrt{57}}$

C. $\frac{23}{\sqrt{57}}$

D. $\frac{21}{\sqrt{38}}$

Correct answer: A

Source solution

Sol. $\mathbf{L_1} : (7\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 2\mathbf{k}) + \lambda(-3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 4\mathbf{k})\ \mathbf{L_2} : (5\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\mathbf{k}) + \lambda(2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\mathbf{k})$

Distance between skew lines

$= \frac{(2\hat{i}+3\hat{j}-2\hat{k})\cdot(2\hat{i}+17\hat{j}-7\hat{k})}{\sqrt{342}}$

$= \frac{69}{\sqrt{342}} = \frac{69}{3\sqrt{38}} = \frac{23}{\sqrt{38}}$

Question 263

If the image of the point $(4,4,3)$ in the line $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-1}{3}$ is (α,β,γ), then $\alpha+\beta+\gamma$ is equal to

A. 9

B. 12

C. 8

D. 7

Correct answer: A

Source solution

Sol. $\frac{\int^{P(4,4,3)}}{Q(2\lambda+1,\lambda+2,3\lambda+11)}\ \frac{x-1}{2}=\frac{y-2}{1}=\frac{z-1}{3}$

$\overrightarrow{PQ}\perp(2\hat{i}+\hat{j}+3\hat{k})$

$\Rightarrow 2(2\lambda-3)+1(\lambda-2)+3(3\lambda-2)=0$

$\Rightarrow 14\lambda-14=0,\lambda=1$

So, $Q(3,3,4)$

Let image in $R(\alpha,\beta,\gamma)$

$\frac{\alpha+4}{2}=3,\frac{\beta+4}{2}=3,\frac{\gamma+3}{2}=4$

$(\alpha,\beta,\gamma)=(2,2,5)$

$\Rightarrow \alpha+\beta+\gamma=9$

Question 264

Let P the point of intersection of the lines $\frac{x-2}{1}=\frac{y-4}{5}=\frac{z-2}{1}$ and $\frac{x-3}{2}=\frac{y-2}{3}=\frac{z-3}{2}$.

Then, the shortest distance of P from the line $4x = 2y = z$ is

A. $\frac{5\sqrt{14}}{7}$

B. $\frac{\sqrt{14}}{7}$

C. $\frac{3\sqrt{14}}{7}$

D. $\frac{6\sqrt{14}}{7}$

Correct answer: C

Source solution

$$\text{Line } L_1: \quad \frac{x-2}{1}=\frac{y-4}{5}=\frac{z-2}{1}=\lambda$$

Hence,

$$x=2+\lambda, \qquad y=4+5\lambda, \qquad z=2+\lambda.$$

Similarly,

$$\text{Line } L_2: \quad \frac{x-3}{2}=\frac{y-2}{3}=\frac{z-3}{2}=\mu$$

gives

$$x=3+2\mu, \qquad y=2+3\mu, \qquad z=3+2\mu.$$

Since the two lines intersect at P,

$$2+\lambda=3+2\mu,$$

$$4+5\lambda = 2+3\mu.$$

From the first equation,

$$\lambda - 2\mu = 1.$$

From the second,

$$5\lambda - 3\mu = -2.$$

Substituting $\lambda = 1 + 2\mu$,

$$5(1+2\mu) - 3\mu = -2$$

$$5 + 7\mu = -2$$

$$\mu = -1.$$

Therefore,

$$\lambda = 1 + 2(-1) = -1.$$

Hence,

$$P = (2-1,\ 4-5,\ 2-1) = (1,-1,1).$$

The line

$$4x = 2y = z$$

can be written as

$$\frac{x}{1} = \frac{y}{2} = \frac{z}{4} = t.$$

Thus it passes through the origin and has direction vector

$$\vec{d} = (1, 2, 4).$$

Let

$$\vec{r} = (1, -1, 1)$$

be the position vector of P.

The shortest distance from a point to a line through the origin is

$$D = \frac{\left|\vec{r} \times \vec{d}\right|}{|\vec{d}|}.$$

Now,

$$\vec{r} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 1 & 2 & 4 \end{vmatrix} = (-6, -3, 3).$$

Therefore,

$$|\vec{r} \times \vec{d}| = \sqrt{(-6)^2 + (-3)^2 + 3^2} = \sqrt{54} = 3\sqrt{6}.$$

Also,

$$|\vec{d}| = \sqrt{1^2 + 2^2 + 4^2} = \sqrt{21}.$$

Hence,

$$D = \frac{3\sqrt{6}}{\sqrt{21}} = 3\sqrt{\frac{2}{7}} = \frac{3\sqrt{14}}{7}.$$

$$\boxed{\frac{3\sqrt{14}}{7}}$$

Hence, the correct option is

$$\boxed{(3)\ \frac{3\sqrt{14}}{7}}.$$

Question 265

If $Q(0,-1,-3)$ is the image of the point P in the plane $3x-y+4z=2$ and R is the point $(3,-1,-2)$, then the area (in sq. units) of $\triangle PQR$ is :

A. $\frac{\sqrt{65}}{2}$

B. $\frac{\sqrt{91}}{4}$

C. $2\sqrt{13}$

D. $\frac{\sqrt{91}}{2}$

Correct answer: D

Source solution

Sol. Image of Q in plane

$\frac{(x-0)}{3}=\frac{(y+1)}{-1}=\frac{z+3}{+4}=\frac{-2(1-12-2)}{9+1+16}=1$

$x=3, y=-2, z=1$

$\mathbf{P}(3,-2,1), \mathbf{Q}(0,-1,-3), \mathbf{R}(3,-1,-2)$

Now area of $\triangle PQR$ is

$$\begin{aligned}\frac{1}{2}|\overrightarrow{PQ}\times\overrightarrow{QR}| &= \frac{1}{2}\left\|\hat{i} \quad \hat{j} \quad \hat{k}3 \quad -1 \quad 43 \quad 0 \quad 1\right\| \\ &= \frac{1}{2}|\{\hat{i}(-1)-\hat{j}(3-12)+\hat{k}(3)\}| \\ &= \frac{1}{2}\sqrt{(1+81+9)} \\ &= \frac{\sqrt{91}}{2}\end{aligned}$$

Question 266

Let the position vectors of two points P and Q be $3\hat{i}-\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}-4\hat{k}$, respectively. Let R and S be two points such that the direction ratios of lines PR and QS are $(4,-1,2)$ and $(-2,1,-2)$, respectively. Let lines PR and QS intersect at T. If the vector $\overrightarrow{TA}$ is perpendicular to both $\overrightarrow{PR}$ and $\overrightarrow{QS}$ and the length of vector $\overrightarrow{TA}$ is $\sqrt{5}$ units, then the modulus of a position vector of A is:

A. $\sqrt{482}$

B. $\sqrt{171}$

C. $\sqrt{5}$

D. $\sqrt{227}$

Correct answer: B

Source solution

Sol. Let points be defined as follows:

$$\mathrm{P}(3,-1,2), \quad \mathrm{Q}(1,2,-4)$$

Direction ratios of the normal to the plane containing points $\mathrm{P}, \mathrm{T}, \mathrm{Q}$ will be proportional to the determinant:

$$\begin{vmatrix} \hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}} \\ 4 & -1 & 2 \\ -2 & 1 & -2 \end{vmatrix}$$

Thus, we have:

$$\frac{\ell}{0}=\frac{m}{4}=\frac{n}{2}$$

For point T, we express the vector $\overrightarrow{\mathrm{PT}}$ as:

$$\overrightarrow{\mathrm{PT}}=\frac{x-3}{4}=\frac{y+1}{-1}=\frac{z-2}{2}=\lambda$$

Similarly, for point Q:

$$\overrightarrow{\mathrm{QT}}=\frac{x-1}{-2}=\frac{y-1}{1}=\frac{z+4}{-2}=\mu$$

The coordinates of point $\mathbf{T}$ can be expressed as:

$$\mathbf{T} : (4\lambda + 3, -\lambda - 1, 2\lambda + 2)$$

And for point $\mathbf{Q}$:

$$\mathbf{T} \cong (2\mu + 1, \mu + 2, -2\mu - 4)$$

Setting the equations equal gives:

$$4\lambda + 3 = -2\mu + 1 \quad \Rightarrow \quad 2\lambda + \mu = -1$$

$$\lambda + \mu = -3$$

Solving these equations:
1. From $2\lambda + \mu = -1$
2. Substitute $\mu = -3 - \lambda$ into the first equation:

$$2\lambda + (-3 - \lambda) = -1 \quad \Rightarrow \quad \lambda - 3 = -1 \quad \Rightarrow \quad \lambda = 2$$

3. Substitute $\lambda = 2$ back to find μ:

$$\mu = -3 - 2 = -5$$

Thus, point $\mathbf{T}$ is:

$$\mathbf{T} : (11, -3, 6)$$

Now, we calculate $\overrightarrow{\mathbf{OA}}$:

$$\overrightarrow{\mathbf{OA}} = (11\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 6\hat{\mathbf{k}}) \pm \left(\frac{2\hat{\mathbf{j}} + \hat{\mathbf{k}}}{\sqrt{5}} \right) \sqrt{5}$$

This simplifies to:

$$\overrightarrow{OA} = (11\hat{i} - 3\hat{j} + 6\hat{k}) \pm (2\hat{j} + \hat{k})$$

Thus, we have two possible vectors:

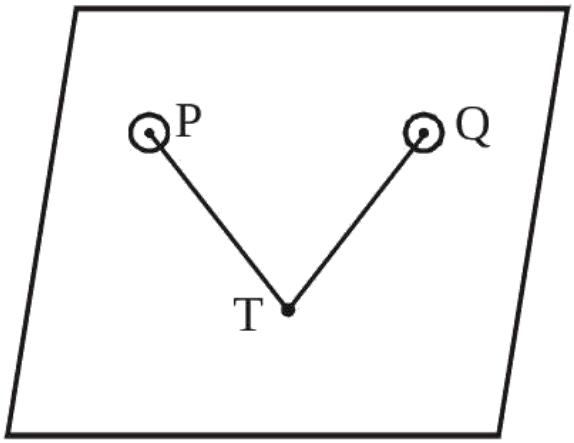
$$\overrightarrow{OA} = 11\hat{i} - \hat{j} + 7\hat{k} \quad \text{or} \quad 9\hat{\mathrm{i}} - 5\hat{\mathrm{j}} + 5\hat{\mathrm{k}}$$

Calculating the magnitudes:

$$|\overrightarrow{OA}| = \sqrt{121 + 1 + 49} = \sqrt{171}$$

or

$$|\overrightarrow{OA}| = \sqrt{81 + 25 + 25} = \sqrt{131}$$



Question 267

Let A be the point of intersection of the lines $\mathbf{L_1 : \frac{x-7}{1} = \frac{y-5}{0} = \frac{z-3}{-1}}$ and $\mathbf{L_2 : \frac{x-1}{3} = \frac{y+3}{4} = \frac{z+7}{5}}$. Let B and C be the point on the lines L_1 and L_2 respectively such that $\mathbf{AB = AC = \sqrt{15}}$. Then the square of the area of the triangle ABC is :

A. 54

B. 63

C. 57

D. 60

Correct answer: A

Source solution

Sol. Angle between both lines

$\cos\theta = \left|\frac{3+0-5}{\sqrt{2}\sqrt{50}}\right|$

$\sin\theta = \frac{2}{10} = \frac{1}{5}$

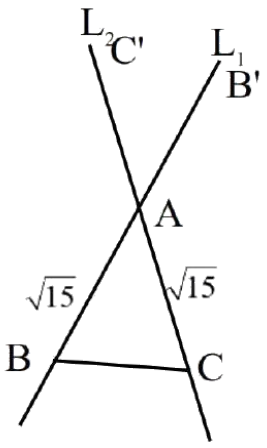
$\sin\theta = \frac{\sqrt{24}}{5}$

area $= \frac{1}{2}ab\sin\theta$

$\frac{1}{2}\sqrt{15}\sqrt{15}\frac{\sqrt{24}}{5}$

square of area $\frac{15.15.24}{4.25}$

option (1)



Question 268

Let the position vectors of three vertices of a triangle be $4\vec{p}+\vec{q}-3\vec{r}, -5\vec{p}+\vec{q}+2\vec{r}$ and $2\vec{p}-\vec{q}+2\vec{r}$. If the position vectors of the orthocenter and the circumcenter of the triangle are $\frac{\vec{p}+\vec{q}+\vec{r}}{4}$ and $\alpha\vec{p}+\beta\vec{q}+\gamma\vec{r}$ respectively, then $\alpha+2\beta+5\gamma$ is equal to:

A. 3

B. 1

C. 6

D. 4

Correct answer: A

Source solution

Sol. We know that:

- Orthocenter $O=\frac{\vec{p}+\vec{q}+\vec{r}}{4}$
- Circumcenter $C=\alpha\vec{p}+\beta\vec{q}+\gamma\vec{r}$
- Centroid $G=\frac{\vec{p}+\vec{q}+\vec{r}}{3}$

Using the relation:

$$2(\alpha\vec{p}+\beta\vec{q}+\gamma\vec{r})+\frac{\vec{p}+\vec{q}+\vec{r}}{4}=3\left(\frac{\vec{p}+\vec{q}+\vec{r}}{3}\right)$$

We simplify:

$$8(\alpha\vec{p}+\beta\vec{q}+\gamma\vec{r})=3(\vec{p}+\vec{q}+\vec{r})$$

From this, we get:

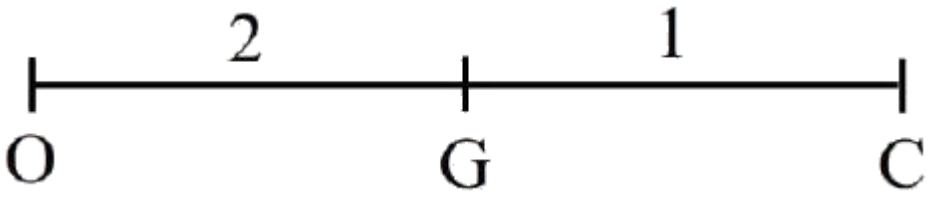
$$8\alpha=3,\quad 8\beta=3,\quad 8\gamma=3$$

Thus,

$$\alpha = \frac{3}{8}, \quad \beta = \frac{3}{8}, \quad \gamma = \frac{3}{8}$$

Now, we calculate:

$$\alpha + 2\beta + 3\gamma = \frac{3}{8} + \frac{6}{8} + \frac{15}{8} = \frac{24}{8} = 3$$



Question 269

Let the line L pass through the point $(0,1,2)$, intersect the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and be parallel to the plane $2x+y-3z=4$. Then the distance of the point $P(1,-9,2)$ from the line L is

A. 9

B. $\sqrt{54}$

C. $\sqrt{69}$

D. $\sqrt{74}$

Correct answer: D

Source solution

$$\text{Let } \overrightarrow{AB}\cdot\vec{n}=0$$

$$\overrightarrow{AB}=(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1+4\lambda)\hat{k}$$

$$\vec{n}=2\hat{i}+\hat{j}-3\hat{k}$$

Calculating the dot product:

$$(1+2\lambda)\cdot 2+(1+3\lambda)\cdot 1+(1+4\lambda)\cdot(-3)=0$$

Expanding this gives:

$$2+4\lambda+1+3\lambda-3-12\lambda=0$$

Combining like terms:

$$5\lambda=0\Rightarrow\lambda=0$$

Now, the line $\overrightarrow{\mathbf{AB}}$ can be expressed as:

$$\overrightarrow{\mathbf{r}} = \hat{\mathbf{j}} + 2\hat{\mathbf{k}} + \mu(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$$

The general form of point Q is:

$$Q(\mu, 1 + \mu, 2 + \mu)$$

Since $\overrightarrow{\mathbf{PQ}} \cdot \overrightarrow{\mathbf{AB}} = 0$:

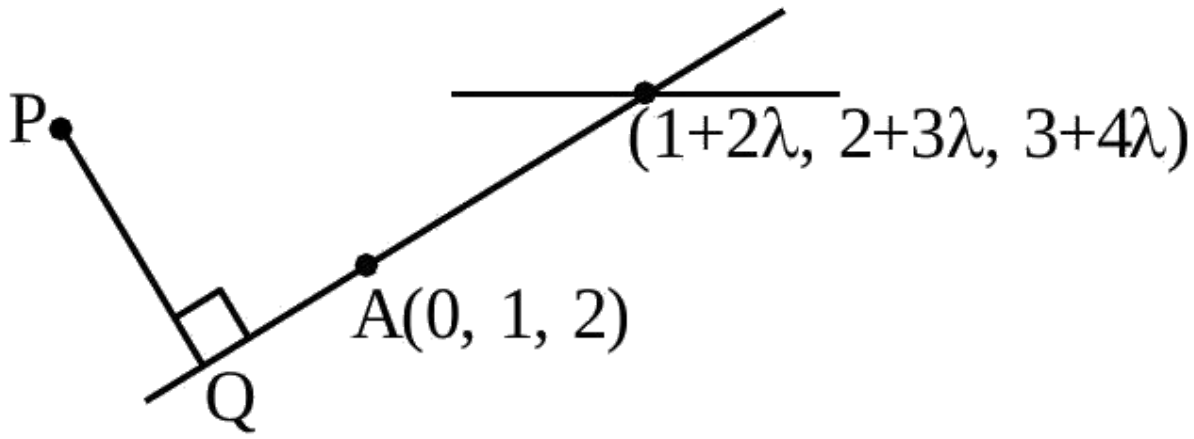
$$(\mu - 1) + (10 + \mu) + \mu = 0$$

Simplifying this:

$$3\mu - 1 + 10 = 0 \Rightarrow 3\mu = -9 \Rightarrow \mu = -3$$

Finally, the distance is calculated as:

$$\text{Distance} = \sqrt{16 + 49 + 9} = \sqrt{74}$$



Question 270

Let (α, β) be the centroid of the triangle formed by the lines $15x - y = 82$, $6x - 5y = -4$ and $9x + 4y = 17$. Then $\alpha + 2\beta$ and $2\alpha - \beta$ are the roots of the equation

A. $x^2 - 7x + 12 = 0$

B. $x^2 - 13x + 42 = 0$

C. $x^2 - 14x + 48 = 0$

D. $x^2 - 10x + 25 = 0$

Correct answer: B

Source solution

Sol.

Upon solving, we find the coordinates of the points as $(6,8)$, $(1,2)$, and $(5,-7)$.

The centroid (α, β) is calculated as follows:

$$\alpha = \frac{6+1+5}{3} = 4$$

$$\beta = \frac{8+2-7}{3} = 1$$

Next, we have the equations:

$$\alpha + 2\beta = 6$$

$$2\alpha - \beta = 7$$

The final answer is:

$$x^2 - 13x + 42 = 0$$

Question 271

Consider the line L passing through the points $(1, 2, 3)$ and $(2, 3, 5)$. The distance of the point $\left(\frac{11}{3}, \frac{11}{3}, \frac{19}{3}\right)$ from the line L along the line $\frac{3x-11}{2} = \frac{3y-11}{1} = \frac{3z-19}{2}$ is equal to :

A. 3

B. 5

C. 4

D. 6

Correct answer: A

Source solution

Sol.

Given the equations:

$$\frac{x-1}{2-1} = \frac{y-2}{3-2} = \frac{z-3}{5-3}$$

This simplifies to:

$$\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{2} = \lambda$$

From this, we can express x, y, and z in terms of λ:

$$x = 1 + \lambda, \quad y = 2 + \lambda, \quad z = 3 + 2\lambda$$

The point B can be represented as:

$$B(1 + \lambda, 2 + \lambda, 3 + 2\lambda)$$

The direction ratios (D.R.) of line segment AB are given by:

$$\left\langle \frac{3\lambda-8}{3}, \frac{3\lambda-5}{3}, \frac{6\lambda-10}{3} \right\rangle$$

Substituting B into the equation:

$$B\left(\frac{5}{3}, \frac{8}{3}, \frac{13}{3}\right)$$

Setting the ratios equal:

$$\frac{3\lambda-8}{3\lambda-5} = \frac{2}{1}$$

Cross-multiplying gives:

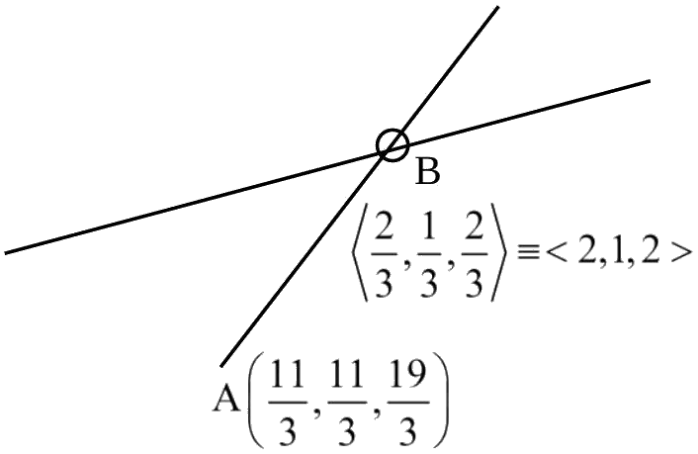
$$3\lambda - 8 = 6\lambda - 10$$

Rearranging the equation:

$$3\lambda = 2 \quad \Rightarrow \quad \lambda = \frac{2}{3}$$

Now, we calculate the length of AB:

$$AB = \frac{\sqrt{36+9+36}}{3} = \frac{9}{3} = 3$$


B
$\left\langle \frac{2}{3}, \frac{1}{3}, \frac{2}{3} \right\rangle \equiv < 2,1,2 >$
$A\left(\frac{11}{3}, \frac{11}{3}, \frac{19}{3}\right)$

Question 272

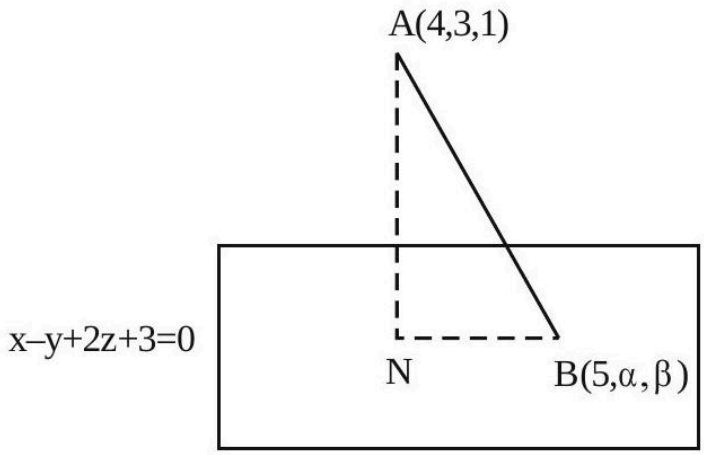
Let the foot of perpendicular from the point $\mathbf{A(4, 3}$, 1) on the plane $P : x - y + 2z + 3 = 0$ be N. If $B(5$, $\alpha, \beta), \alpha, \beta \in \mathbb{Z}$ is a point on plane P such that the area of the triangle ABN in $3\sqrt{2}$, then $\alpha^2 + \beta^2 + \alpha\beta$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 7

Source solution



$\mathbf{AN} = \sqrt{6}$

$5 - \alpha + 2\beta + 3 = 0$

$$\Rightarrow \quad \alpha = 8 + 2\beta \qquad (1)$$

N is given by

$$\frac{x-4}{1}=\frac{y-3}{-1}=\frac{z-1}{2}=\frac{-(4-3+2+3)}{1+1+4}$$

$\Rightarrow$ $x=3, y=4, z=-1$

$\text{N is } (3,4,-1)$

$\text{BN}=\sqrt{4+(\alpha-4)^2+(\beta+1)^2}$

$=\sqrt{4+(2\beta+4)^2+(\beta+1)^2}$

$\text{Area of } \Delta\text{ABN}=\frac{1}{2}\text{AN}\times\text{BN}=3\sqrt{2}$

$\frac{1}{2}\times\sqrt{6}\times\text{BN}=3\sqrt{2}$

$\text{BN}=2\sqrt{3}$

$\Rightarrow$ $4+(2\beta+4)^2+(\beta+1)^2=12$

$(2\beta+4)^2+(\beta+1)^2-8=0$

$5\beta^2+18\beta+9=0$

$(5\beta+3)(\beta+3)=0$

$\alpha=-3$

$\Rightarrow$ $\alpha=2$

$\alpha^2+\beta^2+\alpha\beta=9+4-6=7$

Question 273

Let the line passing through the points $(-1, 2, 1)$ and parallel to the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{4}$ intersect the line $\frac{x+2}{3} = \frac{y-3}{2} = \frac{z-4}{1}$ at the point P. Then the distance of P from the point $Q(4, -5, 1)$ is :

A. 5

B. 4

C. (2) 10

D. (3) $5\sqrt{6}$

Correct answer: D

Source solution

$$\text{Given line } L_1: \quad \frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{4}.$$

Its direction vector is

$$\vec{d}_1 = (2, 3, 4).$$

Therefore, the line passing through $(-1, 2, 1)$ and parallel to L_1 is

$$\frac{x+1}{2} = \frac{y-2}{3} = \frac{z-1}{4} = \lambda.$$

Hence,

$$x = -1 + 2\lambda, \qquad y = 2 + 3\lambda, \qquad z = 1 + 4\lambda.$$

The second line is

$$\frac{x+2}{3} = \frac{y-3}{2} = \frac{z-4}{1} = \mu.$$

Thus,

$$x = -2 + 3\mu, \qquad y = 3 + 2\mu, \qquad z = 4 + \mu.$$

Since the two lines intersect at P,

$$-1 + 2\lambda = -2 + 3\mu, \tag{1}$$

$$2 + 3\lambda = 3 + 2\mu, \tag{2}$$

$$1 + 4\lambda = 4 + \mu. \tag{3}$$

From (3),

$$\mu = 4\lambda - 3.$$

Substituting into (1),

$$-1 + 2\lambda = -2 + 3(4\lambda - 3),$$

$$-1 + 2\lambda = 12\lambda - 11,$$

$$10 = 10\lambda,$$

$$\lambda = 1.$$

Hence,

$$\mu = 4(1) - 3 = 1.$$

Therefore,

$$P = (-1 + 2,\ 2 + 3,\ 1 + 4) = (1, 5, 5).$$

Given

$$Q = (4, -5, 1).$$

Hence,

$$PQ = \sqrt{(4 - 1)^2 + (-5 - 5)^2 + (1 - 5)^2}.$$

$$= \sqrt{3^2 + (-10)^2 + (-4)^2}.$$

$$= \sqrt{9 + 100 + 16}.$$

$$= \sqrt{125} = 5\sqrt{5}.$$

$$\boxed{5\sqrt{6}}$$

Therefore, the distance of P from Q is

$\boxed{5\sqrt{6}}$.

Question 274

Let the vertices Q and R of the triangle PQR lie on the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}, \mathrm{QR} = 5$ and the coordinates of the point P be $(0, 2, 3)$. If the area of the triangle PQR is $\frac{m}{n}$ then :

A. $\mathrm{m} - 5\sqrt{21}\mathrm{n} = 0$

B. $2\,\mathrm{m} - 5\sqrt{21}\mathrm{n} = 0$

C. $5\,\mathrm{m} - 2\sqrt{21}\mathrm{n} = 0$

D. $5\,\mathrm{m} - 21\sqrt{2}\mathrm{n} = 0$

Correct answer: B

Source solution

Sol.

$\mathrm{M}(5\lambda - 3, 2\lambda + 1, 3\lambda - 4)$

Drs of $\mathrm{PM} \Rightarrow 5\lambda - 3, 2\lambda - 1, 3\lambda - 7$

Drs of line $\mathrm{L} \Rightarrow 5, 2, 3$

$\mathrm{PM} \perp \mathrm{L}$

$\Rightarrow (5\lambda - 3)5 + (2\lambda - 1)2 + (3\lambda - 7)3 = 0$

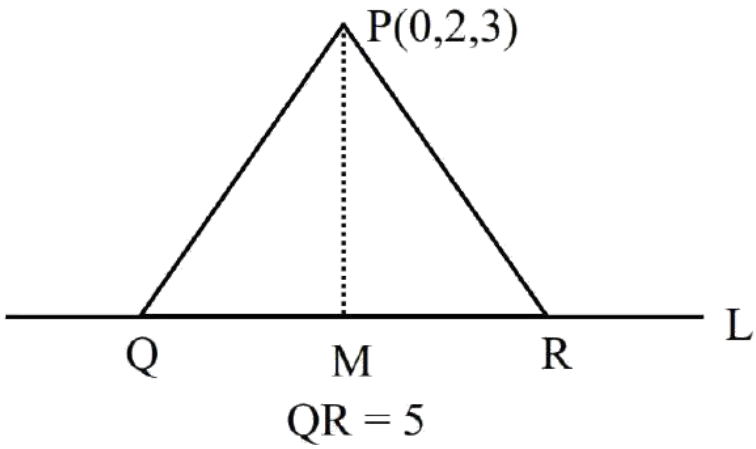
$\Rightarrow \lambda = 1$

$\therefore \mathrm{M}(2, 3, -1)$

$\mathrm{PM} = \sqrt{4 + 1 + 16} = \sqrt{21}$

Area $= \frac{1}{2} \times 5 \times \sqrt{21} = \frac{\mathrm{m}}{\mathrm{n}}$

$2\,\mathrm{m} - 5\sqrt{21}\mathrm{n} = 0$



Question 275

Let (α, β, γ) be mirror image of the point $(2, 3, 5)$ in the line $\frac{x-1}{2} - \frac{y-2}{3} - \frac{z-3}{4}$.
Then $2\alpha + 3\beta + 4\gamma$ is equal to

A. 32

B. 33

C. 31

D. 34

Correct answer: B

Source solution

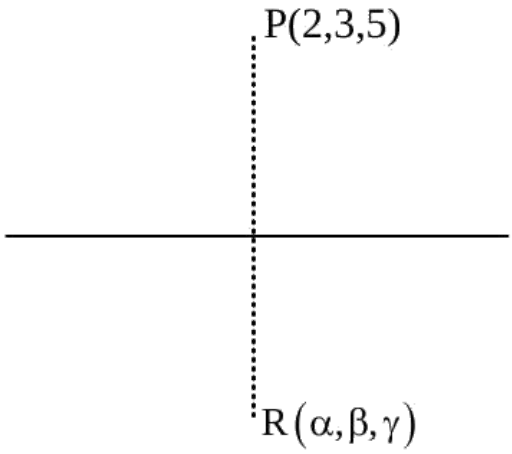
Sol.

$\because \overrightarrow{PR} \perp (2, 3, 4)$

$\therefore \overrightarrow{PR} \cdot (2, 3, 4) = 0$

$(\alpha - 2, \beta - 3, \gamma - 5) \cdot (2, 3, 4) = 0$

$\Rightarrow 2\alpha + 3\beta + 4\gamma = 4 + 9 + 20 = 33$



Question 276

Let P be the point of intersection of the lines

$$\frac{x-2}{1}=\frac{y-4}{5}=\frac{z-2}{1}$$

and

$$\frac{x-3}{2}=\frac{y-2}{3}=\frac{z-3}{2}.$$

Then, the shortest distance of P from the line $4x=2y=z$ is

A. $\frac{5\sqrt{14}}{7}$

B. $\frac{\sqrt{14}}{7}$

C. $\frac{3\sqrt{14}}{7}$

D. $\frac{6\sqrt{14}}{7}$

Correct answer: C

Source solution

$$\text{Let } L_1 \equiv \frac{x-2}{1}=\frac{y-4}{5}=\frac{z-2}{1}=\lambda$$

$$P(\lambda+2,5\lambda+4,\lambda+2)$$

$$L_2 \equiv \frac{x-3}{2}=\frac{y-2}{3}=\frac{z-3}{2}$$

$$P(2\mu+3,3\mu+2,2\mu+3)$$

Setting the equations equal:

$$\lambda+2=2\mu+3 \quad \text{and} \quad 3\mu+2=5\lambda+4$$

From the first equation:

$$\lambda = 2\mu + 1$$

Substituting into the second equation:

$$3\mu = 5(2\mu + 1) + 2$$

$$3\mu = 10\mu + 7$$

$$-7 = 7\mu \quad \Rightarrow \quad \mu = -1$$

$$\lambda = -1$$

Both satisfy P:

$$P(1, -1, 1)$$

Next, consider:

$$L_3 \equiv \frac{x}{1/4} = \frac{y}{1/2} = \frac{z}{1}$$

$$L_3 = \frac{x}{1} = \frac{y}{2} = \frac{z}{4} = k$$

Coordinates of $Q(k, 2k, 4k)$.

Direction ratios of $PQ = \langle k - 1, 2k + 1, 4k - 1 \rangle$.

Since $PQ \perp L_3$:

$$(k - 1) + 2(2k + 1) + 4(4k - 1) = 0$$

$$k - 1 + 4k + 2 + 16k - 4 = 0$$

$$21k - 3 = 0 \quad \Rightarrow \quad k = \frac{1}{7}$$

Thus, the coordinates of Q are:

$$Q\left(\frac{1}{7}, \frac{2}{7}, \frac{4}{7}\right)$$

Now, calculate PQ:

$$PQ = \sqrt{\left(1 - \frac{1}{7}\right)^2 + \left(-1 - \frac{2}{7}\right)^2 + \left(1 - \frac{4}{7}\right)^2}$$

$$= \sqrt{\frac{36}{49} + \frac{81}{49} + \frac{9}{49}} = \sqrt{\frac{126}{49}} = \frac{\sqrt{126}}{7}$$

$$PQ = \frac{3\sqrt{14}}{7}$$

Thus, the answer is:

Option 3 will satisfy.

Question 277

The shortest distance between lines $\mathbf{L_1}$ and $\mathbf{L_2}$, where $L_1 : \frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+4}{2}$ and L_2 is the line passing through the points $\mathbf{A(-4, 4, 3)} \cdot \mathbf{B(-1, 6, 3)}$ and perpendicular to the line $\frac{x-3}{-2} = \frac{y}{3} = \frac{z-1}{1}$, is

A. $\frac{121}{\sqrt{221}}$

B. $\frac{24}{\sqrt{117}}$

C. $\frac{141}{\sqrt{221}}$

D. $\frac{42}{\sqrt{117}}$

Correct answer: C

Source solution

Sol.-

$\mathbf{L_2} = \frac{x+4}{3} = \frac{y-4}{2} = \frac{z-3}{0}$

$\therefore$ S.D $= \frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ 2 & -3 & 23 \\ 2 & 0 & \end{vmatrix}}{\left|\vec{n_1} \times \vec{n_2}\right|}$

$= \frac{\begin{vmatrix} 5 & -5 & -7 \\ 2 & -3 & 23 \\ 2 & 0 & \end{vmatrix}}{\left|\vec{n_1} \times \vec{n_2}\right|}$

$= \frac{141}{|-4\hat{i}+6\hat{j}+13\hat{k}|}$

$= \frac{141}{\sqrt{16+36+169}}$

$= \frac{141}{\sqrt{221}}$

Question 278

The distance of the point $\mathbf{Q}(0, 2, -2)$ form the line passing through the point $\mathbf{P}(5, -4, 3)$ and perpendicular to the lines $\vec{\mathbf{r}} = (-3\hat{\mathbf{i}} + 2\hat{\mathbf{k}}) + \lambda(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}), \quad \lambda \in \mathbb{R}$ and $\vec{\mathbf{r}} = (\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}) + \mu(-\hat{i} + 3\hat{j} + 2\hat{k}), \mu \in \mathbb{R}$ is

A. $\sqrt{86}$

B. $\sqrt{20}$

C. $\sqrt{54}$

D. $\sqrt{74}$

Correct answer: D

Source solution

Sol. A vector in the direction of the required line can be obtained by cross product of

$\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 3 & 5 \\ -1 & 3 & 2 \end{vmatrix}$

$= -9\hat{i} - 9\hat{j} + 9\hat{k}$

Required line,

$\vec{\mathbf{r}} = (5\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) + \lambda'(-9\hat{\mathbf{i}} - 9\hat{\mathbf{j}} + 9\hat{\mathbf{k}})$

$\vec{\mathbf{r}} = (5\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) + \lambda(\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}})$

Now distance of (0, 2, -2)

P.V. of $\mathbf{P} \equiv (5 + \lambda)\hat{\mathbf{i}} + (\lambda - 4)\hat{\mathbf{j}} + (3 - \lambda)\hat{\mathbf{k}}$

$\overrightarrow{\mathbf{AP}} = (5 + \lambda)\hat{\mathbf{i}} + (\lambda - 6)\hat{\mathbf{j}} + (5 - \lambda)\hat{\mathbf{k}}$

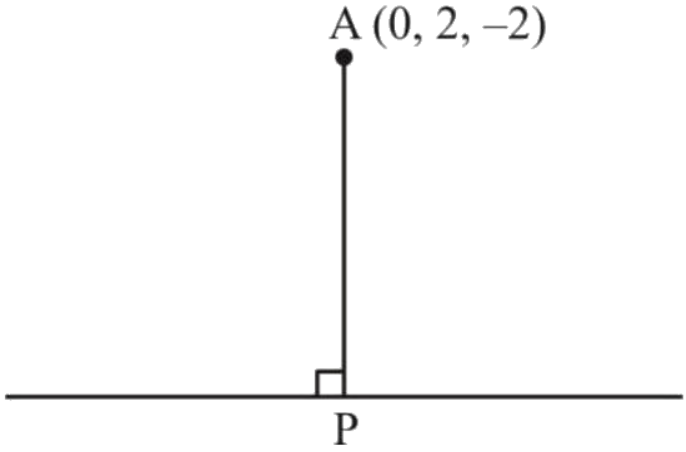
$\overrightarrow{\mathbf{AP}} \cdot (\hat{i} + \hat{j} - \hat{k}) = 0$

$5 + \lambda + \lambda - 6 - 5 + \lambda = 0$

$\lambda = 2$

$|\overrightarrow{\mathbf{AP}}| = \sqrt{49 + 16 + 9}$

$|\overrightarrow{\mathbf{AP}}| = \sqrt{74}$


A (0, 2, –2)
P

Question 279

Let the line L pass through the point $(0, 1, 2)$, intersect the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and be parallel to the plane $2x + y - 3z = 4$. Then the distance of the point $P(1, -9, 2)$ from the line L is

A. 9

B. $\sqrt{54}$

C. $\sqrt{69}$

D. $\sqrt{74}$

Correct answer: D

Source solution

Sol.

$\overrightarrow{AB} \cdot \vec{n}$

$\Rightarrow [(1 + 2\lambda)\hat{i} + (1 + 3\lambda)\hat{j} + (1 + 4\lambda)\hat{k}] \cdot (2\hat{i} + \hat{j} - 3\hat{k})$

$2 + 4\lambda + 1 + 3\lambda - 3 - 12\lambda = 0$

$5\lambda = 0 \Rightarrow \lambda = 0$

Line $\overrightarrow{AB}, \vec{r} = \hat{j} + 2\hat{k} + \mu(\hat{i} + \hat{j} + \hat{k})$

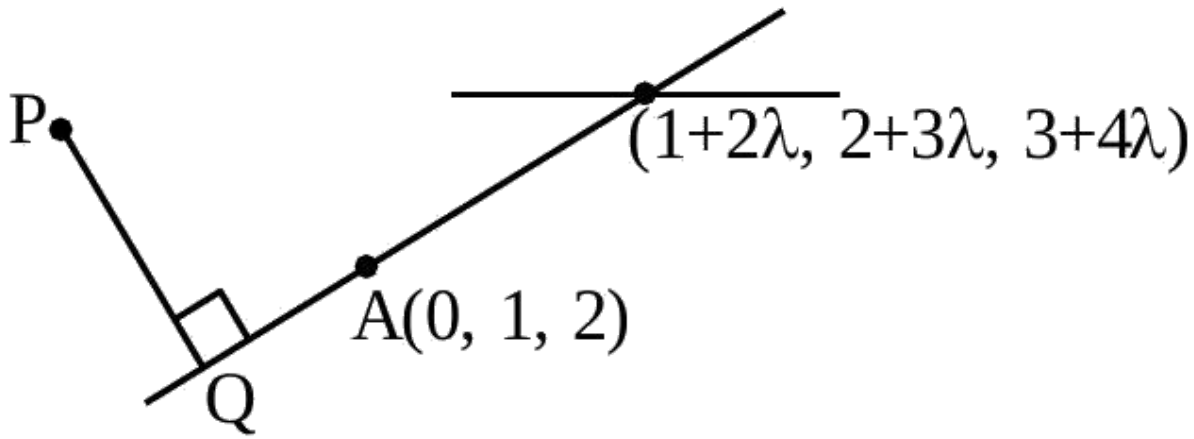
General form: $Q(\mu, 1 + \mu, 2 + \mu)$

$\therefore \overrightarrow{PQ} \cdot \overrightarrow{AB} = 0$

$(\mu - 1) + (10 + \mu) + \mu = 0$

$3\mu = -9 \Rightarrow \mu = -3$

$\therefore$ distance $= \sqrt{16 + 49 + 9} = \sqrt{74}$



Question 280

The vertices B and C of a $\triangle ABC$ lie on the line, $\frac{x+2}{3} = \frac{y-1}{0} = \frac{z}{4}$ such that $BC = 5$ units. Then the area (in sq. units) of this triangle, given that the point $A(1, -1, 2)$, is:

A. $5\sqrt{17}$

B. $\sqrt{34}$

C. 6

D. $2\sqrt{34}$

Correct answer: B

Source solution

$$\text{Area of } \triangle ABC = \frac{1}{2} \times BC \times AD.$$

Given

$$|BC| = 5,$$

so we need the perpendicular distance of A from the line BC.

Let a point D on BC be

$$D = (3\lambda - 2,\ 1,\ 4\lambda).$$

Then

$$\overrightarrow{AD} = (3\lambda - 3)\hat{i} + 2\hat{j} + (4\lambda - 2)\hat{k}.$$

Also, $\overrightarrow{AD}$ and $\overrightarrow{BC}$ are perpendicular, so

$$\overrightarrow{AD} \cdot \overrightarrow{BC} = 0.$$

Hence,

$$(3\lambda - 3)(3) + 2(0) + (4\lambda - 2)(4) = 0.$$

$$9\lambda - 9 + 16\lambda - 8 = 0.$$

$$25\lambda - 17 = 0.$$

$$\lambda = \frac{17}{25}.$$

Therefore,

$$D = \left(3 \cdot \frac{17}{25} - 2,\ 1,\ 4 \cdot \frac{17}{25}\right) = \left(\frac{1}{25},\ 1,\ \frac{68}{25}\right).$$

Now,

$$
\begin{aligned}
&|\overrightarrow{AD}| \\
&\quad = \sqrt{\left(\frac{1}{25} - 1\right)^2 + (2)^2 + \left(\frac{68}{25} - 2\right)^2} \\
&\quad = \sqrt{\left(\frac{-24}{25}\right)^2 + 4 + \left(\frac{18}{25}\right)^2} \\
&\quad = \sqrt{\frac{24^2 + 4(25)^2 + 18^2}{25^2}} \\
&\quad = \sqrt{\frac{576 + 2500 + 324}{25^2}} \\
&\quad = \sqrt{\frac{3400}{25^2}} \\
&\quad = \frac{10\sqrt{34}}{25} \\
&\quad = \frac{2\sqrt{34}}{5}.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
&\text{Area of } \triangle ABC \\
&\qquad = \frac{1}{2} \times |BC| \times |AD| \\
&\qquad = \frac{1}{2} \times 5 \times \frac{2\sqrt{34}}{5} \\
&\qquad = \sqrt{34}.
\end{aligned}
$$

$$
\boxed{\text{Area of } \triangle ABC = \sqrt{34}}
$$

Question 281

The distance of the line

$$\frac{x-2}{2}=\frac{y-6}{3}=\frac{z-3}{4}$$

from the point $(1,4,0)$ along the line

$$\frac{x}{1}=\frac{y-2}{2}=\frac{z+3}{3}$$

is :

A. $\sqrt{17}$

B. $\sqrt{14}$

C. $\sqrt{15}$

D. $\sqrt{13}$

Correct answer: B

Source solution

Let the equation of the parallel line be

$$\frac{x-1}{1}=\frac{y-4}{2}=\frac{z-0}{3}$$

The point of intersection can be expressed as

$$(\lambda+1,2\lambda+4,3\lambda)$$

We also have the parametric representation of another line:

$$(2t+2,3t+6,4t+3)$$

Setting the two expressions equal gives:

$$\lambda + 1 = 2t + 2$$

$$2\lambda + 4 = 3t + 6$$

$$3\lambda = 4t + 3$$

From the first equation, we can express λ in terms of t:

$$\lambda = 2t + 1$$

Substituting λ into the second equation:

$$2(2t + 1) + 4 = 3t + 6$$

This simplifies to:

$$4t + 2 + 4 = 3t + 6$$

Thus,

$$4t + 6 = 3t + 6$$

Solving for t:

$$t = 0$$

Now substituting $t = 0$ back into the parametric equations gives the point of intersection:

$$(2(0)+2, 3(0)+6, 4(0)+3) = (2, 6, 3)$$

Next, we calculate the distance from the point $(1, 4, 0)$ to the point of intersection $(2, 6, 3)$:

$$\text{Distance} = \sqrt{(2-1)^2 + (6-4)^2 + (3-0)^2}$$

Calculating each term:

$$= \sqrt{1^2 + 2^2 + 3^2} = \sqrt{1+4+9} = \sqrt{14}$$

Thus, the final answer is

$$\sqrt{14}$$

Question 282

Let in a $\triangle ABC$, the length of the side AC be 6, the vertex B be $(1, 2, 3)$ and the vertices A, C lie on the line

$$\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}.$$

Then the area (in sq. units) of $\triangle ABC$ is

A. 42

B. 21

C. 56

D. 17

Correct answer: B

Source solution

Let $\mathrm{M}(3\lambda+6, 2\lambda+7, -2\lambda+7)$.

First, we find the vector $\overrightarrow{\mathrm{BM}}$:

$$\overrightarrow{\mathrm{BM}} = (3\lambda+5)\hat{\mathrm{i}} + (2\lambda+5)\hat{\mathrm{j}} + (-2\lambda+4)\hat{\mathrm{k}}$$

Next, we compute the dot product $\overrightarrow{\mathrm{AC}} \cdot \overrightarrow{\mathrm{BM}} = 0$:

$$0 = 3(3\lambda+5) + 2(2\lambda+5) - 2(-2\lambda+4)$$

We also have:

$$\overrightarrow{\mathrm{BM}} = 2\hat{\mathrm{i}} + 3\hat{\mathrm{j}} + 6\hat{\mathrm{k}}$$

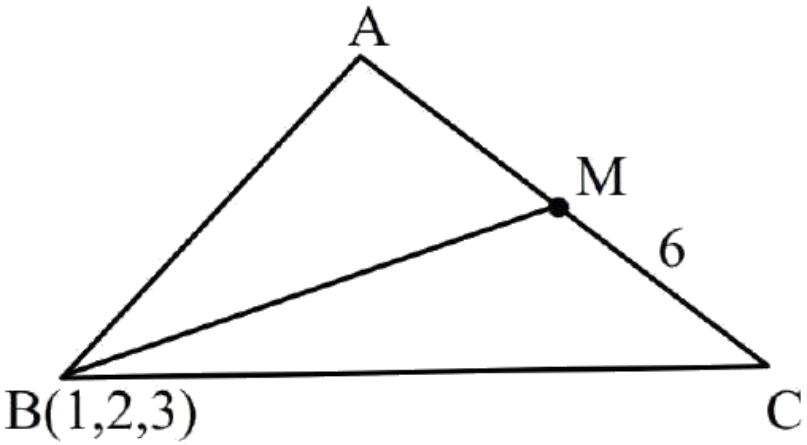
Now, we calculate the magnitude of $\overrightarrow{\mathrm{BM}}$:

$$|\overrightarrow{BM}| = 7$$

Finally, we find the area:

$$\text{Area} = \frac{1}{2} \times 6 \times 7 = 21$$

Thus, the final answer is option (2).



Question 283

Let (α, β) be the centroid of the triangle formed by the lines $15x - y = 82, 6x - 5y = -4$ and $9x + 4y = 17$. Then $\alpha + 2\beta$ and $2\alpha - \beta$ are the roots
of the equation

A. $x^2 - 7x + 12 = 0$

B. $x^2 - 13x + 42 = 0$

C. $x^2 - 14x + 48 = 0$

D. $x^2 - 10x + 25 = 0$

Correct answer: B

Source solution

Sol. upon solving we get coordinates as $(6, 8), (1, 2)$ and $(5, -7)$

So centroid : (α, β) is
$\alpha = \frac{6+1+5}{3} = 4$
$\beta = \frac{8+2-7}{3} = 1$
$\alpha + 2\beta = 6$
$2\alpha - \beta = 7$

Answer. $x^2 - 13x + 42 = 0$

Question 284

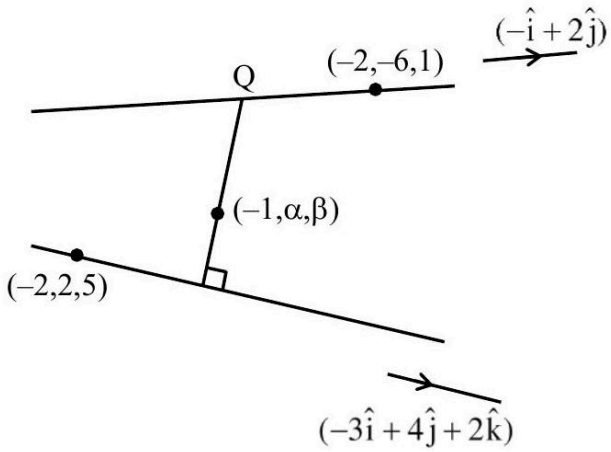
Let the point $(-1, \alpha, \beta)$ lie on the line of the shortest distance between the lines $\frac{x+2}{-3} = \frac{y-2}{4} = \frac{z-5}{2}$ and $\frac{x+2}{-1} = \frac{y+6}{2} = \frac{z-1}{0}$.
Then $(\alpha - \beta)^2$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 25

Source solution



$\mathbf{P}(-3\lambda - 2, 4\lambda + 2, 2\lambda + 5)$
$\mathbf{Q}(-\mu - 2, 2\mu - 6, 1)$
DRS of PQ $= (3\lambda - \mu, 2\mu - 4\lambda - 8, -2\lambda - 4)$
DRS of $PQ = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 0 \\ -3 & 4 & 2 \end{vmatrix}$
$= (4\hat{i} + 2\hat{j} + 2\hat{k})$
OR
(2, 1, 1)
$\frac{3\lambda-\mu}{2} = \frac{2\mu-4\lambda-8}{1} = \frac{-2\lambda-4}{1}$
$\Rightarrow \mu = \lambda + 2 \& 7\lambda = \mu - 8$
$\lambda = -1 \quad \mu = 1$
$\mathbf{Q} : (-3, -4, 1)$
$\mathbf{L_{PQ}} = \frac{x+3}{2} = \frac{y+4}{1} = \frac{z-1}{1}$
$(-1, \alpha, \beta) \Rightarrow 1 = \frac{\alpha+4}{1} = \frac{\beta-1}{1}$

$\Rightarrow \alpha = -3, \beta = 2$
$(\alpha - \beta)^2 = 25$

Question 285

If the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x}{1} = \frac{y}{\alpha} = \frac{z-5}{1}$ is $\frac{5}{\sqrt{6}}$, then the sum of all possible values of α is

A. $\frac{3}{2}$

B. $-\frac{3}{2}$

C. 3

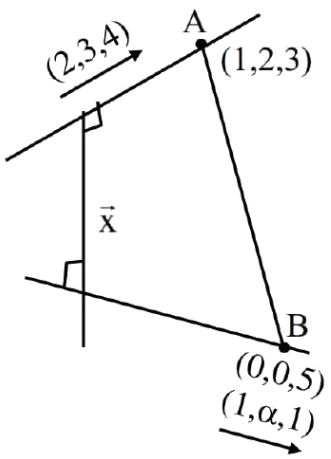
D. -3

Correct answer: D

Source solution

Sol.

$L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$
$L_1 : \frac{x}{1} = \frac{y}{\alpha} = \frac{z-5}{1}$
$\vec{x} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 1 & \alpha & 1 \end{vmatrix} = \hat{i}(3-4\alpha) - \hat{j}(-2) + \hat{k}(2\alpha - 3)$
S.D. $= \left| \frac{\overrightarrow{BA} \cdot \vec{n}}{|\vec{n}|} \right| = \left| \frac{(\hat{i}+2\hat{j}-2\hat{k}) \cdot \vec{n}}{|\vec{n}|} \right|$
$\Rightarrow 6(13 - 8\alpha)^2 = 25\left((4\alpha - 3)^2 + (2\alpha - 3)^2 + 16\right)$
$6\left(64a^2 - 280\alpha + 169\right) = 25\left(20\alpha^2 - 36\alpha + 34\right)$
$\Rightarrow 116\alpha^2 + 348\alpha - 164 = 0$
$\alpha_1 + \alpha_2 = \frac{-348}{116} = -3$



Question 286

If the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z+3}{1}$ and $\frac{x-a}{2}=\frac{y+2}{3}=\frac{z-3}{1}$ intersect at the point P, then the distance of the point P from the plane $z=a$ is :

A. 28

B. 16

C. 10

D. 22

Correct answer: A

Source solution

Sol. 1}
Let $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z+3}{1}=\lambda$
$\mathrm{P}(\lambda+1,2\lambda+2,\lambda-3)$
& $\frac{\mathrm{x}-\mathrm{a}}{2}=\frac{\mathrm{y}+2}{3}=\frac{\mathrm{z}-3}{1}=\mu$
$\mathrm{P}(2\mu+\mathrm{a},3\mu-2,\mu+3)$
$\lambda+1=2\mu+\mathrm{a}$
$2\lambda+2=3\mu-2$
$\lambda-3=\mu+3$
$22+1=2\times16+a$
$2(\mu+6)+2=3\mu-2$
$\lambda=\mu+6$
$\mathrm{a}=23-32$
$2\mu+12=3\mu-4$
$\mathrm{a}=-9$
$\mu=16$
$\therefore\lambda=22$
$\therefore\mathrm{P}(23,46,19)$
Plane is $\mathrm{z}=\mathrm{a}$
$\mathrm{z}=-9$

The distance of p from $\mathrm{z}=-9$ is $19-(-9)=28$

Question 287

If two distinct point Q, R lie on the line of intersection of the planes $-x+2y-z=0$ and $3x-5y+2z=0$ and $PQ=PR=\sqrt{18}$ where the point P is $(1,-2,3)$, then the area of the triangle PQR is equal to

A. $\frac{2}{3}\sqrt{38}$

B. $\frac{4}{3}\sqrt{38}$

C. $\frac{8}{3}\sqrt{38}$

D. $\sqrt{\frac{152}{3}}$

Correct answer: B

Source solution

Sol.

$-x+2y-z=0$

$3x-5y+2z=0$

$\vec{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & 2 & -1\\ 3 & -5 & 2\end{vmatrix}$

$=\hat{i}(-1)-\hat{j}(1)+\hat{k}(-1)$

$\vec{n}=-\hat{i}-\hat{j}-\hat{k}$

Equation of LOI is $\frac{x}{1}=\frac{y}{1}=\frac{z}{1}$

DR: of PT $\rightarrow \alpha-1, \alpha+2, \alpha-3$

DR : of $QR \rightarrow \quad 1, \quad 1,1$

$\Rightarrow(\alpha-1)\times 1+(\alpha+2)\times 1+(\alpha-3)\times 1=0$

$3\alpha=2$

$\alpha=\frac{2}{3}$

$PT^2=\frac{1}{9}+\frac{64}{9}+\frac{49}{9}$

$PT^2=\frac{114}{9}$

$PT=\frac{\sqrt{114}}{3}$

$\cos\theta=\frac{\sqrt{114}}{3}\times\frac{1}{3\sqrt{2}}=\frac{\sqrt{57}}{9}=\frac{\sqrt{19\times 3}}{3\times 3}$

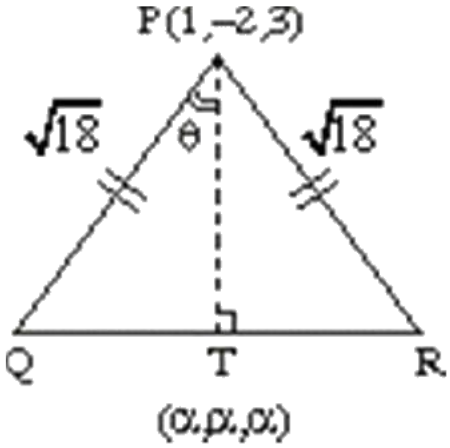
$$=\frac{\sqrt{19}}{3\sqrt{3}}$$

$\cos 2\theta=\frac{2\times 19}{27}-1=\frac{11}{27}$

$\sin 2\theta=\sqrt{1-\left(\frac{11}{27}\right)^2}=\frac{\sqrt{38}\sqrt{16}}{27}$

$$= \frac{4}{27}\sqrt{38}$$

Area $= \frac{1}{2} \times \sqrt{18}\sqrt{18} \times \frac{4}{27}\sqrt{38}$
$= \frac{18}{2} \times \frac{4}{27}\sqrt{38} = \frac{36}{27}\sqrt{38} = \frac{4}{3}\sqrt{38}$



Question 288

Let in a $\triangle ABC$, the length of the side AC be 6 , the vertex B be $(1,2,3)$ and the vertices A, C lie on the line $\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$. Then the area (in sq. units) of $\triangle ABC$ is

A. 42

B. 21

C. 56

D. 17

Correct answer: B

Source solution

Sol.

Let $M(3\lambda+6, 2\lambda+7, -2\lambda+7)$

$$\overrightarrow{BM}=(3\lambda+5)\hat{i}+(2\lambda+5)\hat{j}+(-2\lambda+4)\hat{k}$$

$\overrightarrow{AC} \cdot \overrightarrow{BM}=0=3(3\lambda+5)+2(2\lambda+5)-2(-2\lambda+4)$
$\overrightarrow{BM}=2\hat{i}+3\hat{j}+6\hat{k}$
$|\overrightarrow{BM}|=7$
Area $=\frac{1}{2}\times 6\times 7=21$
Option (2) is the correct answer.

Question 289

If the line, $2x - y + 3 = 0$ is at a distance $\frac{1}{\sqrt{5}}$ and $\frac{2}{\sqrt{5}}$ from the lines $4x - 2y + \alpha = 0$ and $6x - 3y + \beta = 0$, respectively, then the sum of all possible values of α and β is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 30

Source solution

30

$\mathbf{L_1 : 2x - y + 3 = 0}$

$\mathbf{L_2 : 4x - 2y + \alpha = 0}$

$\mathbf{L_3 : 6x - 3y + \beta = 0}$

$$\frac{\left|\frac{\alpha}{2} - 3\right|}{\sqrt{5}} = \frac{1}{\sqrt{5}} \quad \Rightarrow \frac{\alpha}{2} - 3 = 1, -1$$
$$\Rightarrow \alpha = 8, 4$$

$$\frac{\left|\frac{\beta}{3} - 3\right|}{\sqrt{5}} = \frac{2}{\sqrt{5}} \quad \Rightarrow \frac{\beta}{3} - 3 = 2, -2$$
$$\Rightarrow \beta = 15, 3$$

Question 290

The shortest distance between the lines $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-6}{2}$ and $\frac{x-6}{3}=\frac{1-y}{2}=\frac{z+8}{0}$ is ____.

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 14

Source solution

The direction vectors are $(3,2,2)$ and $(3,-2,0)$. Their cross product is $(4,6,-12)$, of magnitude 14. The scalar triple product with displacement $(4,2,-14)$ is 196, so the shortest distance is 14.

Question 291

Let two vertices of triangle ABC be $(2, 4, 6)$ and $(0, -2, -5)$, and its centroid be $(2, 1, -1)$. If the image of third vertex in the plane $x + 2y + 4z = 11$ is (α, β, γ), then $\alpha\beta + \beta\gamma + \gamma\alpha$ is equal to

A. 72

B. 74

C. 76

D. 70

Correct answer: B

Source solution

Given points $A(2, 4, 6)$, $B(0, -2, -5)$, and $G(2, 1, -1)$, we need to find the coordinates of vertex $C(x, y, z)$.

1. **Finding x**:

$$\frac{2 + 0 + x}{3} = 2 \Rightarrow 2 + 0 + x = 6 \Rightarrow x = 4$$

2. **Finding y**:

$$\frac{4 - 2 + y}{3} = 1 \Rightarrow 4 - 2 + y = 3 \Rightarrow y = 1$$

3. **Finding z**:

$$\frac{6 - 5 + z}{3} = -1 \Rightarrow 6 - 5 + z = -3 \Rightarrow z = -4$$

Thus, the coordinates of vertex C are $C(4, 1, -4)$.

Next, we find the image of vertex C in the plane, denoted as (α, β, γ):

$$\frac{\alpha - 4}{1} = \frac{\beta - 1}{2} = \frac{\gamma + 4}{4} = \frac{-2(4 + 2 - 16 - 11)}{21}$$

Calculating the right-hand side:

$$4+2-16-11=-21 \Rightarrow -2(-21)=42 \Rightarrow \frac{42}{21}=2$$

Now, we have:

$$\frac{\alpha-4}{1}=2 \Rightarrow \alpha=6$$

$$\frac{\beta-1}{2}=2 \Rightarrow \beta-1=4 \Rightarrow \beta=5$$

$$\frac{\gamma+4}{4}=2 \Rightarrow \gamma+4=8 \Rightarrow \gamma=4$$

Thus, the image coordinates are $(\alpha,\beta,\gamma)=(6,5,4)$.

Finally, we calculate:

$$\alpha\beta+\beta\gamma+\gamma\alpha=6\cdot5+5\cdot4+4\cdot6=30+20+24=74$$

The final answer is:

$$\alpha\beta+\beta\gamma+\gamma\alpha=74$$

Question 292

Let a straight line L pass through the point $\mathbf{P(2,-1,3)}$ and be perpendicular to the lines $\frac{x-1}{2}=\frac{y+1}{1}=\frac{z-3}{-2}$ and $\frac{x-3}{1}=\frac{y-2}{3}=\frac{z+2}{4}$. If the line L intersects the yz -plane at the point Q , then the distance between the points P and Q is :

A. 2

B. $\sqrt{10}$

C. 3

D. $2\sqrt{3}$

Correct answer: C

Source solution

Sol. Vector parallel to 'L'

$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & -2\\ 1 & 3 & 4\end{vmatrix}=10\hat{i}-10\hat{j}+5\hat{k}$

$=5(2\hat{\mathrm{i}}-2\hat{\mathrm{j}}+\hat{\mathrm{k}})$

Equation of 'L'

$\frac{\mathrm{x}-2}{2}=\frac{\mathrm{y}+1}{-2}=\frac{z-3}{1}=\lambda$ (say)

Let $\mathbf{Q}(2\lambda+2,-2\lambda-1,\lambda+3)$

$\Rightarrow 2\lambda+2=0\Rightarrow\lambda=-1$

$\Rightarrow \mathbf{Q}(0,1,2)$

$\mathbf{d}(\mathbf{P},\mathbf{Q})=3$

Question 293

If the two lines $l_1 : \frac{x-2}{3} = \frac{y+1}{-2}, z = 2$ and $l_2 : \frac{x-1}{1} = \frac{2y+3}{\alpha} = \frac{z+5}{2}$ perpendicular, then an angle between the lines l_2 and $l_3 : \frac{1-x}{3} = \frac{2y-1}{-4} = \frac{z}{4}$ is :

A. $\cos^{-1}\left(\frac{29}{4}\right)$

B. $\sec^{-1}\left(\frac{29}{4}\right)$

C. $\cos^{-1}\left(\frac{2}{29}\right)$

D. $\cos^{-1}\left(\frac{2}{\sqrt{29}}\right)$

Correct answer: B

Source solution

Sol. $l_1 : \frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-2}{0}$

$l_2 : \frac{x-1}{1} = \frac{y+3/2}{\alpha/2} = \frac{z+5}{2}$

$l_3 : \frac{x-1}{-3} = \frac{y-1/2}{-2} = \frac{z-0}{4}$

$l_1 \perp l_2 \Rightarrow \frac{|3-\alpha+0|}{\sqrt{13}\sqrt{1+\frac{\alpha^2}{4}+4}} = 0 \Rightarrow \alpha = 3$

angle between $l_2 \& l_3$

$\cos\theta = \frac{|1\times(-3)+(-2)(\alpha/2)+2\times4|}{\sqrt{1+4+\frac{\alpha^2}{4}}\sqrt{9+16+4}}$

$\cos\theta = \frac{|-3-\alpha+8|}{\sqrt{5+\frac{\alpha^2}{4}}\sqrt{29}}$

put $\alpha = 3$

$\cos\theta = \frac{2}{\sqrt{\frac{29}{4}}\sqrt{29}} = \frac{4}{29}$

$\theta = \cos^{-1}\left(\frac{4}{29}\right) \Rightarrow \theta = \sec^{-1}\left(\frac{29}{4}\right)$

Mathematical Reasoning

Questions 294-300 / 7 selected / 2.3% practice weight / Historical appendix

Question 294

The compound statement $(\sim (P \wedge Q)) \vee ((\sim P) \wedge Q) \Rightarrow ((\sim P) \wedge (\sim Q))$ is equivalent to

A. $(\sim Q) \vee P$

B. $((\sim P) \vee Q) \wedge (\sim Q)$

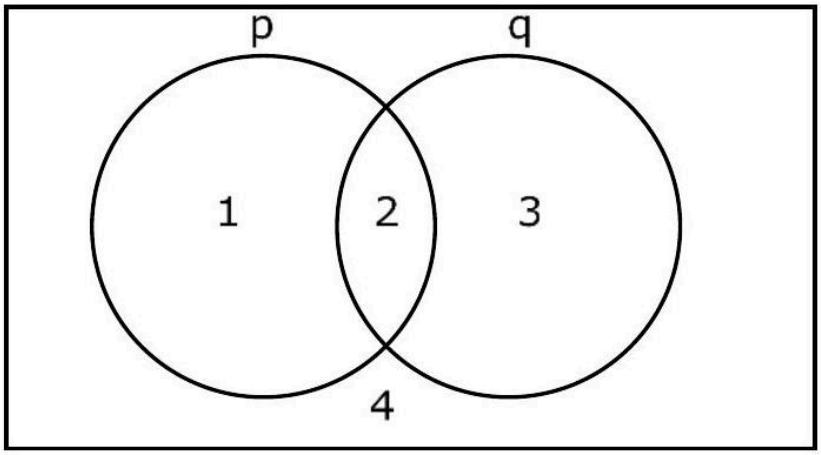
C. $(\sim P) \vee Q$

D. $((\sim P) \vee Q) \wedge ((\sim Q) \vee P)$

Correct answer: D

Source solution

$$(\sim (\mathrm{p} \wedge \mathrm{q}) \vee (\sim \mathrm{p} \wedge \mathrm{q})) \rightarrow \sim \mathrm{p} \wedge \sim \mathrm{q}$$



$$(1+2+4)+2 \Rightarrow 4$$
$$(1+2+4) \Rightarrow 4 \qquad = \sim (1+2+4)+4$$
$$= 3+4$$

Ans. 4

8o. The distance of the point $(7, -3, -4)$ from the plane passing through the points $(2, -3, 1), (-1, 1, -2)$ and $(3, -4, 2)$ is :

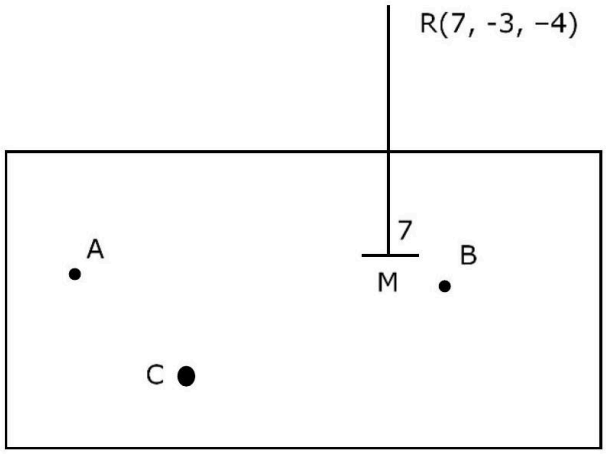
(1) 5

(2) 4

(3) $5\sqrt{2}$

(4) $4\sqrt{2}$

Sol. (3)



$$\overline{n}_p = \overline{AB} \times \overline{DC} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 3 & -4 & 3 \\ -1 & 1 & -1 \end{vmatrix} = \langle 1, 0, -1 \rangle$$

Equation of plane

$$P: \quad 1(x-2) + 0(y+3) - (z-1) = 0$$
$$P: \quad x - z - 1 = 0$$

$$d(RM) = \left| \frac{7+4-1}{\sqrt{2}} \right| = 5\sqrt{2}$$

Question 295

The statement $(p \wedge (\sim q)) \Rightarrow (p \Rightarrow (\sim q))$ is

A. a tautology

B. a contradiction

C. equivalent to $p \vee q$

D. equivalent to $(\sim p) \vee (\sim q)$

Correct answer: A

Source solution

$(p \wedge (\sim q)) \Rightarrow (p \Rightarrow (\sim q))$

P	q	$\sim q$	$p \wedge \sim q$	$p \Rightarrow \sim q$	$(p \wedge \sim q) \Rightarrow (p \Rightarrow \sim q)$
T	T	F	F	F	T
T	F	T	T	T	T
F	T	F	F	T	T
F	F	T	F	T	T

Tautology

Question 296

(S1) $(p \Rightarrow q) \vee (p \wedge (\sim q))$ is a tautology
(S2) $((\sim p) \Rightarrow (\sim q)) \wedge ((\sim p) \vee q)$ is a contradiction.
Then

A. both (S1) and (S2) are correct

B. only (S1) is correct

C. only (S2) is correct

D. both (S1) and (S2) are wrong

Correct answer: B

Source solution

$S_1 : (P \Rightarrow q) V (P \wedge (\sim q))$

P	q	$P \Rightarrow q$	$\sim q$	$P \wedge \sim q$	$(P \Rightarrow q) V (P \wedge \sim q)$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	T	F	T

S_1 is a tautology
$S_2 : ((\sim P) \Rightarrow (\sim q)) \wedge ((\sim P) \vee q)$

$\sim \mathrm{P}$	$\sim \mathrm{q}$	$\sim \mathrm{P} \Rightarrow \sim \mathrm{q}$	$\sim \mathrm{P} \mathrm{v} \mathrm{q}$	$((\sim \mathrm{P}) \Rightarrow (\sim \mathrm{q})) \Lambda (\sim \mathrm{P}) \mathrm{v} \mathrm{q})$
F	F	T	T	T
F	T	T	F	F
T	F	F	T	F
T	T	T	T	T

S_2 is not a contradiction

Question 297

The negation of the expression $q \vee ((\sim q) \wedge p)$ is equivalent to

A. $(\sim p) \vee (\sim q)$

B. $p \wedge (\sim q)$

C. $(\sim p) \vee q$

D. $(\sim p) \wedge (\sim q)$

Correct answer: D

Source solution

$$
\begin{aligned}
& \sim (q \vee (\sim q) \wedge p) \\
& = \sim q \wedge (q \vee \sim p) \\
& = (\sim q \wedge q) \vee (\sim q \wedge \sim p) \\
& = \mathrm{F} \vee (\sim q \wedge \sim p) = (\sim q) \wedge (\sim p)
\end{aligned}
$$

Question 298

The statement $B \Rightarrow ((\sim A) \vee B)$ is equivalent to :

A. $A \Rightarrow (A \Leftrightarrow B)$

B. $A \Rightarrow ((\sim A) \Rightarrow B)$

C. $A \wedge B$

D. $A \vee B$

The original exam question has multiple correct choices and a conflicting printed ALLEN answer label. Independent truth-table verification establishes that original options 2, 3 and 4 are equivalent. In this corrected single-answer practice version, original options 1 and 2 are preserved; options 3 and 4 are replaced by A AND B and A OR B. Only option 2 is correct. The original defective choices remain in the review evidence.

Correct answer: B

Source solution

The target is $\neg B \vee \neg A \vee B$, a tautology. Option 2 is $\neg A \vee A \vee B$, also a tautology. Option 1 reduces to $\neg A \vee B$ and is false at A=true, B=false. Option 3 is false at A=false, B=false, and option 4 is also false there. Therefore only option 2 is equivalent for every truth assignment.

Question 299

Which of the following statements is a tautology ?

A. $p \vee (p \wedge q)$

B. $(\mathrm{p} \wedge (\mathrm{p} \rightarrow \mathrm{q})) \rightarrow \sim \mathrm{q}$

C. $(\mathrm{p} \wedge \mathrm{q}) \rightarrow (\sim (\mathrm{p}) \rightarrow \mathrm{q})$

D. $\mathrm{p} \rightarrow (\mathrm{p} \wedge (\mathrm{p} \rightarrow \mathrm{q}))$

Correct answer: C

Source solution

(i) $\mathrm{p} \rightarrow (\mathrm{p} \wedge (\mathrm{p} \rightarrow \mathrm{q}))$
$(\sim p) \vee (p \wedge (\sim p \vee q))$
$(\sim p) \vee (f \vee (p \wedge q))$
$\sim p \vee (p \wedge q) = (\sim p \vee p) \wedge (\sim p \vee q)$
$= \sim p \vee q$
(ii) $(p \wedge q) \rightarrow (\sim p \rightarrow q)$
$\sim (p \wedge q) \vee (p \vee q) = t$
$\{a, b, d\} \vee \{a, b, c\} = V$
Tautology
(iii) $(p \wedge (p \rightarrow q)) \rightarrow \sim q$
$\sim (p \wedge (\sim p \vee q)) \vee \sim q = \sim (p \wedge q) \vee \sim q = \sim p \vee \sim q$
Not tautology
(iv) $pV(p \wedge q) = p$

Not tautology.
So, option (2) is correct.

Question 300

Consider the following statements:

P : I have fever
Q: I will not take medicine
R : I will take rest.
The statement "If I have fever, then I will take medicine and I will take rest" is equivalent to:

A. $((\sim \mathrm{P}) \vee \sim \mathrm{Q}) \wedge ((\sim \mathrm{P}) \vee \mathrm{R})$

B. $(\mathrm{P} \vee \mathrm{Q}) \wedge ((\sim \mathrm{P}) \vee \mathrm{R})$

C. $((\sim \mathrm{P}) \vee \sim \mathrm{Q}) \wedge ((\sim \mathrm{P}) \vee \sim \mathrm{R})$

D. $(\mathrm{P} \vee \sim \mathrm{Q}) \wedge (\mathrm{P} \vee \sim \mathrm{R})$

Correct answer: A

Source solution

$$\begin{aligned} & \mathrm{P} \rightarrow (\sim \mathrm{Q} \wedge \mathrm{R}) \\ & \sim \mathrm{P} \vee (\sim \mathrm{Q} \wedge \mathrm{R}) \\ & \Rightarrow ((\sim \mathrm{P}) \vee (\sim \mathrm{Q})) \wedge ((\sim \mathrm{P}) \vee \mathrm{R}) \end{aligned}$$
