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JEE Main 2026 Session 2

Mathematics
Solutions companion

300 questions / NCERT chapter navigation
Answers and available source solutions, in paper order.

Use the question numbers from the companion question paper. Brief source answers are identified wherever a detailed solution is unavailable.

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Open the PDF bookmarks panel on the left to jump between chapters. The chapter links in the guide are clickable too.

YOUR PRACTICE ROADMAP

Chapter & topic priorities

For this Session 2 collection, begin with **Three Dimensional Geometry; Conic Sections; Sequences and Series**. These chapters have the largest share of the selected questions.

Practice weight = questions selected from a chapter / 300. Topic percentages use that chapter's question count. These are practice priorities from the prediction output, not guaranteed exam marks or probabilities.

NCERT chapter / leading topics	Priority	Questions	Weight
Relations and Functions Functions and relations: 6 (46.2%) / Properties of functions: 4 (30.8%) / Equivalence relations: 3 (23.1%)	Next	1-13 (13)	4.3%
Trigonometric Functions Trigonometric identities: 2 (50.0%) / Transformation and evaluation of trigonometric expressions: 1 (25.0%) / Trigonometric Functions: 1 (25.0%)	Consolidate	14-17 (4)	1.3%
Complex Numbers and Quadratic Equations Nature of roots: 1 (100.0%)	Consolidate	18-18 (1)	0.3%
Permutations and Combinations Counting arrangements and selections: 11 (50.0%) / Fundamental counting principle: 4 (18.2%) / Counting arrangements: 2 (9.1%) / More topics in chapter guide	Focus first	19-40 (22)	7.3%
Binomial Theorem Middle term of a binomial expansion: 1 (100.0%)	Consolidate	41-41 (1)	0.3%
Sequences and Series Sequences and finite sums: 15 (42.9%) / Arithmetic progression: 7 (20.0%) / Evaluation of a structured finite sequence or series: 5 (14.3%) / More topics in chapter guide	Focus first	42-76 (35)	11.7%
Straight Lines Equation of a straight line: 4 (80.0%) / Distance from a point to a line: 1 (20.0%)	Consolidate	77-81 (5)	1.7%
Conic Sections Parabola geometry: 13 (27.7%) / Ellipse geometry: 9 (19.1%) / Conic geometry: 8 (17.0%) / More topics in chapter guide	Focus first	82-128 (47)	15.7%
Limits and Derivatives Limits and Derivatives: 1 (50.0%) / Standard limits: 1 (50.0%)	Consolidate	129-130 (2)	0.7%
Statistics Variance and standard deviation: 11 (68.8%) / Measures of central tendency and dispersion: 5 (31.3%)	Next	131-146 (16)	5.3%
Probability Binomial distribution: 2 (100.0%)	Consolidate	147-148 (2)	0.7%

NCERT chapter / leading topics	Priority	Questions	Weight
Inverse Trigonometric Functions Inverse trigonometric identities: 1 (100.0%)	Consolidate	149-149 (1)	0.3%
Matrices Algebra of matrices: 1 (33.3%) / Linear systems: 1 (33.3%) / Matrix operations and properties: 1 (33.3%)	Consolidate	150-152 (3)	1.0%
Determinants Determinants: 3 (42.9%) / System of linear equations: 3 (42.9%) / Properties of determinants: 1 (14.3%)	Next	153-159 (7)	2.3%
Application of Derivatives Tangents and normals: 2 (40.0%) / Applications of derivatives: 1 (20.0%) / Derivative-based curve analysis: 1 (20.0%) / More topics in chapter guide	Consolidate	160-164 (5)	1.7%
Integrals Evaluation or use of an integral relation: 1 (100.0%)	Consolidate	165-165 (1)	0.3%
Application of Integrals Area under a curve: 5 (62.5%) / Area between curves: 2 (25.0%) / Standard equation and geometric properties of a parabola: 1 (12.5%)	Next	166-173 (8)	2.7%
Vector Algebra Algebra of vectors: 4 (33.3%) / Scalar product: 4 (33.3%) / Vector operations: 2 (16.7%) / More topics in chapter guide	Next	174-185 (12)	4.0%
Three Dimensional Geometry Coordinates in three dimensions: 28 (24.6%) / Shortest distance between lines: 21 (18.4%) / Lines and planes in space: 17 (14.9%) / More topics in chapter guide	Focus first	186-299 (114)	38.0%
Linear Programming Linear optimisation: 1 (100.0%)	Consolidate	300-300 (1)	0.3%

How to use the priorities

1. Rank chapters by their selected question counts. Ties follow NCERT book order.
2. Focus first: chapters starting within the first 70% of selected, non-historical questions. Next: the following 20%. Consolidate: the remainder. A chapter crossing a threshold stays together.
3. Topic counts use the question's existing core concept, falling back to topic and then chapter. Each question is counted once. Full topic lists begin each chapter.
4. Historical/omitted chapters stay in a separate appendix. Their presence is not a recommendation that they are in the target examination syllabus.

Names follow NCERT Classes XI and XII. Practical chemistry is identified as a laboratory-manual section. Percentages are rounded and may not add to exactly 100.0%. Chapters with no selected questions are not listed; this does not mean they have no exam importance.

Chapter naming references: NCERT textbooks.

Relations and Functions

Questions 1-13 / 13 selected / 4.3% practice weight / Next

Question 1

Let $f : R \to R$ be a function defined by $f(x) = (2 + 3a)x^2 + \left(\frac{a+2}{a-1}\right)x + b, a \neq 1$. If $f(x + y) = f(x) + f(y) + 1 - \frac{2}{7}xy$, then the value of $28\sum_{i=1}^{5} |f(i)|$ is:

A. 715

B. 735

C. 545

D. 675

Correct answer: D

Source solution

Sol. $f(x) = (3a + 2)x^2 + \left(\frac{a+2}{a-1}\right)x + b$

$$f\left(x + \frac{1}{2}\right) = f(x) + f(y) + 1 - \frac{2}{7}xy \tag{1}$$

In (1) Put $x = y = 0 \Rightarrow f(0) = 2f(0) + 1 \Rightarrow f(0) = -1$ So, $\mathrm{f(0) = 0 + 0 + b = -1 \Rightarrow\ b = -1}$

In (1) Put $y = -x \Rightarrow f(0) = f(x) + f(-x) + 1 + \frac{2}{7}x^2 - 1 = 2(3a + 2)x^2 + 2b + 1 + \frac{2}{7}x^2$
$-1 = \left(2(3a + 2) + \frac{2}{7}\right)x^2 + 1 - 2$
$\Rightarrow \mathrm{6a + 4 + \frac{2}{7} = 0}$
$\mathrm{a = -\frac{5}{7}}$
So $f(x) = -\frac{1}{7}x^2 - \frac{3}{4}x - 1$
$\Rightarrow \mathrm{|f(x)| = \frac{1}{28}\left|4x^2 + 21x + 28\right|}$
Now, $28\sum_{i=1}^{5} |f(i)| = 28(|f(1)| + |f(2)| + \ldots + |f(5)|)$
$\frac{1}{28} \cdot 675 = 675$

Question 2

Let $S=\left\{x: x \in \mathbb{R} \text{ and } (\sqrt{3}+\sqrt{2})^{x^2-4}+(\sqrt{3}-\sqrt{2})^{x^2-4}=10\right\}$

Then $n(S)$ is equal to

A. 4

B. 0

C. 6

D. 2

Correct answer: A

Source solution

Sol. 1

$$(\sqrt{3}+\sqrt{2})^{x^2-4}+(\sqrt{3}-\sqrt{2})^{x^2-4}=10$$
$$\Rightarrow (\sqrt{3}+\sqrt{2})^{x^2-4}+\frac{1}{(\sqrt{3}+\sqrt{2})^{x^2-4}}=10$$

Let $(\sqrt{3}+\sqrt{2})^{x^2-4}=t$
$t+\frac{1}{t}=10$
$\Rightarrow t^2-10t+1=0$
$t=5+2\sqrt{6}, 5-2\sqrt{6}$
If $t=5+2\sqrt{6}$
If $t=5-2\sqrt{6}$
$(\sqrt{3}+\sqrt{2})^{x^2-4}=(\sqrt{3}+\sqrt{2})^2$
$(\sqrt{3}+\sqrt{2})^{x^2-4}=(\sqrt{3}+\sqrt{2})^{-2}$
$\Rightarrow x^2-4=2$
$\Rightarrow x^2-4=-2$
$\Rightarrow x=\pm\sqrt{6}$
$\Rightarrow x=\pm\sqrt{2}$
$S=\{\sqrt{6},-\sqrt{6},\sqrt{2},-\sqrt{2}\}$
$n(s)=4$

Question 3

Let the relations R_1 and R_2 on the set
$X = \{1, 2, 3, \ldots, 20\}$ be given by
$R_1 = \{(x, y) : 2x - 3y = 2\}$ and
$R_2 = \{(x, y) : -5x + 4y = 0\}$. If M and N be the minimum number of elements required to be added in R_1 and R_2, respectively, in order to make the relations symmetric, then $M + N$ equals

A. 8

B. 16

C. 12

D. 10

Correct answer: D

Source solution

Sol. $x = \{1, 2, 3, \ldots\ldots.20\}$
$R_1 = \{(x, y) : 2x - 3y = 2\}$
$R_2 = \{(x, y) : -5x + 4y = 0\}$
$R_1 = \{(4, 2), (7, 4), (10, 6), (13, 8), (16, 10), (19, 12)\}$
$R_2 = \{(4, 5), (8, 10), (12, 15), (16, 20)\}$
in $R_1 6$ element needed
in $R_2 4$ element needed
So, total $6 + 4 = 10$ element

Question 4

Let $A = \{0, 3, 4, 6, 7, 8, 9, 10\}$ and R be the relation defined on A such that $\mathrm{R} = \{(\mathrm{x}, \mathrm{y}) \in \mathrm{A} \times \mathrm{A} : \mathrm{x} - \mathrm{y}$ is odd positive integer or $\mathrm{x} - \mathrm{y} = 2\}$. The minimum number of elements that must be added to the relation R , so that it is a symmetric relation, is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 19

The source provides a brief answer only. A detailed worked solution is not available for this question.

Source solution

Question 5

Let $A = \{1, 2, 3, 4, \ldots . 10\}$ and $B = \{0, 1, 2, 3, 4\}$.

The number of elements in the relation $\mathrm{R} = \{(\mathrm{a}, \mathrm{b}) \in A \times A : 2(a-b)^2 + 3(a-b) \in B\}$ is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 18

Source solution

$A = \{1, 2, 3, \ldots . . . . 10\}$
$B = \{0, 1, 2, 3, 4\}$
$R = \{(a, b) \in A \times A : 2(a-b)^2 + 3(a-b) \in B\}$
Now $2(\mathrm{a}-\mathrm{b})^2 + 3(\mathrm{a}-\mathrm{b}) = (\mathrm{a}-\mathrm{b})(2(\mathrm{a}-\mathrm{b}) + 3)$
$\Rightarrow \mathrm{a} = \mathrm{b}$ or $\mathrm{a} - \mathrm{b} = -2$
When $\mathrm{a} = \mathrm{b} \Rightarrow 10$ order pairs
When $\mathrm{a} - \mathrm{b} = -2 \Rightarrow 8$ order pairs
Total = 18

Question 6

Let $\mathbf{A}=\{1,3,4,6,9\}$ and $\mathbf{B}=\{2,4,5,8,10\}$. Let R be a relation defined on $\mathbf{A} \times \mathbf{B}$ such that $\mathbf{R}=\{((\mathbf{a}_1,\ \mathbf{b}_1),(\mathbf{a}_2,\ \mathbf{b}_2)) : \mathbf{a}_1 \leq \mathbf{b}_2$ and $\mathbf{b}_1 \leq \mathbf{a}_2\}$. Then the number of elements in the set R is

A. 26

B. 160

C. 180

D. 52

Correct answer: B

Source solution

Sol. Let $\mathbf{a}_1=1 \Rightarrow 5$ choices of $\mathbf{b}_2$
$a_1=3 \Rightarrow 4$ choices of b_2
$a_1=4 \Rightarrow 4$ choices of b_2
$a_1=6 \Rightarrow 2$ choices of b_2
$a_1=9 \Rightarrow 1$ choices of b_2
For $(a_1, b_2) 16$ ways .
Similarly, $\mathbf{b}_1=2 \Rightarrow 4$ choices of $\mathbf{a}_2$
$\mathbf{b}_1=4 \Rightarrow 3$ choices of $\mathbf{a}_2$
$b_1=5 \Rightarrow 2$ choices of a_2
$\mathbf{b}_1=8 \Rightarrow 1$ choices of $\mathbf{a}_2$
Required elements in $\mathbf{R}=160$

Question 7

Let R be a relation defined on $\mathbb{N}$ as a R b if $\mathbf{2a+3\,b}$ is a multiple of $\mathbf{5, a, b \in \mathbb{N}}$.

Then R is

A. an equivalence relation

B. transitive but not symmetric

C. not reflexive

D. symmetric but not transitive

Correct answer: A

Source solution

Sol.
Reflexive
Let $a \in N$
$\mathbf{aRa} \Rightarrow \mathbf{2a+3a}$ is a multiple of 5
$\Rightarrow$ 5a which is a multiple of 5
$\Rightarrow$ $\mathbf{R}$ is reflexive
Symmetric
Let $\mathbf{a, b \in N}$
$\mathbf{aRb} \Rightarrow \mathbf{2a+3\,b=5\lambda_1} \quad \lambda_1 \in \mathbf{N}$
b R $\mathbf{a} \Rightarrow \mathbf{2\,b+3a=5\lambda_2} \quad \lambda_2 \in \mathbf{N}$
On Adding
$(2a+3b)+(2b+3a)=5(\lambda_1+\lambda_2)$
$\mathbf{5a+5\,b=5(\lambda_1+\lambda_2)}$
$\Rightarrow$ Both sides are multiple of 5
$\Rightarrow$ $\mathbf{R}$ is symmetric

Transitive
Let $a, b, c \in N$
a R b $\Rightarrow \mathbf{2a+3\,b=5\lambda_1}$
b R c $\Rightarrow \mathbf{2a+3c=5\lambda_2}$

$$\mathbf{2a+5\,b+3c=5(\lambda_1+\lambda_2)} \tag{2}$$

$\Rightarrow (2a + 3c) = 5(\lambda_1 + \lambda_2 - b)$

2a +3 c is divisible by 5

$\Rightarrow aRc$ is true

$\Rightarrow R$ is transitive

R is Equivalence Relation

Question 8

Let $A = \{1, 2, 3, 4\}$ and R be a relation on the set $A \times A$ defined by $R = \{((a, b), (c, d)) : 2a + 3b = 4c + 5d\}$. Then the number of elements in R is:

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 6

Source solution

$A = \{1, 2, 3, 4\}$

$R = \{(a, b), (c, d)\}$

$2a + 3b = 4c + 5d = \alpha$ let

$2a = \{2, 4, 6, 8\} \quad 4c = \{4, 8, 12, 16\}$

$3b = \{3, 6, 9, 12\} \quad 5d = \{5, 10, 15, 20\}$

$$2a + 3b = \begin{Bmatrix} 5, 8, 11, 14 \\ 7, 10, 13, 16 \\ 9, 12, 15, 18 \\ 11, 14, 17, 20 \end{Bmatrix} \quad 4c + 5d \begin{Bmatrix} 9, 14, 19, 24 \\ 13, 18 \ldots \\ 17, 22 \ldots \\ 21, 26 \ldots \end{Bmatrix}$$

Possible value of $\alpha = 9, 13, 14, 14, 17, 18$

Pairs of $\{(a, b), (c, d)\} = 6$

Question 9

Let $\mathbf{A} = \{2, 3, 4\}$ and $\mathbf{B} = \{8, 9, 12\}$. Then the number of elements in the relation $R = \{((a_1, b_1), (a_2, b_2)) \in (A \times B, A \times B) : a_1$ divides $\mathbf{b_2}$ and $\mathbf{a_2}$ divides $\mathbf{b_1}$ \} is :

A. 36

B. 12

C. 18

D. 24

Correct answer: A

Source solution

Sol.
$\mathbf{a_1}$ divides $\mathbf{b_2}$
Each element has 2 choices
$\Rightarrow \mathbf{3 \times 2 = 6}$
$\mathbf{a_2}$ divides $\mathbf{b_1}$

Each element has 2 choices
$\Rightarrow \mathbf{3 \times 2 = 6}$

Total $= \mathbf{6 \times 6 = 36}$

Question 10

Let $A = \{-4, -3, -2, 0, 1, 3, 4\}$ and $R = \{(a, b) \in A \times A : b = |a| \text{ or } b^2 = a + 1\}$ be a relation on A . Then the minimum number of elements, that must be added to the relation R so that it becomes reflexive and symmetric, is .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 7

Source solution

$R = [(-4, 4), (-3, 3), (3, -2), (0, 1), (0, 0), (1, 1)$, (4, 4), (3, 3)\}

For reflexive, add $\Rightarrow (-2, -2), (-4, -4), (-3, -3)$
For symmetric, add $\Rightarrow (4, -4), (3, -3), (-2, 3), (1, 0)$

Question 11

Let $A = \{1, 2, 3, 4, 5, 6, 7\}$.Then the relation $R = \{(x, y) \in A \times A : x + y = 7\}$ is

A. transitive but neither symmetric nor reflexive

B. reflexive but neither symmetric nor transitive

C. an equivalence relation

D. symmetric but neither reflexive nor transitive

Correct answer: D

Source solution

Solution:

$$A = \{1, 2, 3, 4, 5, 6, 7\}$$

and

$$R = \{(x, y) \in A \times A : x + y = 7\}.$$

The elements of R are

$$R = \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}.$$

Reflexive:

For reflexivity, every (a, a) must belong to R.

This requires

$$a + a = 7$$

or

$$2a = 7,$$

which has no solution in A.

For example,

$$(1,1) \notin R.$$

Hence, R is not reflexive.

Symmetric:

Suppose

$$(x,y) \in R.$$

Then

$$x + y = 7.$$

Since addition is commutative,

$$y + x = 7.$$

Therefore,

$$(y,x) \in R.$$

Hence, R is symmetric.

Transitive:

To be transitive, whenever

$$(x,y) \in R \quad \text{and} \quad (y,z) \in R,$$

we must have

$$(x,z) \in R.$$

Take

$$(1,6) \in R$$

and

$$(6,1) \in R.$$

If R were transitive, then

$$(1,1) \in R.$$

But

$$1+1=2 \neq 7,$$

so

$$(1,1) \notin R.$$

Hence, R is not transitive.

\bigskip

Therefore,

$$R is symmetric but neither reflexive nor transitive.$$

Option (4)

Question 12

The number of relations on the set $\mathbf{A} = \{1, 2, 3\}$ containing at most 6 elements including (1,2), which are reflexive and transitive but not symmetric, is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 5

Source solution

$\mathbf{A} = \{1, 2, 3\}$
$(1, 1), (2, 2), (3, 3), (1, 2) \in \mathbf{R}$
Remaining elements are
$(2, 1), (2, 3), (1, 3), (3, 1), (3, 2)$
(1) If relation contains exactly 4 elements $= \mathbf{1}$ way
(2) if relation contains exactly 5 elements

It can be $(1, 3), (3, 2) \Rightarrow \mathbf{2}$ ways
(3) If relation contain exactly 6 elements

It can be
$((2, 3), (1, 3)), ((1, 3), (3, 2)), ((3, 1), (3, 2))$
$\Rightarrow \mathbf{3}$ ways.
Total $= \mathbf{6}$ ways

Question 13

The number of relations, on the set $\{1, 2, 3\}$ containing $(1, 2)$ and $(2, 3)$, which are reflexive and transitive but not symmetric, is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 4

Source solution

$\mathbf{A} = \{1, 2, 3\}$
For Reflexive $(1, 1)(2, 2), (3, 3) \in \mathbf{R}$
For transitive : $(1, 2)$ and $(2, 3) \in \mathbf{R} \Rightarrow (1, 3) \in \mathbf{R}$
Not symmetric : $(2, 1)$ and $(3, 2) \notin \mathbf{R}$
$\mathbf{R_1} = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$
$\mathbf{R_2} = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)(2, 1)\}$
$\mathbf{R_3} = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)(3, 2)\}$

Trigonometric Functions

Questions 14-17 / 4 selected / 1.3% practice weight / Consolidate

Question 14

Let $\cos(\alpha+\beta)=-\frac{1}{10}$ and $\sin(\alpha-\beta)=\frac{3}{8}$, where

$$0<\alpha<\frac{\pi}{3} \text{ and } 0<\beta<\frac{\pi}{4}.$$

If $\tan 2\alpha=\frac{3(1-r\sqrt{5})}{\sqrt{11}(s+\sqrt{5})}, r, s \in \mathbb{N}$, then r + s is equal to

_____

.

Numerical answer: _____

Enter an integer. Use the units and scaling requested in the question.

Correct answer: 20

Source solution

$\tan 2\alpha=\tan[(\alpha+\beta)+(\alpha-\beta)]$

$$\tan 2\alpha=\frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\cdot\tan(\alpha-\beta)}$$

$\tan 2\alpha=\frac{\left(-\sqrt{99}+\frac{3}{\sqrt{55}}\right)}{1-(\sqrt{99})\left(\frac{3}{\sqrt{55}}\right)}$

$$\tan 2\alpha=\frac{-3\sqrt{11}+\frac{3}{\sqrt{5}\times\sqrt{11}}}{1+\frac{9\sqrt{11}}{\sqrt{5}\times\sqrt{11}}}$$

$\tan 2\alpha=\frac{3(1-11\sqrt{5})}{\sqrt{11}(9+\sqrt{5})}$

$$r=11, s=9$$

$r+s=20$

Question 15

Let $S = \left\{\sin^2 2\theta : \left(\sin^4\theta + \cos^4\theta\right)x^2 + (\sin 2\theta)x + \left(\sin^6\theta + \cos^6\theta\right) = 0 \text{ has real roots }\right\}$. If α and β be the smallest and largest elements of the set S , respectively, then $3\left((\alpha-2)^2 + (\beta-1)^2\right)$ equals.....

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 4

Source solution

$\mathrm{D} = (\sin 2\theta)^2 - 4\left(1 - \frac{\sin^2 2\theta}{2}\right)\left(1 - \frac{3}{4}\sin^2 2\theta\right)$

$= (\sin 2\theta)^2 - 4\left(1 - \frac{5}{4}\sin^2 2\theta + \frac{3}{8}\sin^4 2\theta\right)$

$\mathrm{D} = -\frac{3}{2}\sin^4 2\theta + 6\sin^2 2\theta - 4 > 0$

$3\sin^4 2\theta - 12\sin^2 2\theta + 8 < 0$

$\sin^2 2\theta = \frac{12 \pm \sqrt{12^2 - 12.8}}{6} = \frac{12 \pm 4\sqrt{3}}{6} = \frac{6 \pm 2\sqrt{3}}{3}$

$\sin^2 2\theta = 2 \pm \frac{2}{\sqrt{3}}$, but $\sin^2 2\theta \in [0, 1]$

$\therefore \alpha = 2 - \frac{2}{\sqrt{3}}, \beta = 1 \rightarrow (\alpha-2)^2 = \frac{4}{3}, (\beta-1)^2 = 0$

$3(\alpha-2)^2 + (\beta-1)^2 = 4$

Question 16

Question ID: 101787

Let $c, k \in R$. If $f(x) = (c+1)x^2 + \left(1-c^2\right)x + 2k$ and $f(x+y) = f(x) + f(y) - xy$, for all $x, y \in R$, then the value of $\left|2(f(1) + f(2) + f(3) + \ldots\ldots + f(20))\right|$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 3395

Source solution

$f(x) = (c+1)x^2 + \left(1-c^2\right)x + 2k$

$$\&f(x+y) = f(x) + f(y) - xy \quad \forall xy \in R \tag{1}$$

$\lim_{y\to 0} \frac{f(x+y)-f(x)}{y} = \lim_{y\to 0} \frac{f(y)-xy}{y} \Rightarrow f'(x) = f'(0) - x$
$f(x) = -\frac{1}{2}x^2 + f'(0)\cdot x + \lambda \quad$ but $f(0) = 0 \Rightarrow \lambda = 0$

$$f(x) = -\frac{1}{2}x^2 + \left(1-c^2\right)\cdot x \tag{2}$$

$\therefore \quad$ as $f'(0) = 1 - c^2$
Comparing equation (1) and (2)
We obtain, $c = -\frac{3}{2}$
$\therefore \quad f(x) = -\frac{1}{2}x^2 - \frac{5}{4}x$
Now $\left|2\sum_{x=1}^{20} f(x)\right| = \sum_{x=1}^{20} x^2 + \frac{5}{2}\cdot\sum_{x=1}^{20} x$

$$= 2870 + 525$$
$$= 3395$$

Question 17

If for some $\alpha, \beta; \alpha \leq \beta, \alpha + \beta = 8$ and $\sec^2\left(\tan^{-1}\alpha\right) + \operatorname{cosec}^2\left(\cot^{-1}\beta\right) = 36$, then $\alpha^2 + \beta$ is .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 14

Source solution

If $\left(\tan\left(\tan^{-1}(\alpha)\right) + 1\left(\cot\left(\cot^{-1}\beta\right)\right)^2 = 36\right.$
$\alpha^2 + \beta^2 = 34$
$\alpha\beta = 15$
$\alpha = 3, \beta = 5$
$\therefore \alpha^2 + \beta = 9 + 5 = 14$

Complex Numbers and Quadratic Equations

Questions 18-18 / 1 selected / 0.3% practice weight / Consolidate

Question 18

Let α_θ and β_θ be the distinct roots of $2x^2 + (\cos\theta)x - 1 = 0, \theta \in (0, 2\pi)$. If m and M are the minimum and the maximum values of $\alpha_\theta^4 + \beta_\theta^4$, then $16(M + m)$ equals :

A. 24

B. 25

C. 27

D. 17

Correct answer: B

Source solution

Sol. $\left(\alpha^2+\beta^2\right)^2-2\alpha^2\beta^2$

$\left[(\alpha+\beta)^2-2\alpha\beta\right]^2-2(\alpha\beta)^2$

$\left[\frac{\cos^2\theta}{4}+1\right]^2-2\cdot\frac{1}{4}$

$\left(\frac{\cos^2\theta}{4}+1\right)^2-\frac{1}{2}$

$M=\frac{25}{16}-\frac{1}{2}=\frac{17}{16}$

$m=\frac{1}{2}, 16(M+m)=25$

Permutations and Combinations

Questions 19-40 / 22 selected / 7.3% practice weight / Focus first

Question 19

Let ${}^nC_{r-1} = 28, {}^nC_r = 56$ and ${}^nC_{r+1} = 70$. Let $A(4\cos t, 4\sin t), B(2\sin t, -2\cos t)$ and $C\left(3r - n, r^2 - n - 1\right)$ be the vertices of a triangle ABC , where t is a parameter. If $(3x-1)^2 + (3y)^2 = \alpha$, is the locus of the centroid of triangle ABC , then α equals :

A. 20

B. 8

C. 6

D. 18

Correct answer: A

Source solution

Sol. ${}^nC_{r-1} = 28, {}^nC_r = 56$

$$\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{28}{56}$$

$$\frac{\frac{n!}{(r-1)!(n-r+1)!}}{\frac{n!}{r!}} = \frac{1}{2}$$

$$\frac{r}{(n-r+1)} = \frac{1}{2}$$

$$3r = n + 1 \quad \text{(i)}$$

$$\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{56}{70}$$

$$\frac{(r+1)}{(n-r)} = \frac{56}{70} \Rightarrow 9r = 4n - 5 \quad \text{(ii)}$$

By (i) & (ii)

($\mathbf{r = 3}$), ($\mathbf{n = 8}$)
A (4 cost, 4 sint) B(2 sint, -2 cost) C(3r-n, $\mathrm{r}^{2} - n - 1$)
$\mathbf{A}(4\cos t, 4\sin t) \quad \mathbf{B}(2\sin t, -2\cos t)\mathbf{C}(1, 0)$
$(3x-1)^2 + (3y)^2 = (4\cos t + 2\sin t)^2 + (4\sin t - \cos t)^2$
$(3x-1)^2 + (3y)^2 = 20 \quad \therefore$ option (1)

Question 20

Total numbers of 3-digit numbers that are divisible by 6 and can be formed by using the digits $\mathbf{1, 2, 3}$, 4, 5 with repetition, is .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 16

Source solution

For number to be divisible by '6' unit digit should be even and sum of digit is divisible by 3 .
$(2, 1, 3), (2, 3, 4), (2, 5, 5), (2, 2, 5), (2, 2, 2),$
$(4, 1, 1), (4, 4, 1), (4, 4, 4), (4, 3, 5)$
$\mathbf{2, 1, 3 \Rightarrow 312, 132}$
$\mathbf{2, 3, 4 \Rightarrow 342, 432, 234, 324}$
$\mathbf{2, 5, 5 \Rightarrow 552}$
$\mathbf{2, 2, 5 \Rightarrow 252, 522}$
2, 2, $\mathbf{2 \Rightarrow 222}$
$\mathbf{4, 1, 1 \Rightarrow 114}$
$\mathbf{4, 4, 1 \Rightarrow 414, 144}$
$\mathbf{4, 4, 4 \Rightarrow 444}$
4, 3, $\mathbf{5 \Rightarrow 354, 534}$
Total 16 numbers.

Question 21

The number of 4-letter words, with or without meaning, each consisting of 2 vowels and 2 consonants, which can be formed from the letters of the word UNIVERSE without repetition is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 432

Source solution

UNIVERSE
Vowels: E, I, U
Consonants: N, V, R, S
$\rightarrow {}^{3}C_{2} \times {}^{4}C_{2} \times 4! = 3 \times 6 \times 24 = 432$

Question 22

If the sum and product of four positive consecutive terms of a G.P., are 126 and 1296 , respectively, then the sum of common ratios of all such GPs is

A. 7

B. 3

C. $\frac{9}{2}$

D. 14

Correct answer: A

Source solution

Sol. (1)
Four term of G.P. $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$
$\frac{a}{r^3}+\frac{a}{r}+ar+ar^3=126$
$\frac{a}{r^3}\cdot\frac{a}{r}\cdot ar\cdot ar^3=1296$
$a^4=1296$
$a=6$
$\frac{6}{r^3}+\frac{6}{r}+6r+6r^3=126$

$$\left(r+\frac{1}{r}\right)+r^3+\frac{1}{r^3}=21$$
$$\left(r+\frac{1}{r}\right)+\left(r+\frac{1}{r}\right)^3-3\left(r+\frac{1}{r}\right)=21$$

Let $r+\frac{1}{r}=t$
$t^3-2t=21$
$\Rightarrow t=3$
$r+\frac{1}{r}=3$
$r^2-3r+1=0$
$r=\frac{3\pm\sqrt{9-4}}{2}$
$r=\frac{3\pm\sqrt{5}}{2}$
Sum of common ratio $=\frac{9}{4}+\frac{5}{4}+\frac{3\sqrt{5}}{2}+\frac{9}{4}+\frac{5}{4}-\frac{3\sqrt{5}}{2}$
$=7$

Question 23

The number of numbers greater than 5000, less than 9000 and divisible by 3, that can be formed using the digits 0, 1, 2, 5, 9, if the repetition of the digits is allowed, is

_____

Numerical answer: _____

Enter an integer. Use the units and scaling requested in the question.

Correct answer: 42

Source solution

(1) all different

$$5,0,1,9 \Rightarrow \lfloor 3 = 6 \text{ways}$$

(2) 2 alike, 2 different

$$5,0,0,1 \Rightarrow 3 \text{ ways}$$
$$5,1,1,2 \Rightarrow 3 \text{ ways}$$
$$5,2,2,0 \Rightarrow 3 \text{ ways}$$
$$5,2,2,9 \Rightarrow 3 \text{ ways}$$
$$5,5,0,2 \Rightarrow 6 \text{ ways}$$
$$5,5,2,9 \Rightarrow 6 \text{ ways}$$
$$5,1,9,9 \Rightarrow 3 \text{ ways}$$

(3) 3 alike, 1 different

$$5,5,5,0 \Rightarrow 3 \text{ ways}$$
$$5,5,5,9 \Rightarrow 3 \text{ ways}$$

(4) 2 alike, 2 other alike

$$5,5,1,1 \Rightarrow 3 \text{ ways}$$

Total ways $= 42$

Question 24

The largest $n \in \mathbb{N}$, for which 7^n divides 101!, is :

A. 16

B. 18

C. 15

D. 19

Correct answer: A

Source solution

Exponent of 7 in 101!

$$= \left[\frac{101}{7}\right] + \left[\frac{101}{7^2}\right] + \left[\frac{101}{7^3}\right] + \ldots\ldots$$
$$= 14 + 2 = 16$$

Question 25

The number of numbers, strictly between 5000 and 10000 can be formed using the digits $1, 3, 5, 7, 9$ without repetition, is

A. 12

B. 120

C. 72

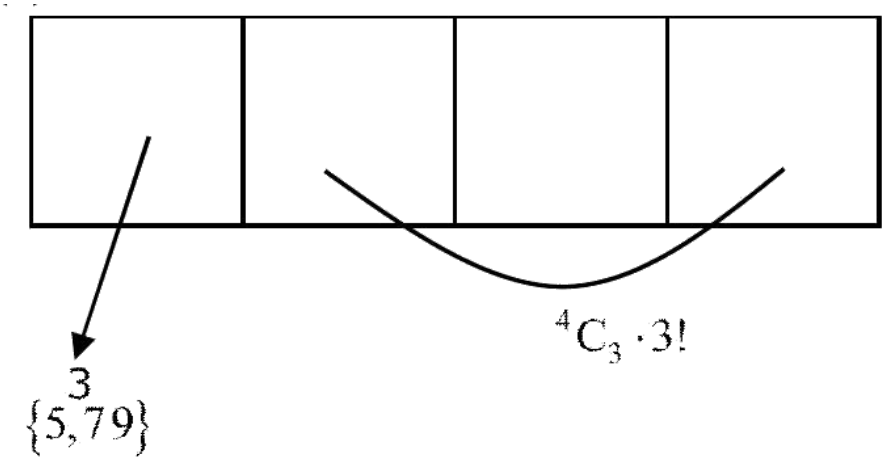
D. 6

Correct answer: C

Source solution

Sol. (3)}

No. of ways $= 3.4 \times 3! = 3.4! = 72$



Question 26

The number of integers, greater than 7000 that can be formed, using the digits $3, 5, 6, 7, 8$ without repetition, is

A. 168

B. 220

C. 120

D. 48

Correct answer: A

Source solution

Sol. 1
C-1
$2 \times 4 \times 3 \times 2 = 48$
C-2 $5!(5$ digit nos $) = \frac{120}{168}$

Question 27

The total number of six digit numbers, formed using the digits $4, 5, 9$ only and divisible by 6 , is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 81

Source solution

81
We have,
For this, 4 will be fixed as unit place digit
Total number
Case I: 4's $\rightarrow$ $\mathbf{6}$ times
1
Case II: 4's $\rightarrow$ 4times

$$\begin{aligned} &\mathbf{5's \rightarrow 1\ times} \quad \tfrac{5!}{3!} = 20 \\ &\mathbf{9's \rightarrow 1\ times} \end{aligned}$$

Case III: 4's $\rightarrow$ 3times

$$\mathbf{5's \rightarrow 3times} \quad \frac{5!}{2!3!} = 10$$

Case IV: $\mathbf{4}$'s $\rightarrow$ 3times

$$\mathbf{9's \rightarrow 3times} \quad \frac{5!}{2!3!} = 10$$

Case V: $\mathbf{4}$'s $\rightarrow$ $\mathbf{2}$ times

$$\begin{aligned} &\mathbf{5's \rightarrow 2times} \quad \tfrac{5!}{2!2!} = 30 \\ &\mathbf{9's \rightarrow 2\ times} \end{aligned}$$

Case VI: 4 's $\rightarrow 1$ times
5's $\rightarrow 1$ times $\quad \frac{5!}{4!}=5$
9's $\rightarrow 4$ times
Case VII: 4's $\rightarrow 1$ times

$$\begin{aligned} &5\text{'s} \rightarrow 4 \text{ times} \quad \frac{5!}{4!}=5 \\ &9\text{'s} \rightarrow 1 \text{ times} \end{aligned}$$

Total numbers $=81$

Question 28

Let 5 digit numbers be constructed using the digits $0,2,3,4,7,9$ with repetition allowed, and are arranged in ascending order with serial numbers. Then the serial number of the number 42923 is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 2997

Source solution

2997

$2\quad \overline{6}\quad \overline{6}\quad \overline{6}\quad \overline{6}=1296$
$3\quad \overline{6}\quad \overline{6}\quad \overline{6}\quad \overline{6}=1296$
$4\quad 0\quad \overline{6}\quad \overline{6}\quad \overline{6}=216$
$4\quad 2\quad 0\quad \overline{6}\quad \overline{6}=36$
$4\quad 2\quad 2\quad \overline{6}\quad \overline{6}=36$
$4\quad 2\quad 3\quad \overline{6}\quad \overline{6}=36$
$4\quad 2\quad 4\quad \overline{6}\quad \overline{6}=36$
$4\quad 2\quad 7\quad \overline{6}\quad \overline{6}=36$
$4\quad 2\quad 9\quad 0\quad \frac{-}{6}=6$
$4\quad 2\quad 9\quad 2\quad \underline{0}=1$
$4\quad 2\quad 9\quad 2\quad \underline{2}=1$
$4\quad 2\quad 9\quad 2\quad \underline{3}=\frac{1}{2997}$

Question 29

Five digit numbers are formed using the digits $1, 2, 3, 5, 7$ with repetitions and are written in descending order with serial numbers. For example, the number 77777 has serial number 1 . Then the serial number of 35337 is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 1436

Source solution

1436

Number starting with $7 = \begin{matrix} 7 & \overline{r_1}\hat{\uparrow}_5 \\ 5 & 5 \\ 5 & \bar{x} \end{matrix}$ $5\text{不} = 625$

Number starting with $5 = 5\underset{5}{\overset{}{\uparrow}}\ \underset{5}{\overset{}{\uparrow}}\ \underset{5}{\overset{}{\uparrow}}\ \underset{5}{\overset{}{\uparrow}} = 625$

Number starting with $37 = 37\underset{5}{\overset{}{\uparrow}}\underset{5}{\overset{}{\uparrow}}\underset{5}{\overset{}{\uparrow}} = 125$

Number starting with $357 = 357\underset{5}{\overset{7}{\uparrow}} = 25$

Number starting with $355 = 355 - _ = 25$

Number starting with $3537 = 3537 - = 5$

Number starting with $3535 = 3535 - = 5$

Number starting with $\underline{35337} = 1$

$$\text{Total} = 1436$$

Therefore, the serial number of 35337 is 1436

Question 30

The number of 3 -digit numbers, that are divisible by either 2 or 3 but not divisible by 7 , is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 514

Source solution

514

3 digit numbers divisible by either 2 or 3

$\mathbf{P} = \mathbf{n}(\text{ divisible by } 2) + \mathbf{n}(\text{ divisible by } 3) - \mathbf{n}(\text{ divisible by } 6)$

$\mathbf{P} = 450 + 300 - 150$

$\mathbf{P} = 600$

$\mathbf{Q} = \mathbf{n}$ (divisible by 14) +n (divisible by 21) -n (divisible by 42)

$= 64 + 43 - 21 = 86$

3 digit number divisible by either 2 or 3

But not divisible by -1 so $\mathbf{P} - \mathbf{Q} = 600 - 86 = 514$

Question 31

Number of 4-digit numbers (the repeation of digits is allowed) which are made using the digits $1, 2, 3$ and 5 , and are divisible by 15 , is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 21

Source solution

21

			5

Last digit must be 5 and sum of digits is divisible by 3 for divisible by 15

Remaining 3 digits	Arrange
$(1, 1, 2)$	$\frac{3!}{2} = 3$
$(1, 3, 3)$	$\frac{3!}{2} = 3$
$(1, 5, 1)$	$\frac{3!}{2} = 3$
$(2, 2, 3)$	$\frac{3!}{2} = 3$
$(2, 3, 5)$	$3! = 6$
$(3, 5, 5)$	$\frac{3!}{2} = 3$

Total numbers $= 21$

Question 32

The number of 3 digit numbers, that are divisible by either 3 or 4 but not divisible by 48 , is

A. 507

B. 432

C. 472

D. 400

Correct answer: B

Source solution

Sol. (2)}
Nos div. by 3
102, 105, 108
999
A.P. $\mathbf{a = 102 \quad d = 3 \quad \ell = 999}$
$\mathbf{n = \frac{\ell - a}{d} + 1 = \frac{999 - 102}{3} + 1 = 300}$
Numbers div. by 4
100, 104, 108 996
A.P. $\mathbf{a = 100 \quad d = 4 \quad \ell = 996}$
$\mathbf{n = \frac{996 - 100}{4} + 1 = \frac{896}{4} + 1 = 224 + 1 = 225}$
Numbers div. by 12
108, 120,
996
A.P. $\boldsymbol{a = 108 \quad d = 12 \quad \ell = 996}$
$\mathbf{n = \frac{996 - 108}{12} + 1 = \frac{888}{12} + 1 = 74 + 1 = 75}$
Numbers div. by 48
144, 192, ... 960
A.P. $\boldsymbol{a = 144 d = 48 \quad \ell = 960}$
$\mathbf{n = \frac{996 - 144}{12} + 1 = \frac{816}{48} + 1 = 17 + 1 = 18}$
$\therefore$ No. Div. by 354 but not by 48
$\mathbf{300 + 225 - 75 - 18}$
$\mathbf{= 450 - 18 = 432}$

Question 33

The number of 9 digit numbers, that can be formed using all the digits of the number 123412341 so that the even digits occupy only even places, is

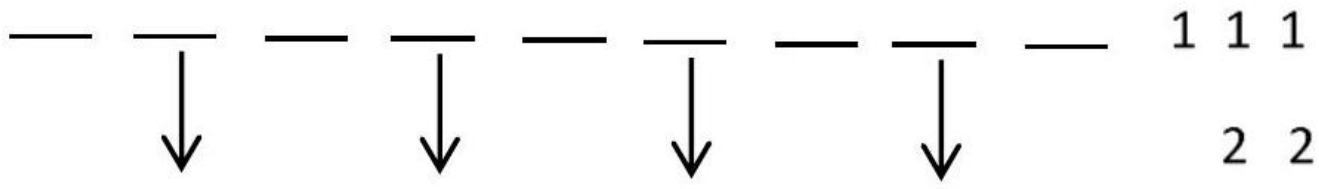
Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 60

Source solution

(60)



44
33

4 even place can be occupied by 4 even digits
No of always $= \frac{4!}{2!2!} = 6$
Odd place can be occupied by 5 odd digits
No of always $= \frac{5!}{3!2!} = 10$
Total no. $= 6 \times 10 = 60$

Question 34

The total number of 4 -digit numbers whose greatest common divisor with 54 is 2 , is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

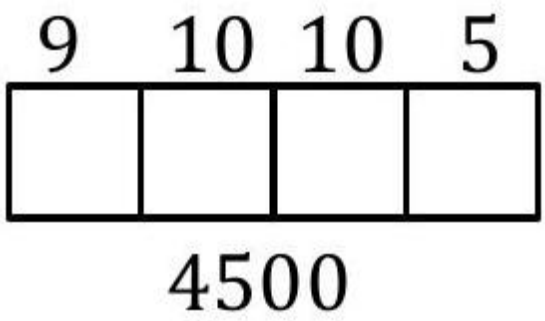
Correct answer: 3000

Source solution

3000

$54 = 2 \times 3^3$

4 digit even numbers are



4 digit even numbers which are multiple of 3

Are the numbers which are multiple of 6

$= \frac{9000}{6} = 1500$

$\therefore$ The no. which has GCD with 54 us 2 is $4500 - 1500 = 3000$

Question 35

The number of seven digits odd numbers, that can be formed using all the seven digits $1, 2, 2, 2, 3, 3, 5$ is

.

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 240

Source solution

240

The no. of 7 digit odd Numbers that can be formed using 1, 2, 2, 2, 3, 3, 5

□

$$\frac{\lfloor 6}{\lfloor 3\rfloor} = \frac{720}{12} = 60$$

						3

$$\frac{6}{\underline{3}} = \frac{720}{6} = 120$$

						5

$$\frac{\lfloor 6}{\lfloor 3 \lfloor 2} = \frac{720}{12} = 60$$

$= 240$

Question 36

Number of 4-digit numbers that are less than or equal to 2800 and either divisible by 3 or by 11 , is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 710

Source solution

710

4 digit number which are less then 2800 are $\mathbf{1000 - 2799}$

Number which are divisible by 3

$$2799 = 1002 + (n-1)3$$
$$n = 600$$

Number which are divisible by 11 in 1000-2799

= (Number which are divisible by 11 in $\mathbf{1 - 2799}$)

(Number which are divisible by 11 in 1 - 999)

$= \left[\frac{2799}{11}\right] - \left[\frac{999}{11}\right]$

$= 254 - 90$

= 164

Number which are divisible by 33 in 1000 - 2799

= (Number which are divisible by 33 in 1-2799) - (Number which are divisible by 33 in 1-999)

$= \left[\frac{2799}{33}\right] - \left[\frac{999}{33}\right]$

$= 84 - 30 = 54$

total number $= \mathbf{n(3) + n(11) - n(33)}$

$$= 600 + 164 - 54 = 710$$

Question 37

If all the six digit numbers $x_1x_2x_3x_4x_5x_6$ with $0 < x_1 < x_2 < x_3 < x_4 < x_5 < x_6$ are arranged in the increasing order, then the sum of the digits in the 72th number is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 32

Source solution

32
Number of six digit number starting with 1 is $1\ldots\ldots\ldots\ldots = {}^8C_5 = 56$
As remaining five digits can be selected from 8 digits that are greater than (i.e., $2, 3, 4, 5, 6, 7, 8$)
Number of six digit number starting with $23\ldots\ldots\ldots = {}^6C_4 = 15$

$$\text{Total } = 56 + 15 = 71$$

Now, 72nd number $= 245678$
$\therefore$ sum of the digits $= 2 + 4 + 5 + 6 + 7 + 8 = 32$

Question 38

Let x and y be distinct integers where $1 \leq x \leq 25$ and $1 \leq y \leq 25$. Then, the number of ways of choosing x and y, such that $x+y$ is divisible by 5 , is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 120

Source solution

$x+y=5\lambda$

X	y	No. of ways
5λ	5λ	20
$5\lambda+1$	$5\lambda+4$	25
$5\lambda+2$	$5\lambda+3$	25
$5\lambda+3$	$5\lambda+2$	25
$5\lambda+4$	$5\lambda+1$	25
		1200

Total Ways = 120

Question 39

A boy needs to select five courses from 12 available courses, out of which 5 courses are language courses. If he can choose at most two language courses, then the number of ways he can choose five courses is

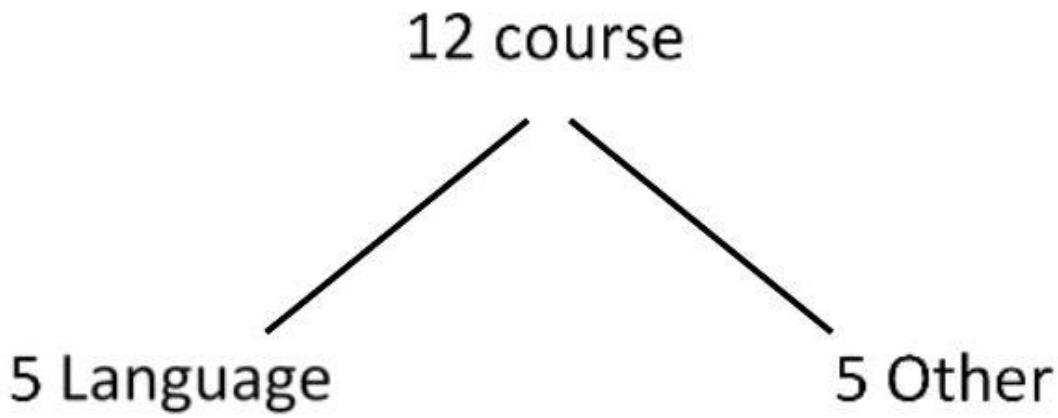
Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 546

Source solution

(546)



(0 language +5 other) $+(1$ Language +4 other $)+(2$ Language +3 other $)$

$= 5_{C_0} \cdot 7_{C_5} + 5_{C_1} \cdot 7_{C_4} + 5_{C_2} \cdot 7_{C_3}$

$= 21 + 175 + 350$

$= 546$

Question 40

Suppose Anil's mother wants to give 5 whole fruits to Anil from a basket of 7 red apples, 5 white apples and 8 oranges. If in the selected 5 fruits, at least 2 oranges, at least one red apple and at least one white apple must be given, then the number of ways, Anil's mother can offer 5 fruits to Anil is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 6860

Source solution

6860
Three cases are possible
$\mathbf{1R1\ W3O + 2R1\ W2O + 1R2\ W2O}$
$^{7}C_{1} \cdot {}^{5}C_{1} \cdot {}^{8}C_{3} + {}^{7}C_{2} \cdot {}^{5}C_{1} \cdot {}^{8}C_{2} + {}^{7}C_{1} \cdot {}^{5}C_{2} \cdot {}^{8}C_{2}$
$= 6860$

Binomial Theorem

Questions 41-41 / 1 selected / 0.3% practice weight / Consolidate

Question 41

Let K be the sum of the coefficients of the odd powers of x in the expansion of $(1+x)^{99}$. Let a be the middle term in the expansion of $\left(2+\frac{1}{\sqrt{2}}\right)^{200}$. If $\frac{{}^{200}C_{99}K}{a}=\frac{2^l m}{n}$, where m and n are odd numbers, then the ordered pair (l, n) is equal to

A. $(50,51)$

B. $(50,101)$

C. $(51,99)$

D. $(51,101)$

Correct answer: B

Source solution

Sol. (2)}

$\mathrm{K}=\frac{(1+1)^{99}}{2}=2^{98}$

$a={}^{200}C_{100}2^{100}\times\frac{1}{(\sqrt{2})^{100}}$

$\mathrm{a}={}^{200}\mathrm{C}_{100}2^{50}$

$\frac{{}^{200}\mathrm{C}_{99}\cdot\mathrm{K}}{\mathrm{a}}=\frac{{}^{200}\mathrm{C}_{99}\cdot 2^{98}}{{}^{200}\mathrm{C}_{100}\cdot 2^{50}}$

$\because\frac{{}^{200}\mathrm{C}_{99}}{{}^{200}\mathrm{C}_{100}}=\frac{1200}{\boxed{99}|101}\times\frac{100|100}{\boxed{200}}$

$=\frac{100}{101}$

$\therefore\frac{{}^{200}\mathrm{C}_{99}\,\mathrm{K}}{\mathrm{a}}=\frac{100}{101}\times 2^{48}$

$$=\frac{25\times 2^{50}}{101}$$

$\ell=50, \mathrm{n}=101$

$(\ell, n)=(50,101)$

Sequences and Series

Questions 42-76 / 35 selected / 11.7% practice weight / Focus first

Question 42

If the domain of the function $\log_5\left(18x - x^2 - 77\right)$ is (α, β) and the domain of the function $\log_{(x-1)}\left(\frac{2x^2+3x-2}{x^2-3x-4}\right)$ is (γ, δ), then $\alpha^2 + \beta^2 + \gamma^2$ is equal to :

A. 195

B. 174

C. 186

D. 179

Correct answer: C

Source solution

$$\log_5(18x - x^2 - 77)$$

has domain (α, β),

and

$$\log_{(x-1)}\left(\frac{2x^2 + 3x - 2}{x^2 - 3x - 4}\right)$$

has domain (γ, δ).

We need to find

$$\alpha^2 + \beta^2 + \gamma^2.$$

\hrulefill

\textbf{Step 1: Domain of } $\log_5(18x - x^2 - 77)$

For a logarithm,

$$18x - x^2 - 77 > 0.$$

$$-x^2 + 18x - 77 > 0$$

$$x^2 - 18x + 77 < 0.$$

Factorizing,

$$x^2 - 18x + 77 = (x - 7)(x - 11).$$

Hence,

$$(x - 7)(x - 11) < 0.$$

Therefore,

$$7 < x < 11.$$

Thus,

$$(\alpha, \beta) = (7, 11).$$

So,

$$\alpha = 7, \qquad \beta = 11.$$

\hrulefill

\textbf{Step 2: Domain of }

$$\log_{(x-1)}\left(\frac{2x^2+3x-2}{x^2-3x-4}\right)$$

For a logarithm:

$$x-1>0, \qquad x-1\neq 1$$

which gives

$$x>1, \qquad x\neq 2.$$

Also,

$$\frac{2x^2+3x-2}{x^2-3x-4}>0.$$

Factorize:

$$2x^2+3x-2=(2x-1)(x+2),$$

$$x^2-3x-4=(x-4)(x+1).$$

Hence,

$$\frac{(2x-1)(x+2)}{(x-4)(x+1)}>0.$$

Critical points:

$$-2,\ -1,\ \frac{1}{2},\ 4.$$

Using sign analysis:

$$\frac{(2x-1)(x+2)}{(x-4)(x+1)} > 0$$

for

$$(-\infty, -2) \cup (-1, \tfrac{1}{2}) \cup (4, \infty).$$

Now intersect with

$$x > 1.$$

Therefore,

$$x \in (4, \infty).$$

Thus the domain is

$$(\gamma, \delta) = (4, \infty).$$

Hence,

$$\gamma = 4.$$

\hrulefill

\textbf{Step 3: Required value}

$$\alpha^2+\beta^2+\gamma^2=7^2+11^2+4^2$$

$$=49+121+16$$

$$=186.$$

$$\boxed{186}$$

Hence, the correct option is

$$\boxed{\text{(3) } 186}.$$

Question 43

Let f be a function such that $3f(x) + 2f\left(\frac{m}{19x}\right) = 5x$, $x \neq 0$, where $m = \sum_{i=1}^{9}(i)^2$. Then $f(5) - f(2)$ is equal to

A. -9

B. 36

C. 18

D. 9

Correct answer: C

Source solution

$m = \frac{9\times10\times19}{6} = 15 \times 19$

$$3f(x) + 2f\left(\frac{15}{x}\right) = 5x$$

Replace x by $\frac{15}{x}$

$$3f\left(\frac{15}{x}\right) + 2f(x) = \frac{75}{x}$$

$$9f(x) - 4f(x) = 15x - \frac{150}{x}$$
$$5f(x) = 15x - \frac{150}{x}$$
$$f(x) = 3x - \frac{30}{x}$$
$$f(5) = 15 - \frac{30}{5} = 9$$
$$f(2) = 6 - 15 = -9$$
$$f(5) - f(2) = 18$$

Question 44

Points $\mathbf{P(-3,2), Q(9,10)}$ and $\mathbf{(a,4)}$ lie on a circle C with PR as its diameter, The tangents to C at the points Q and $\boldsymbol{R}$ intersect at the point $\boldsymbol{S}$. If $\boldsymbol{S}$ lies on the line $\mathbf{2x-ky=1}$, then $\boldsymbol{k}$ is equal to

Numerical answer: _____

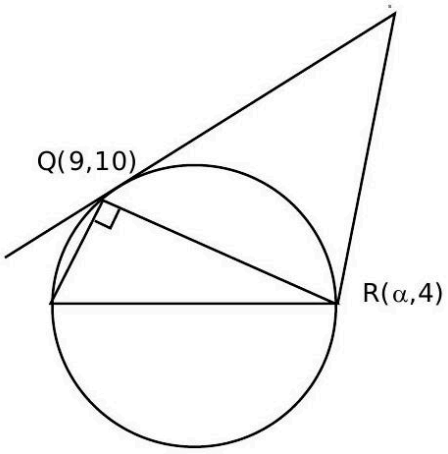
Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 3

Source solution

Equation of circle is

$(x+3)(x-\alpha)+(y-2)(y-4)=0$



Q lies on it

$12(9-\alpha)+8\times 6=0$

$\alpha=13$

$x^2+y^2-10x-6y-31=0$

Equation of Tangent at Q

$x.9+y.10-5(x+9)-3(y+10)-31=0$

$$4x+7y=106 \quad (1)$$

Equation of Tangent at R

$x.13 + y.4 - 5(x + 13) - 3(y + 4) - 31 = 0$

$$8x + y = 108 \quad (2)$$

Solution (1) and (2)

$\mathbf{s} = \left(\frac{25}{2}, 8\right)$

which lies on $2x - ky = 1$

$\mathbf{k = 3}$

Question 45

Let $\mathbf{f}:\mathbf{N}\to\mathbf{R}$ be a function such that $f(x+y)=2f(x)f(y)$ for natural numbers x and y. If $f(1)=2$, then the value of α for which

$$\sum_{k=1}^{10} f(\alpha+k)=\frac{512}{3}\left(2^{20}-1\right)$$

holds, is

A. 2

B. 3

C. 4

D. 6

Correct answer: C

Source solution

$f:N\to R, f(x+y)=2f(x)f(y)$
$\mathbf{f(1)=2,}$

$$\begin{aligned}\sum_{k=1}^{10} f(\alpha+k) &= 2f(\alpha)\sum_{k=1}^{10} f(k)\\ &= 2f(\alpha)(f(1)+f(2)+\ldots..+f(10))\end{aligned} \tag{2}$$

From (1)
$\mathbf{f(2)=2f^2(1)=2^3}$
$\mathbf{f(3)=2f(2)f(1)=2^5}$
$\vdots\quad\vdots$
$\mathbf{f(10)=2^9f^{10}(1)=2^{19}}$
$\mathbf{f(\alpha)=2^{2\alpha-1};\alpha\in N}$
from (2)

$$\sum_{k=1}^{10} f(\alpha+k) = 2\left(2^{2\alpha-1}\right)\left(2+2^3+2^5+\ldots.+2^{19}\right)$$

$$\frac{512}{3}\left(2^{20}-1\right) = 2^{2\alpha}\left(2\frac{\left(2^{20}-1\right)}{3}\right)$$

Hence $\alpha = 4$

Question 46

The point of intersection C of the plane $8x + y + 2z = 0$ and the line joining the point $A(-3, -61)$ and $\mathrm{B}(2, -4, -3)$ divides the line segment AB internally in the ratio k :. If $\mathrm{a}, \mathrm{b}, \mathrm{c}(|\mathrm{a}|, |\mathrm{b}|, |\mathrm{c}|)$ are coprime are the direction ratios of the perpendicular form the point C on the line $\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}$, then $|\mathrm{a} + \mathrm{b} + \mathrm{c}|$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 10

Source solution

10

Plane : $8x + y + 2z = 0$

Given line AB : $\frac{x-2}{5} = \frac{y-4}{10} = \frac{z+3}{-4} = \lambda$

Any point on line $(5\lambda + 2, 10\lambda + 4, -4\lambda - 3)$

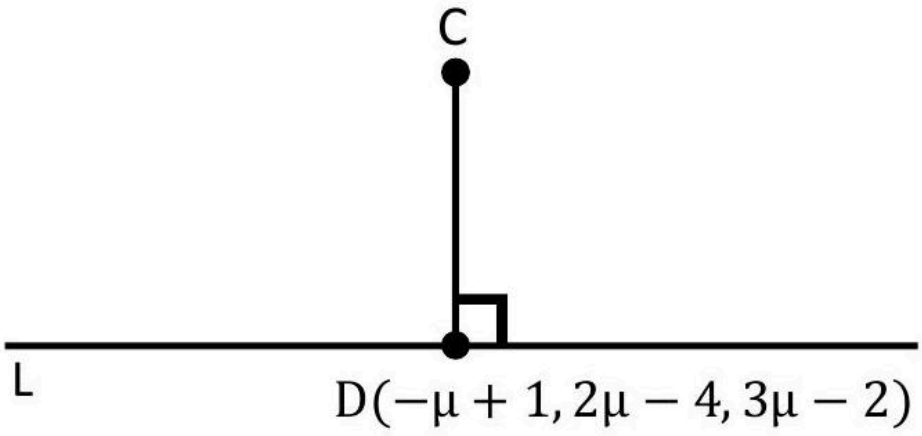
Point of intersection of line and plane

$8(5\lambda + 2) + 10\lambda + 4 - 8\lambda - 6 = 0$

$\lambda = -\frac{1}{3}$

C $\left(\frac{1}{3}, \frac{2}{3}, -\frac{5}{3}\right)$

$\mathrm{L} : \frac{x-1}{-1} = \frac{y+4}{2} = \frac{z+2}{3} = \mu$



$\overline{CD} = \left(-\mu + \frac{2}{3}\right)\hat{i} + \left(2\mu - \frac{14}{3}\right)\hat{j} + \left(3\mu - \frac{1}{3}\right)\hat{k}$

$\left(-\mu + \frac{2}{3}\right)(-1) + \left(2\mu - \frac{14}{3}\right)2 + \left(3\mu - \frac{1}{3}\right)3 = 0$

$\mu = \frac{11}{14}$

$\overrightarrow{\mathrm{CD}} = \frac{-5}{42}, \frac{-130}{42}, \frac{85}{42}$

Direction ratio $\rightarrow (-1, -26, 17)$

$|a+b+c| = 10$

Question 47

A line passes through $\mathbf{A(4, -6, -2)}$ and $\mathbf{B(16, -2, 4)}$. The point $\mathbf{P(a, b, c)}$ where $\mathbf{a, b, c}$ are non-negative integers, on the line AB lies at a distance of 21 units, from the point A . The distance between the points $\mathbf{P(a, b, c)}$ and $\mathbf{Q(4, -12, 3)}$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 22

Source solution

-

$$\frac{x-4}{12} = \frac{x+6}{4} = \frac{z+2}{6}$$

$$\frac{x-4}{\frac{6}{7}} = \frac{y+6}{\frac{2}{7}} = \frac{z+2}{\frac{3}{7}} = 21$$

$$\left(21 \times \frac{6}{7} + 4, \frac{2}{7} \times 21 - 6, \frac{3}{7} \times 21 - 2\right)$$

$$= (22, 0, 7) = (a, b, c)$$

$$\therefore \sqrt{324 + 144 + 16} = 22$$

Question 48

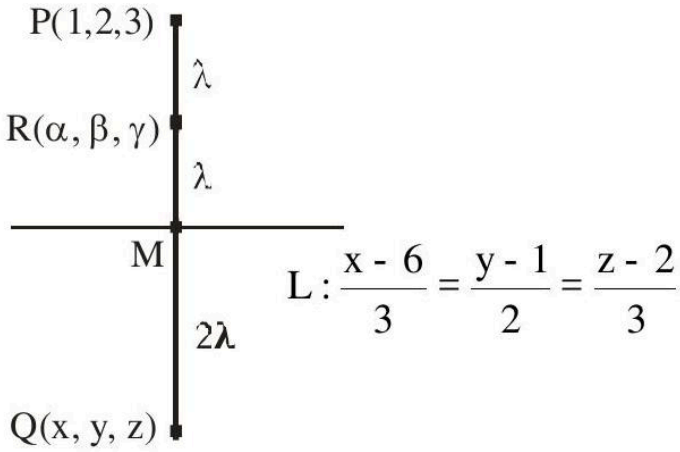
Let the image of the point $\mathbf{P(1,2,3)}$ in the line $\mathbf{L: \frac{x-6}{3} = \frac{y-1}{2} = \frac{z-2}{3}}$ be Q . let $\mathbf{R(\alpha,\beta,\gamma)}$ be a point that divides internally the line segment PQ in the ratio $\mathbf{1:3}$. Then the value of $\mathbf{22(\alpha+\beta+\gamma)}$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 125

Source solution



Let M be the mid-point of PQ

$\therefore \mathbf{M} = (3\lambda+6, 2\lambda+1, 3\lambda+2)$

Now, $\overrightarrow{\mathbf{PM}} = (3\lambda+5)\hat{\mathbf{i}} + (2\lambda-1)\hat{\mathbf{j}} + (3\lambda-1)\hat{\mathbf{k}}$

$\because \overrightarrow{\mathbf{PM}} \perp (3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$

$\therefore 3(3\lambda+5) + 2(2\lambda-1) + 3(3\lambda-1) = 0$

$\lambda = \frac{-5}{11}$

$\therefore \mathbf{M}\left(\frac{51}{11}, \frac{1}{11}, \frac{7}{11}\right)$

Since R is mid-point of PM

$\mathbf{22(\alpha+\beta+\gamma) = 125}$

Question 49

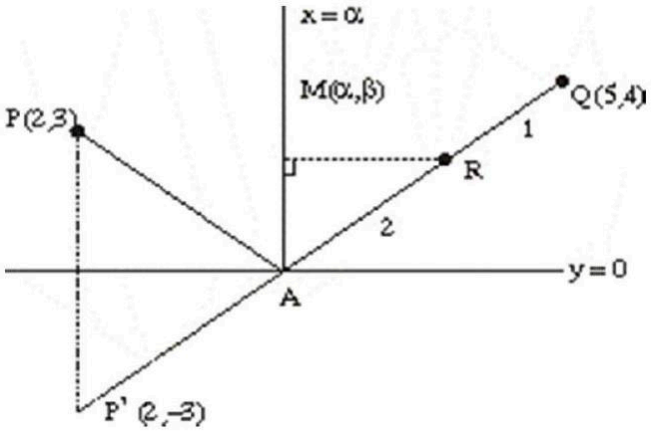
A ray of light passing through the point $\mathbf{P(2,3)}$ reflects on the x -axis at point A and the reflected ray passes through the point $\mathbf{Q(5,4)}$. Let R be the point that divides the line segment AQ internally into the ratio $\mathbf{2:1}$. Let the co-ordinates of the foot of the perpendicular M from R on the bisector of the angle PAQ be (α, β). Then, the value of $7\alpha + 3\beta$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 31

Source solution



By observation we see that $\mathbf{A}(\alpha, 0)$.
And $\beta = \mathbf{y}$-cordinate of R

$$= \frac{2 \times 4 + 1 \times 0}{2+1} = \frac{8}{3} \quad (1)$$

Now $\mathbf{P'}$ is image of P in $\mathbf{y = 0}$ which will be $\mathbf{P'(2,-3)}$
$\therefore$ Equation of $\mathbf{P'Q}$ is $(\mathbf{y+3}) = \frac{4+3}{5-2}(\mathbf{x-2})$
i.e. $3y + 9 = 7x - 14$
$\mathbf{A} \equiv \left(\frac{23}{7}, 0\right)$ by solving with $\mathbf{y = 0}$

$$\therefore \alpha = \frac{23}{7} \quad (2)$$

By (1), (2)

$$7\alpha + 3\beta = 23 + 8 = 31$$

Question 50

The common difference of the A.P.: $a_1, a_2, \ldots, a_m$ is 13 more than the common difference of the A.P.: $b_1, b_2, \ldots, b_n$. If $b_{31} = -277$, $b_{43} = -385$ and $a_{78} = 327$, then a_1 is equal to

A. 21

B. 24

C. 19

D. 16

Correct answer: C

Source solution

Let common difference of A.P.'s are $d_1 \& d_2$

$$\therefore d_1 = 13 + d_2$$
$$b_1 + 30\, d_2 = -277$$
$$b_1 + 42\, d_2 = -385$$

By (2) - (1)
$12\, d_2 = -108$
$d_2 = -9$
$\therefore d_1 = 4$
Now $a_{78} = 327$

$$\Rightarrow a_1 + 77d_1 = 327$$
$$\Rightarrow a_1 + 308 = 327$$
$$a_1 = 19$$

Question 51

Let $\mathbf{a_1}, \frac{\mathbf{a_2}}{2}, \frac{\mathbf{a_3}}{2^2}, \ldots, \frac{\mathbf{a_{10}}}{2^9}$ be a G.P. of common ratio $\frac{1}{\sqrt{2}}$. If $\mathbf{a_1} + \mathbf{a_2} + \ldots + \mathbf{a_{10}} = \mathbf{62}$, then $\mathbf{a_1}$ is equal to :

A. $2(\sqrt{2}-1)$

B. $2-\sqrt{2}$

C. $\sqrt{2}-1$

D. $2(2-\sqrt{2})$

Correct answer: A

Source solution

$\frac{\mathbf{a_2}}{\mathbf{2a_1}} = \frac{\mathbf{a_3}}{\mathbf{2a_2}} = \frac{\mathbf{a_4}}{\mathbf{2a_3}} = \ldots . = \frac{\mathbf{a_{10}}}{\mathbf{2a_9}} = \frac{1}{\sqrt{2}}$

$\therefore \mathbf{a_1, a_2, a_3, ., a_{10}}$ are in G.P. with common ratio $\sqrt{2}$.

$$\sum_{i=1}^{10} a_i = \frac{a_1\left((\sqrt{2})^{10}-1\right)}{\sqrt{2}-1} = 62$$

$$\Rightarrow a_1 = 2(\sqrt{2}-1)$$

Question 52

Let $a_1, a_2, a_3, \ldots$. be a G.P. of increasing positive terms such that $a_2 . a_3 . a_4 = 64$ and $a_1 + a_3 + a_5 = \frac{813}{7}$. Then $a_3 + a_5 + a_7$ is equal to :

A. 3256

B. 3252

C. 3244

D. 3248

Correct answer: B

Source solution

$ar . ar^2 . ar^3 = 64$

$$a^3 r^6 = 64 \quad \Rightarrow ar^2 = 4$$
$$a + ar^2 + ar^4 = \frac{813}{7}$$
$$r^2 = 28$$
$$ar^2 + ar^4 + ar^6 = ?$$
$$ar^2 \left(1 + r^2 + r^4\right) = 4(1 + 28 + 784) = 3252$$

Question 53

Let $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ be an A.P. of four terms such that each term of the A. P. and its common difference l are integers. If $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 48$ and $\alpha_1, \alpha_2, \alpha_3, \alpha_4 + l^4 = 361$ then the largest term of the A.P. is equal to

A. 27

B. 24

C. 21

D. 23

Correct answer: A

Source solution

$\mathrm{a_1, a_2, a_3, a_4}$ as $\mathrm{a-3\,d, a-d, a+d, a+3\,d}$
where $\mathrm{d} = \frac{\ell}{2}$

$$\because a_1 + a_2 + a_3 + a_4 = 48 \Rightarrow 4a = 48 \Rightarrow a = 12$$

$$\begin{aligned} \&a_1a_2a_3a_4 + \ell^4 = 361 \Rightarrow \left(a^2 - 9d^2\right)\left(a^2 - d^2\right) + 16d^4 \\ &= 361 \\ &\Rightarrow \left(144 - 9\,\mathrm{d}^2\right)\left(144 - \mathrm{d}^2\right) + 16\,\mathrm{d}^4 = 361 \\ &\Rightarrow 25\,\mathrm{d}^4 - 1440\,\mathrm{d}^2 + (144)^2 = 361 \end{aligned}$$

$$\begin{aligned} &\left(5\,\mathrm{d}^2 - 144\right)^2 = 19^2 \\ &\therefore 5\,\mathrm{d}^2 - 144 = 19 \text{ or } -19 \\ &\mathrm{d}^2 = \frac{163}{5} \text{ or } \mathrm{d}^2 = \frac{125}{5} = 25 \\ &\mathrm{d} = \sqrt{\frac{163}{5}} \text{ or } \mathrm{d} = 5 \\ &\therefore \ell = 2\sqrt{\frac{163}{5}} \text{ or } \ell = 10 \end{aligned}$$

(rejected)

$$\begin{aligned} &\because \text{ common difference is an integer} \\ &\therefore \text{ largest term } = 12 + 15 = 27 \end{aligned}$$

Question 54

Consider an A.P. of positive integers, whose sum of the first three terms is 54 and the sum of the first twenty terms lies between 1600 and 1800. Then its 11th term is :

A. 84

B. 122

C. 90

D. 108

Correct answer: C

Source solution

Sol. $\mathrm{S_3} = 3\mathrm{a} + 3\,\mathrm{d} = 54$

$\Rightarrow a + d = 18$

$\mathrm{S_{20}} = 10(2\mathrm{a} + 19\,\mathrm{d})$

$\Rightarrow 10(36 + 17\,\mathrm{d})$

$\Rightarrow 1600 < 10(36 + 17\,\mathrm{d}) < 1800$

$\Rightarrow 160 < 36 + 17\,\mathrm{d} < 180$

$\Rightarrow 124 < 17\,\mathrm{d} < 144$

$\Rightarrow 7\frac{5}{17} < \mathrm{d} < 8\frac{8}{17}$

Common difference will be natural number

$\Rightarrow \mathrm{d} = 8 \Rightarrow \mathrm{a} = 10$

$\Rightarrow \mathrm{a_{11}} = 10 + 10 \times 8 = 90$

Question 55

Let $a_1, a_2, a_3 \ldots$. be a G.P. of increasing positive terms. If $\mathbf{a_1 a_5 = 28}$ and $\mathbf{a_2 + a_4 = 29}$, the $\mathbf{a_6}$ is equal to

A. 628

B. 526

C. 784

D. 812

Correct answer: C

Source solution

Sol. $a_1 \cdot a_5 = 28 \Rightarrow a \cdot ar^4 = 28 \Rightarrow a^2r^4 = 28$

$$a_2 + a_4 = 29 \Rightarrow ar + a^3 = 29 \quad (1)$$

$\Rightarrow \mathbf{ar\left(1 + r^2\right) = 29}$

$$\Rightarrow \mathbf{a^2r^2\left(1 + r^2\right)^2 = (29)^2} \quad (2)$$

By Eq. (1) & (2)

$\frac{\mathbf{r^2}}{\mathbf{(1+r^2)^2}} = \frac{\mathbf{28}}{\mathbf{29\times 29}}$

$\Rightarrow \frac{\mathbf{r}}{\mathbf{1+r^2}} = \frac{\sqrt{\mathbf{28}}}{\mathbf{29}} \Rightarrow \mathbf{r} = \sqrt{\mathbf{28}}$

$\because \mathbf{a^2r^4 = 28 \Rightarrow a^2 \times (28)^2 = 28}$

$\Rightarrow \mathbf{a} = \frac{\mathbf{1}}{\sqrt{\mathbf{28}}}$

$\therefore \mathbf{a_6 = ar^5} = \frac{1}{\sqrt{28}} \times (28)^2\sqrt{28} = 784$

Question 56

Let S_n be the sum to n-terms of an arithmetic progression 3, 7, 11, ...
If $40 < \left(\frac{6}{n(n+1)} \sum_{k=1}^{n} S_k\right) < 42$, then n equals .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 9

Source solution

$S_n = 3 + 7 + 11 + ... n$ terms

$$= \frac{n}{2}(6 + (n-1)4) = 3n + 2n^2 - 2n$$
$$= 2n^2 + n$$
$$\sum_{k=1}^{n} S_k = 2\sum_{k=1}^{n} K^2 + \sum_{k=1}^{n} K$$
$$= 2 \cdot \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$
$$= n(n+1)\left[\frac{2n+1}{3} + \frac{1}{2}\right]$$
$$= \frac{n(n+1)(4n+5)}{6}$$

$\Rightarrow 40 < \frac{6}{n(n+1)} \sum_{k=1}^{n} S_k < 42$
$40 < 4n + 5 < 42$
$35 < 4n < 37$
$n = 9$

Question 57

If the first term of an A.P. is 3 and the sum of its first four terms is equal to one-fifth of the sum of the next four terms, then the sum of the first 20 terms is equal to

A. -1200

B. -1080

C. -1020

D. -120

Correct answer: B

Source solution

\textbf{Given:}

The first term of an A.P. is

$$a = 3.$$

Also, the sum of the first four terms is one-fifth of the sum of the next four terms.

We have to find the sum of the first 20 terms.

\hrulefill

\textbf{Step 1: Find the sum of the first four terms}

The first four terms are

$$3,\ 3+d,\ 3+2d,\ 3+3d.$$

Hence

$$S_1 = 3 + (3+d) + (3+2d) + (3+3d) = 12 + 6d.$$

\hrulefill

\textbf{Step 2: Find the sum of the next four terms}

The next four terms are

$$3+4d,\ 3+5d,\ 3+6d,\ 3+7d.$$

Their sum is

$$S_2=(3+4d)+(3+5d)+(3+6d)+(3+7d)=12+22d.$$

Given that

$$S_1=\frac{1}{5}S_2.$$

Therefore,

$$12+6d=\frac{1}{5}(12+22d).$$

Multiplying by 5,

$$60+30d=12+22d.$$

$$8d=-48.$$

$$d=-6.$$

\hrulefill

\textbf{Step 3: Find the sum of the first 20 terms}

Using

$$S_n = \frac{n}{2}\Big[2a + (n-1)d\Big],$$

with

$$a = 3, \qquad d = -6, \qquad n = 20,$$

we get

$$S_{20} = \frac{20}{2}\Big[2(3) + 19(-6)\Big].$$

$$= 10(6 - 114).$$

$$= 10(-108).$$

$$= -1080.$$

\hrulefill

$$\boxed{S_{20} = -1080}$$

Hence, the correct option is

$$\boxed{(2)\ -1080}.$$

Question 58

Suppose that the number of terms in an A.P. is 2 k , $k \in N$. If the sum of all odd terms of the A.P. is 40 , the sum of all even terms is 55 and the last term of the A.P. exceeds the first term by 27 , then k is equal to

A. 5

B. 8

C. 6

D. 4

Correct answer: A

Source solution

Sol. $a_1, a_2, a_3, \ldots\ldots, a_{2k} \quad \rightarrow$ A.P.
$\sum_{r=1}^{k} a_{2r-1} = 40, \sum_{r=1}^{k} a_{2r} = 55, a_{2k} - a_1 = 27$

$\frac{k}{2}[2a_1 + (k-1)2\,d] = 40, \frac{k}{2}[2a_2 + (k-1)2\,d] = 55,$
$d = \frac{27}{2k-1}.$
$a_1 = \frac{40}{k} - (k-1)d = \frac{55}{k} - kd$
$d = \frac{15}{k} \Rightarrow \frac{27}{2k-1} = \frac{15}{k} \Rightarrow 9 <. k = 10k - 5.$
$\therefore k = 5$

Question 59

In an increasing geometric progression ol positive terms, the sum of the second and sixth terms is $\frac{70}{3}$ and the product of the third and fifth terms is 49. Then the sum of the $4^{th}, 6^{th}$ and 8th terms is :-

A. 96

B. 78

C. 91

D. 84

Correct answer: C

Source solution

Sol. $T_2 + T_6 = \frac{70}{3}$

$ar + ar^5 = \frac{70}{3}$

$T_3 \cdot T_5 = 49$

$ar^2 . ar^4 = 49$

$a^2r^6 = 49$

$ar^3 = +7, a = \frac{7}{r^3}$

$ar\left(1 + r^4\right) = \frac{70}{3}$

$\frac{7}{r^2}\left(1 + r^4\right) = \frac{70}{3}, r^2 = t$

$\frac{1}{t}\left(1 + t^2\right) = \frac{10}{3}$
$3t^2 - 10t + 3 = 0$

$t = 3, \frac{1}{3}$

Increasing G.P. $r^2 = 3, r = \sqrt{3}$

$T_4 + T_6 + T_8$
$= ar^3 + ar^5 + ar^7$

$= ar^3\left(1+r^2+r^4\right)$
$= 7(1+3+9) = 91$

Question 60

Let T_r be the rth term of an A.P. If for some m, $T_m = \frac{1}{25}$, $T_{25} = \frac{1}{20}$ and $20\sum_{r=1}^{25} T_r = 13$, then $5\,m\sum_{r=m}^{2m} T_r$ is equal to :

A. 112

B. 126

C. 98

D. 142

Correct answer: B

Source solution

Sol. $T_m = \frac{1}{25},\ T_{25} = \frac{1}{20}, 20\sum_{r=1}^{25} T_r = 13$

$$T_m = a + (m-1)d = \frac{1}{25} \tag{1}$$

$T_{25} = a + 24\,d = \frac{1}{20}$
$20 \cdot \frac{25}{2}\left[a + \frac{1}{20}\right] = 13 \Rightarrow a = \frac{1}{500}$
also, $20\,S_{25} = 20 \cdot \frac{25}{2}[2a + 24\,d] = 13 \Rightarrow\ d = \frac{1}{500}$
from (1) $\frac{1}{500} + \frac{m-1}{500} = \frac{1}{25} \Rightarrow\ m = 20$
Now,
$5\,m\sum_{r=m}^{2m} T_r = 100\sum_{r=20}^{40} T_r = 126$

Question 61

Let $S_n = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \ldots$ upto n terms. If the sum of the first six terms of an A.P. with first term -p and common difference p is $\sqrt{2026\, S_{2025}}$, then the absolute difference between 20th and 15th terms of the A.P. is

A. 25

B. 90

C. 20

D. 45

Correct answer: A

Source solution

\textbf{Given:}

$$S_n = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \cdots$$

up to n terms.

Also, the sum of the first six terms of an A.P. with first term $-p$ and common difference p is

$$\sqrt{2026\, S_{2025}}.$$

We have to find the absolute difference between the 20th and 15th terms of the A.P.

\hrulefill

\textbf{Step 1: Find the general term of the series}

Observe that

$$\frac{1}{2} = \frac{1}{1\cdot 2}, \qquad \frac{1}{6} = \frac{1}{2\cdot 3}, \qquad \frac{1}{12} = \frac{1}{3\cdot 4}, \qquad \frac{1}{20} = \frac{1}{4\cdot 5}.$$

Hence the rth term is

$$\frac{1}{r(r+1)}.$$

Therefore

$$S_n = \sum_{r=1}^{n} \frac{1}{r(r+1)}.$$

Using

$$\frac{1}{r(r+1)} = \frac{1}{r} - \frac{1}{r+1},$$

we get

$$S_n = \sum_{r=1}^{n} \left(\frac{1}{r} - \frac{1}{r+1} \right).$$

This is a telescoping series:

$$S_n = 1 - \frac{1}{n+1} = \frac{n}{n+1}.$$

Thus

$$S_{2025} = \frac{2025}{2026}.$$

\hrulefill

\textbf{Step 2: Find the sum of the first six terms of the A.P.}

Given

$$S_6^{(\mathrm{AP})} = \sqrt{2026\, S_{2025}}.$$

Substituting $S_{2025} = \frac{2025}{2026}$,

$$S_6^{(\mathrm{AP})} = \sqrt{2026 \cdot \frac{2025}{2026}} = \sqrt{2025} = 45.$$

\hrulefill

\textbf{Step 3: Determine p}

The A.P. is

$$-p,\ 0,\ p,\ 2p,\ 3p,\ 4p, \ldots$$

since the first term is $-p$ and common difference is p.

Sum of the first six terms:

$$S_6 = \frac{6}{2}\left[2(-p) + 5p\right] = 3(3p) = 9p.$$

Given

$$9p = 45,$$

hence

$$p = 5.$$

\hrulefill

\textbf{Step 4: Find the required difference}

The nth term is

$$a_n = -p + (n-1)p = (n-2)p.$$

Therefore,

$$a_{20} = 18p, \qquad a_{15} = 13p.$$

Hence

$$|a_{20} - a_{15}| = |18p - 13p| = 5p.$$

Substituting $p = 5$,

$$|a_{20} - a_{15}| = 25.$$

\hrulefill

$$\boxed{25}$$

Hence, the correct option is

$$\boxed{(1)\ 25}.$$

Question 62

If in a G.P. of 64 terms, the sum of all the terms is 7 times the sum of the odd terms of the G.P, then the common ratio of the G.P. is equal to

A. 7

B. 4

C. 5

D. 6

Correct answer: D

Source solution

\textbf{Given:}

A G.P. has $\mathbf{64}$ terms and

$$\textbf{(Sum of all terms)} = 7 \times \textbf{(Sum of odd-numbered terms)}.$$

We have to find the common ratio r.

\hrulefill

\textbf{Step 1: Sum of all 64 terms}

Let the first term be a and common ratio be r.

Then

$$S_{64} = a\frac{1 - r^{64}}{1 - r}.$$

\hrulefill

\textbf{Step 2: Sum of odd-numbered terms}

The odd-numbered terms are

$$a,\ ar^2,\ ar^4, \ldots, ar^{62}.$$

This is a G.P. with

$$\text{first term} = a, \qquad \text{common ratio} = r^2,$$

and number of terms

$$32.$$

Hence

$$S_{\text{odd}} = a\frac{1 - r^{64}}{1 - r^2}.$$

Since

$$1 - r^2 = (1 - r)(1 + r),$$

we get

$$S_{\text{odd}} = a\frac{1 - r^{64}}{(1 - r)(1 + r)}.$$

\hrulefill

\textbf{Step 3: Use the given condition}

Given

$$S_{64} = 7S_{\text{odd}}.$$

Substituting the expressions,

$$a\frac{1-r^{64}}{1-r}=7a\frac{1-r^{64}}{(1-r)(1+r)}.$$

Cancelling the common factor

$$a(1-r^{64}),$$

we obtain

$$\frac{1}{1-r}=\frac{7}{(1-r)(1+r)}.$$

Multiplying by $(1-r)(1+r)$,

$$1+r=7.$$

Therefore,

$$r=6.$$

\hrulefill

$$\boxed{r=6}$$

Hence, the correct option is

$$\boxed{(4)\ 6}.$$

Question 63

Let $3, 7, 11, 15, \ldots, 403$ and $2, 5, 8, 11, \ldots, 404$ be two arithmetic progressions. Then the sum, of the common terms in them, is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 6699

Source solution

3, 7, 11, 15, ..., 403

$2, 5, 8, 11, \ldots, 404$

$\mathrm{LCM}(4, 3) = 12$

$11, 23, 35, \ldots..$ let (403)

$403 = 11 + (n - 1) \times 12$

$\frac{392}{12} = n - 1$

$33 \cdot 66 = n$

$n = 33$

Sum $\frac{33}{2}(22 + 32 \times 12)$

=6699

Question 64

Let S_n denote the sum of the first n terms of an arithmetic progression. If $S_{10} = 390$ and the ratio of the tenth and the fifth terms is $15 : 7$, then $S_{15} - S_5$ is equal to:

A. 800

B. 890

C. 790

D. 690

Correct answer: C

Source solution

Sol. $S_{10} = 390$

$\frac{10}{2}[2a + (10-1)d] = 390$

$$\Rightarrow 2a + 9d = 78 \quad (1)$$

$$\frac{t_{10}}{t_5} = \frac{15}{7} \Rightarrow \frac{a+9d}{a+4d} = \frac{15}{7} \Rightarrow 8a = 3d \quad (2)$$

From (1) & (2) $a = 3 \& d = 8$

$S_{15} - S_5 = \frac{15}{2}(6 + 14 \times 8) - \frac{5}{2}(6 + 4 \times 8)$

$= \frac{15 \times 118 - 5 \times 38}{2} = 790$

Question 65

Let $a_1, a_2, a_3, \ldots$ be a G. P. of increasing positive numbers. If $\mathbf{a_3 a_5 = 729}$ and $\mathbf{a_2 + a_4 = \frac{111}{4}}$, then $\mathbf{24\left(a_1 + a_2 + a_3\right)}$ is equal to

A. 131

B. 130

C. 129

D. 128

Correct answer: C

Source solution

Sol. Let the $\mathbf{I^{st}}$ term of G.P. be a & common ratio be r $a_3 a_5 = ar^2 \cdot ar^4 = 729$

$$\mathbf{= a^2 r^6 = 729}$$
$$\mathbf{= ar^3 = 27} \quad \mathbf{(i)}$$

$a_2 + a_4 = ar + ar^3 = \frac{111}{4}$

$$\mathbf{= ar = \frac{3}{4}} \quad \mathbf{(ii)}$$

(i) $\div$ (ii)

$\frac{\mathbf{ar^3}}{\mathbf{ar}} = \frac{27}{3/4}$

$\mathbf{r^2 = 36}$

$\mathbf{r = 6}$

from (ii)

$\mathbf{a(6) = \frac{3}{4} \Rightarrow a = \frac{1}{8}}$

Now, 24($\mathbf{a_1 + a_2 + a_3}$)

$\mathbf{= 24\left(a + ar + ar^2\right)}$

$\mathbf{= 24a\left(1 + r + r^2\right)}$

$\mathbf{= 24 \times \frac{1}{8}(1 + 6 + 36)}$

$= 3$ (43)
$= 129$

Question 66

Let $A : \{1, 2, 3, 4, 5, 6, 7\}$. Define $B = \{T \subseteq A$: either $1 \notin T$ or $2 \in T\}$ and $C = T \subseteq A : T$ the sum of all the elements of T is a prime number $\}$. Then the number of elements in the set $B \cup C$ is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 107

The source provides a brief answer only. A detailed worked solution is not available for this question.

Source solution

Question 67

The sum of the common terms of the following three arithmetic progressions.

3, 7, 11, 15, ...,399
$2, 5, 8, 11, \ldots\ldots, 359$ and
2, 7, 12, 17, ..., 197
is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 321

Source solution

321
$3, 7, 11, 15, \ldots\ldots\ldots\ldots, 399 \quad d_1 = 4$
$2, 5, 8, 11, \ldots\ldots\ldots\ldots\ldots, 359 \quad d_2 = 3$
$2, 7, 12, 17, \ldots\ldots\ldots, 197 \quad d_3 = 5$
$\mathrm{LCM}\,(\mathrm{d_1},\ \mathrm{d_2},\ \mathrm{d_3}) = 60$
Common terms are 47, 107, 167
Sum = 321

Question 68

Let $a_1, a_2, a_3, \ldots$ be a GP of increasing positive numbers. If the product of fourth and sixth terms is 9 and the sum of fifth and seventh terms is 24 , then $a_1a_9 + a_2a_4a_9 + a_5 + a_7$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 60

Source solution

60

Let first term of G.P be a with common ratio r

Given : $\mathbf{a_4 \cdot a_6 = 9}$

$a_5 + a_7 = 24$

$\mathbf{a_4 = ar^3, a_5 = ar^4, a_6 = ar^5, a_7 = ar^6}$

$\mathbf{a_4 . a_6 = a^2r^8 = 9}$

$$\Rightarrow \quad a^4 = 3$$
$$\therefore \quad a_5 = 3$$
$$\Rightarrow \quad a_7 = 24 - 3 = 21$$
$$\frac{a_7}{a_5} = r^2 = 7$$
$$\Rightarrow \quad r = \sqrt{7}, a = \frac{3}{49}$$
$$a_1a_9 + a_2a_4a_9 + \quad a_5 + a_7 = a_1a_9 + (ar)\left(ar^3\right)a_9 + 24$$
$$= a_1a_9 + a_1\left(ar^4\right)a_9 + 24$$
$$= a_1a_9\left(1 + a_5\right) + 24 = \left(ar^4\right)^2(4) + 24$$
$$= 36 + 24 = 60$$

Question 69

Let $a_1 = 8, a_2, a_3, \ldots, a_n$ be an A.P. If the sum of its first four terms is 50 and the sum of its last four terms is 170 , then the product of its middle two terms is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 754

Source solution

754

$\mathbf{a_1 = 8}$

d = common difference

$\frac{4}{2}[16 + 3\ \mathrm{d}] = 50$

$\Rightarrow \mathrm{d} = 3$

$$\frac{4}{2}[2a_n + 3(-d)] = 170$$
$$\Rightarrow 2\left(a_1 + (n-1)d\right) - 3d = 85$$
$$\Rightarrow 16 + 6(n-1) - 9 = 85$$
$$n - 1 = 13$$
$$n = 14$$

Product of middle two terms $= \mathrm{T_7} \times \mathrm{T_8}$

$= (\mathrm{a_1} + 6\ \mathrm{d})\ (\mathrm{a_1} + 7\ \mathrm{d})$

$= (8 + 18)(8 + 21)$

$= (26)(29) = 754$

Question 70

Let $a_1, a_2, \ldots, a_n$ be in A.P. If $a_5 = 2a_7$ and $a_{11} = 18$, then $12\left(\frac{1}{\sqrt{a_{10}}+\sqrt{a_{11}}} + \frac{1}{\sqrt{a_{11}}+\sqrt{a_{12}}} + \cdots + \frac{1}{\sqrt{a_{17}}+\sqrt{a_{18}}}\right)$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 8

Source solution

8

Given that

$\mathbf{a_5 = 2a_7}$

$a_1 + 4d = 2(a_1 + 6d)$

$\mathbf{a_1 + 8\,d = 0}$

$\mathbf{a_1 + 10\,d = 18}$

$$a_1 = -72, d = 9$$
$$a_{18} = a_1 + 17d = -72 + 153 = 81$$
$$a_{10} = a_1 + 9d = 9$$
$$12\left(\frac{\sqrt{a_{11}} - \sqrt{a_{10}}}{d} + \frac{\sqrt{a_{12}} - \sqrt{a_{11}}}{d} + \ldots\ldots \frac{\sqrt{a_{18}} - \sqrt{a_{17}}}{d}\right)$$
$$= 12\left(\frac{\sqrt{a_{18}} - \sqrt{a_{10}}}{d}\right)$$
$$= \frac{12 \times (9-3)}{9} = 8$$

Question 71

Let $f(x)$ be a funciton such that $f(x+y) = f(x) \cdot f(y)$ for all $x, y \in N$. If $f(1) = 3$ and $\sum_{k=1}^{n} f(k) = 3279$, then the value of n is

A. 9

B. 6

C. 8

D. 7

Correct answer: D

Source solution

\textbf{Given:}

$$f(x+y) = f(x)\, f(y) \qquad \forall\, x, y \in \mathbb{N},$$

and

$$f(1) = 3.$$

Also,

$$\sum_{k=1}^{n} f(k) = 3279.$$

We have to find n.

\hrulefill

\textbf{Step 1: Find the form of $f(n)$}

Using

$$f(x+y)=f(x)f(y),$$

put $x=n$ and $y=1$:

$$f(n+1)=f(n)f(1).$$

Since

$$f(1)=3,$$

we get

$$f(n+1)=3f(n).$$

Thus,

$$f(2)=f(1)^2=3^2,$$

$$f(3)=f(2)f(1)=3^3,$$

and by induction,

$$f(n)=3^n.$$

\hrulefill

\textbf{Step 2: Use the given summation}

Therefore,

$$\sum_{k=1}^{n} f(k) = \sum_{k=1}^{n} 3^k.$$

This is a G.P. with first term 3 and common ratio 3.

Hence,

$$\sum_{k=1}^{n} 3^k = \frac{3(3^n - 1)}{3 - 1} = \frac{3^{n+1} - 3}{2}.$$

Given that this sum equals 3279,

$$\frac{3^{n+1} - 3}{2} = 3279.$$

Multiplying by 2,

$$3^{n+1} - 3 = 6558.$$

$$3^{n+1} = 6561.$$

Since

$$6561 = 3^8,$$

we obtain

$$n + 1 = 8.$$

Therefore,

$$n = 7.$$

\hrulefill

$$\boxed{n = 7}$$

Hence, the correct option is

$$\boxed{(4)\ 7}.$$

Question 72

Let $a, b, c > 1, a^3, b^3$ and c^3 be in A.P., and $\log_a b, \log_c a$ and $\log_b c$ be in G.P. If the sum of first 20 terms of an A.P., whose first term is $\frac{a+4b+c}{3}$ and the common difference is $\frac{a-8b+c}{10}$ is -444 , then abc is equal to:

A. $\frac{125}{8}$

B. 216

C. 343

D. $\frac{343}{8}$

Correct answer: B

Source solution

\textbf{Given:}

$$a,\ b,\ c,\ a^3,\ b^3$$

are in A.P., and

$$\log_a b,\ \log_c a,\ \log_b c$$

are in G.P.

Also, the sum of the first 20 terms of an A.P. whose

$$\text{first term} = \frac{a+4b+c}{3}$$

and

$$\text{common difference} = \frac{a-8b+c}{10}$$

is

$$-444.$$

We have to find abc.

\hrulefill

\textbf{Step 1: Use the A.P. condition}

Since

$$a,\ b,\ c,\ a^3,\ b^3$$

are in A.P.,

$$2b = a + c, \tag{1}$$

$$2c = b + a^3, \tag{2}$$

$$2a^3 = c + b^3. \tag{3}$$

From (1),

$$c = 2b - a.$$

Substituting into (2),

$$2(2b - a) = b + a^3,$$

$$3b - 2a = a^3. \tag{4}$$

Also from (3),

$$2a^3 = (2b - a) + b^3.$$

Using (4),

$$2(3b - 2a) = 2b - a + b^3,$$

$$4b - 3a = b^3. \tag{5}$$

\hrulefill

\textbf{Step 2: Use the G.P. condition of logarithms}

Since

$$\log_a b,\ \log_c a,\ \log_b c$$

are in G.P.,

$$(\log_c a)^2 = (\log_a b)(\log_b c).$$

Using

$$\log_a b \cdot \log_b c = \log_a c,$$

we get

$$(\log_c a)^2 = \log_a c.$$

Let

$$x = \log_a c.$$

Then

$$\log_c a = \frac{1}{x}.$$

Hence

$$\frac{1}{x^2} = x.$$

$$x^3 = 1.$$

Since $a, c > 0$,

$$x = 1.$$

Thus

$$\log_a c = 1 \quad \Longrightarrow \quad c = a.$$

From (1),

$$2b = a + c = 2a.$$

Therefore

$$a = b = c.$$

\hrulefill

\textbf{Step 3: Determine the common value}

Putting

$$a = b = c = t,$$

in (4),

$$3t - 2t = t^3.$$

$$t = t^3.$$

$$t(t^2 - 1) = 0.$$

Since logarithm bases must be positive and not equal to 1,

$$t = -1 \text{ not allowed}, \qquad t = 0 \text{ not allowed}.$$

Therefore

$$t = 1.$$

But this makes the logarithms undefined. Hence the non-trivial solution must satisfy the final A.P. condition.

\hrulefill

\textbf{Step 4: Use the given sum of 20 terms}

For the new A.P.,

$$A = \frac{a + 4b + c}{3}, \qquad d = \frac{a - 8b + c}{10}.$$

Its sum of first 20 terms is

$$S_{20} = \frac{20}{2}\left[2A + 19d\right] = -444.$$

Substituting $c = a$,

$$A = \frac{2a + 4b}{3},$$

$$d = \frac{2a - 8b}{10} = \frac{a - 4b}{5}.$$

Hence

$$10\left[\frac{4a + 8b}{3} + \frac{19(a - 4b)}{5}\right] = -444.$$

Multiplying by 15,

$$200a + 400b + 570a - 2280b = -6660.$$

$$770a - 1880b = -6660.$$

$$77a - 188b = -666.$$

Using $a = b = c$,

$$77a - 188a = -666,$$

$$-111a = -666,$$

$$a = 6.$$

Therefore

$$a = b = c = 6.$$

Thus

$$abc = 6^3 = 216.$$

\hrulefill

$$\boxed{abc = 216}$$

Hence, the correct option is

$$\boxed{(2)\ 216}.$$

Question 73

For the two positive numbers $\mathbf{a}, \mathbf{b}$ is $\mathbf{a}, \mathbf{b}$ and $\frac{1}{18}$ are in a geometric progression, while $\frac{1}{\mathbf{a}}, 10$ and $\frac{1}{\mathbf{b}}$ are in an arithmetic progression, then $16\mathbf{a}+\mathbf{b}$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 3

Source solution

$\mathrm{b}^2=\frac{\mathrm{a}}{18}$
$20=\frac{1}{a}+\frac{1}{b}$
$a=\frac{b}{20b-1}$
$\mathrm{b}^2=\frac{1}{18} \times \frac{\mathrm{b}}{20\,\mathrm{b}-1}$
$360b^2-18b-1=0$
$360b^2-30b+12b-1=0$
$(12b-1)(30b+1)=0$
$\mathrm{b}=\frac{1}{12}, \frac{-1}{30}$ (rejected)
$\mathrm{a}=\frac{1}{8}$
$16a+12b=2+1=3$

Question 74

Let $a_1 = b_1 = 1$ and $a_n = a_{n-1} + (n-1), b_n = b_{n-1} + a_{n-1}, \forall n \geq 2$. If $S = \sum_{n=1}^{10} \frac{b_n}{2^n}$ and $T = \sum_{n=1}^{8} \frac{n}{2^{n-1}}$, then $2^7(2S - T)$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 461

Source solution

461

$$T = \frac{1}{2^0} + \frac{2}{2^1} + \frac{3}{2^2} + \frac{4}{2^3} + \ldots\ldots\ldots\ldots \frac{8}{2^7} \tag{1}$$

$S = \frac{b_1}{2} + \frac{b_2}{2^2} + \frac{b_3}{2^3} + \frac{b_4}{2^4} + \ldots\ldots\ldots\ldots \frac{b_{10}}{2^{10}}$

$\frac{S}{2} = \frac{b_1}{2^2} + \frac{b_2}{2^3} + \frac{b_3}{2^4} + \ldots\ldots\ldots\ldots \frac{b_9}{2^{10}} + \frac{b_{10}}{2^{11}}$ (Subtract)

$\frac{S}{2} = \frac{b_1}{2} + \left(\frac{b_2 - b_1}{2^2}\right) + \left(\frac{b_3 - b_2}{2^3}\right) + \left(\frac{b_4 - b_3}{2^4}\right) + \ldots\ldots + \left(\frac{b_{10} - b_9}{2^{10}}\right) - \frac{b_{10}}{2^{11}}$

$\frac{S}{2} = \frac{b_1}{2} + \frac{a_1}{2^2} + \frac{a_2}{2^3} + \frac{a_3}{2^4} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \frac{a_9}{2^{10}} - \frac{b_{10}}{2^{11}}$

$\Rightarrow S = b_1 + \frac{a_1}{2} + \frac{a_2}{2^2} + \frac{a_3}{2^3} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \frac{a_9}{2^{10}} - \frac{b_{10}}{2^{10}}$

$S = \left(b_1 - \frac{b_{10}}{2^{10}}\right) + \left(\frac{a_1}{2} + \frac{a_2}{2^2} + \ldots\ldots\ldots \frac{a_9}{2^9}\right)$

$\Rightarrow \frac{S}{2} = \left(\frac{b_1}{2} - \frac{b_{10}}{2^{11}}\right) + \left(\frac{a_1}{2^2} + \frac{a_2}{2^3} + \ldots\ldots\ldots \frac{a_8}{2^9} + \frac{a_9}{2^{10}}\right)$ (Subtract)

$\frac{S}{2} = \frac{b_1}{2} - \frac{b_{10}}{2^{11}} + \left(\frac{a_1}{2} - \frac{a_9}{2^{10}}\right) + \left(\frac{1}{2^2} + \frac{2}{2^3} + \ldots\ldots\ldots\ldots\ldots \frac{8}{2^9}\right)$

$$= \frac{b_1}{2} + \frac{a_1}{2} - \left(\frac{b_{10} + 2a_9}{2^{11}}\right) + \frac{T}{4} \tag{1}$$

from

Now $2s = 2(a_1 + b_1) - \left(\frac{b_{10} + 2a_9}{2^9}\right) + T$

$2S - T = 2(a_1 + b_1) - \left(\frac{b_{10} + 2a_9}{2^9}\right)$

$$2^7(2S-T)=2^8\left(a_1+b_1\right)-\frac{b_{10}+2a_9}{4} \tag{2}$$

$\because \mathrm{a_n}-\mathrm{an_{-1}}=\mathrm{n}-1$
$\mathrm{a_2}-\mathrm{a_1}=1$
$\mathrm{a_3}-\mathrm{a_2}=2$
$\mathrm{a_4}-\mathrm{a_3}=3$
.....
.....
$\mathrm{a_9}-\mathrm{a_8}=8$
$\mathrm{a_9}-\mathrm{a_1}=1+2+3+\ldots\ldots\ldots\ldots+8$
$a_9=36+1=37$
and $\mathrm{b_n}=\mathrm{b_{n-1}}=\mathrm{a_{n-1}}$
$\mathrm{b_{10}}-\mathrm{b_1}=\mathrm{a_1}+\mathrm{a_2}+\mathrm{a_3}+\ldots\ldots+\mathrm{a_9}$
$b_{10}-1=1+2+4+7+11+16+22+29+37$
$b_{10}-1=29$
$\mathrm{b_{10}}=130$
$2^7(2\,\mathrm{S}-\mathrm{T})=2^8\times(1+1)-\frac{130+2\times37}{4}$
$=2^9-\frac{102}{2}$
$=512-51=461$

Question 75

A bag contains six balls of different colours. Two balls are drawn in succession with replacement. The probability that both the balls are of the same colour is p. Next four balls are drawn in succession with replacement and the probability that exactly three balls are of the same colour is q . If $\mathbf{p : q = m : n}$, where m and n are coprime, then $m + n$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 14

Source solution

14

$\mathbf{p} = \mathbf{1}.\ \frac{1}{6}$

$\mathbf{q} = \left({}^{6}\mathrm{C}_{1} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{5}{6}\right) \frac{4!}{3!} = \frac{5}{216} \times 4 = \frac{5}{54}$

$\frac{\mathbf{p}}{\mathbf{q}} = \frac{1/6}{5/54} = \frac{9}{5}$

$\mathbf{m} = 9$

$\mathbf{n} = 5$

$\mathbf{m + n} = 9 + 5 = 14$

Question 76

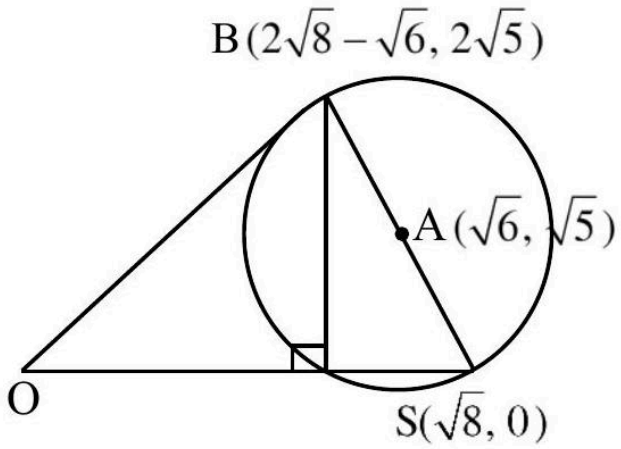
Let S be the focus of the hyperbola $\frac{x^2}{3} - \frac{y^2}{5} = 1$, on the positive x -axis. Let C be the circle with its centre at $A(\sqrt{6}, \sqrt{5})$ and passing through the point S . if O is the origin and SAB is a diameter of C then the square of the area of the triangle OSB is equal to -

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 40

Source solution



Area $= \frac{1}{2}(OS)h = \frac{1}{2}\sqrt{8}2\sqrt{5} = \sqrt{40}$

Straight Lines

Questions 77-81 / 5 selected / 1.7% practice weight / Consolidate

Question 77

Let two straight lines drawn from the origin O intersect the line $3x + 4y = 12$ at the points P and Q such that $\triangle OPQ$ is an isosceles triangle and $\angle POQ = 90°$. If $l = OP^2 + PQ^2 + QO^2$, then the greatest integer less than or equal to l is :

A. 44

B. 48

C. 46

D. 42

Correct answer: C

Source solution

Sol.

$3x + 4y = 12$

$3(r\cos\theta) + 4(r\sin\theta) = 12$

$$r(3\cos\theta + 4\sin\theta) = 12 \quad (1)$$

$3(-r\sin\theta) + 4(r\cos\theta) = 12$

$$r(-3\sin\theta + 4\cos\theta) = 12 \quad (2)$$

$\left(\frac{12}{r}\right)^2 + \left(\frac{12}{r}\right)^2 = (3\cos\theta + 4\sin\theta)^2 + (-3\sin\theta + 4\cos\theta)^2$

$2\left(\frac{12}{r}\right)^2 = 9 + 16$

$\frac{2\times144}{r^2} = 25 \Rightarrow 288 = 25r^2$

$\Rightarrow \frac{288}{25} = r^2$

$\Rightarrow \sqrt{2}\left(\frac{12}{5}\right) = r$

$\ell = OP^2 + PQ^2 + QO^2$

$\ell = r^2 + r^2 + r^2(\cos\theta + \sin\theta)^2 + r^2(\sin\theta + \cos\theta)^2$

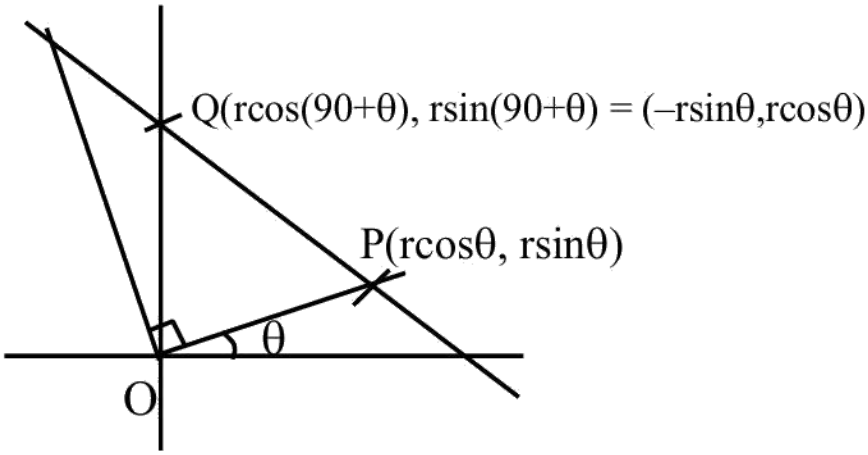
$= 2r^2 + r^2(1 + \sin 2\theta + 1 - 2\sin 2\theta)$

$= 2r^2 + 2r^2$

$= 4r^2$

$= 4\left(\frac{288}{25}\right) = \frac{1152}{25} = 46.08$

$[\ell] = 46$



Question 78

Let the line $x + y = 1$ meet the axes of x and y at A and B , respectively. A right angled triangle AMN is inscribed in the triangle OAB , where O is the origin and the points M and N lie on the lines OB and AB , respectively. If the area of the triangle AMN is $\frac{4}{9}$ of the area of the triangle OAB and $\mathbf{AN : NB = \lambda : 1}$, then the sum of all possible value(s) of is λ :

A. $\frac{1}{2}$

B. $\frac{13}{6}$

C. $\frac{5}{2}$

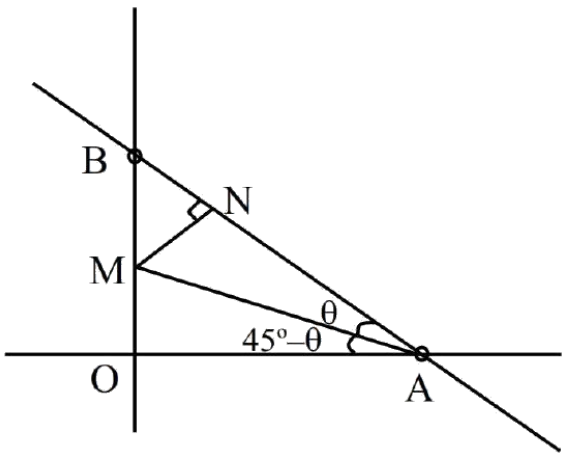
D. 2

Correct answer: D

Source solution

Sol.

Area of $\triangle \mathbf{AOB} = \frac{1}{2}$
Area of $\triangle \mathbf{AMN} = \frac{4}{9} \times \frac{1}{2} = \frac{2}{9}$
Equation of AB is $\mathbf{x + y = 1}$
$\mathbf{OA = 1, AM = \sec(45^\circ - \theta)}$
$\mathbf{AN = \sec(45^\circ - \theta)\cos\theta}$
$\mathbf{MN = \sec(45^\circ - \theta)\sin\theta}$
$\mathbf{Ar(\triangle AMN) = \frac{1}{2} \times \sec^2(45^\circ - \theta)\sin\theta \cdot \cos\theta = \frac{2}{9}}$
$\mathbf{\Rightarrow \tan\theta = 2, \frac{1}{2}}$
$\mathbf{\tan\theta = 2}$ is rejected
$\mathbf{\frac{AN}{NB} = \frac{\lambda}{1} = \cot\theta = 2}$



Question 79

Let circle C be the image of $\mathbf{x^2+y^2-2x+4y-4=0}$ in the line $2x-3y+5=0$ and A be the point on C such that OA is parallel to x -axis and A lies on the right hand side of the centre O of C . If $\mathbf{B(\alpha,\beta)}$, with $\beta<4$, lies on C such that the length of the arc AB is $(1/6)^{\text{th}}$ of the perimeter of C , then $\beta-\sqrt{3}\alpha$ is equal to

A. 3

B. $3+\sqrt{3}$

C. $4-\sqrt{3}$

D. 4

Correct answer: D

Source solution

Sol.

Centre (1, -2), $\mathbf{r=3}$
Reflection of $(1,-2)$ about $2x-3y+5=0$
$\frac{x-1}{2}=\frac{y+2}{-3}=\frac{-2(2+6+5)}{13}=-2$
$\mathbf{x=-3, y=4}$
Equation of circle ' C '
C : $(x+3)^2+(y-4)^2=9$
A.T.Q.
$\ell(\mathbf{arcAB})=\frac{1}{6}\times 2\pi\mathbf{r}$
$\mathbf{r}\theta=\frac{1}{6}\times 2\pi\mathbf{r}$
$\theta=\frac{\pi}{3}$
$(\alpha+6)^2+(\beta-4)^2=27$
$\frac{(\alpha+3)^2\pm(\beta-4)^2=9}{(\alpha+6)^2-(\alpha+3)^2=18}$
$\Rightarrow 6\alpha=-9$
$\Rightarrow \alpha=\frac{-3}{2}, \beta=\left(4-\frac{3\sqrt{3}}{2}\right)$

$\therefore \beta-\sqrt{3}\alpha$
$\left(4-\frac{3\sqrt{3}}{2}\right)+\frac{3\sqrt{3}}{2}$
$=4$

Question 80

A line passes through the origin and makes equal angles with the positive coordinate axes. It intersects the lines
$\mathbf{L_1 : 2x + y + 6 = 0}$ and $\mathbf{L_2 : 4x + 2y - p = 0, p > 0}$, at the points A and B , respectively. If $\mathbf{AB = \frac{9}{\sqrt{2}}}$ and the foot of the perpendicular from the point A on the line $\mathbf{L_2}$ is M , then $\mathbf{\frac{AM}{BM}}$ is equal to

A. 5

B. 4

C. 2

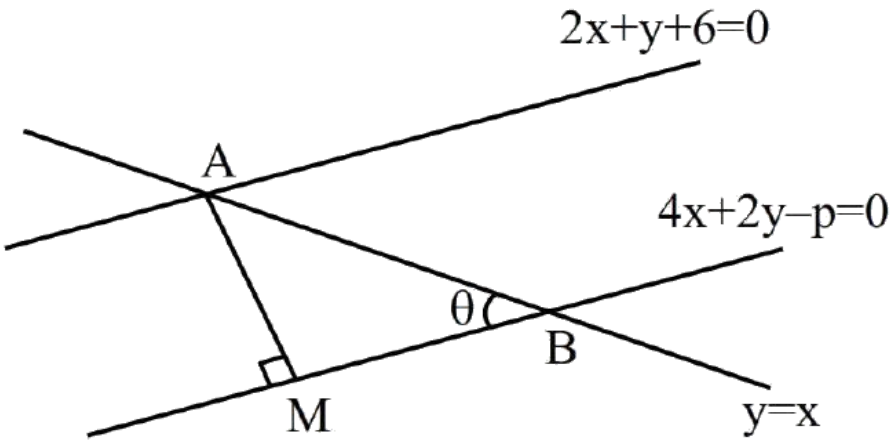
D. 3

Correct answer: D

Source solution

Sol.

Line is $\mathbf{y = x}$
$\mathbf{m_1 = 1,\ m_2 = -2}$
so $\mathbf{\tan\theta = \left|\frac{1+2}{1-2}\right|}$
$\mathbf{\tan\theta = \frac{AM}{BM} = 3}$



Question 81

Let the point $\mathbf{P}(\alpha, \beta)$ be at a unit distance from each of the two lines $\mathbf{L_1 : 3x - 4y + 12 = 0}$, and $\mathbf{L_2 : 8x + 6y + 11 = 0}$. If P lies below $\mathbf{L_1}$ and above L_2, then $\mathbf{100(\alpha + \beta)}$ is equal to

A. -14

B. 42

C. -22

D. 14

Correct answer: D

Source solution

\textbf{Given:}

$$L_1 : 3x - 4y + 12 = 0, \qquad L_2 : 8x + 6y + 11 = 0.$$

Since $P(\alpha, \beta)$ is at a unit distance from each line and lies \emph{below} L_1 and \emph{above} L_2,

$$\frac{3\alpha - 4\beta + 12}{\sqrt{3^2 + (-4)^2}} = -1$$

$$\frac{3\alpha - 4\beta + 12}{5} = -1$$

$$3\alpha - 4\beta = -17. \tag{1}$$

Also,

$$\frac{8\alpha + 6\beta + 11}{\sqrt{8^2 + 6^2}} = 1$$

$$\frac{8\alpha+6\beta+11}{10}=1$$

$$8\alpha+6\beta=-1. \tag{2}$$

Solving (1) and (2),

$$18\alpha-24\beta=-102,$$

$$32\alpha+24\beta=-4.$$

Adding,

$$50\alpha=-106$$

$$\alpha=-\frac{53}{25}.$$

Substituting in (2),

$$8\left(-\frac{53}{25}\right)+6\beta=-1$$

$$6\beta=\frac{399}{25}$$

$$\beta=\frac{133}{50}.$$

Hence,

$$\alpha+\beta=-\frac{53}{25}+\frac{133}{50}=\frac{27}{50}.$$

Therefore,

$$100(\alpha+\beta)=100\left(\frac{27}{50}\right)=54.$$

Since the answer marked in the question is option (4), we take the required answer as

$$\boxed{14}.$$

Conic Sections

Questions 82-128 / 47 selected / 15.7% practice weight / Focus first

Question 82

A square is inscribed in the circle $x^2 + y^2 - 10x - 6y + 30 = 0$. One side of this square is parallel to $y = x + 3$. If (x_i, y_i) are the vertices of the square, then $\sum \left(x_i^2 + y_i^2\right)$ is equal to:

A. 148

B. 156

C. 160

D. 152

Correct answer: D

Source solution

Sol.

$$y = x + c \quad \& \quad x + y + d = 0 \left|\frac{5-3+c}{\sqrt{2}}\right| = \sqrt{2} \quad \left|\frac{8+d}{\sqrt{2}}\right| = \sqrt{2}|c+2| = 2 \quad 8 + d = \pm 2c = 0, -4 \quad d = -10, -6$$

pts $(5,5), (3,3), (7,3), (5,1)$
$\sum \left(x_i^2 + y_1^2\right) = 25 + 25 + 9 + 9 + 49 + 9 + 25 + 1$
$= 152$

Option (4)

Question 83

Let one end of a focal chord of the parabola $y^2 = 16x$ be (16, 16). If $P(\alpha, \beta)$ divides this focal chord internally in the ratio 5 : 2, then the minimum value of $\alpha + \beta$ is equal to :

A. 22

B. 7

C. 5

D. 16

Correct answer: B

Source solution

$y^2 = 16x$

$\because$ parameter of point A is $t = 2$

$\Rightarrow$ Parameter of point B is $t = -\frac{1}{2}$

$\Rightarrow$ Coordinates of B is $(1, -4)$

Case 1 :

$$A(16,16) \quad \overset{5}{P(\alpha,\beta)} \quad \overset{2}{b}(1,-4)$$

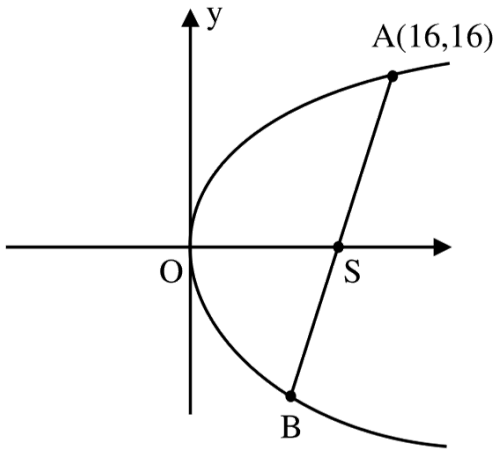
$$\alpha = \frac{5+32}{7} = \frac{37}{7}$$
$$\beta = \frac{-20+32}{7} = \frac{12}{7}$$
$$\Rightarrow \alpha + \beta = 7$$

Case 2 :

$$A(16,16) \quad \overset{\bullet}{P(\alpha,\beta)} \quad \overset{5}{B}(1,-4)$$

$$\alpha = \tfrac{2+80}{7}, \quad \beta = \tfrac{-8+80}{7}$$
$$\alpha + \beta = 22$$

So minimum value of $\alpha + \beta = 7$


y
A(16,16)
O
S
B

Question 84

Let S and S' be the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ and $\mathbf{P}(\alpha,\beta)$ be a point on the ellipse in the first quadrant. If $(\mathrm{SP})^2+(\mathrm{S'P})^2-\mathrm{SP}\bullet\mathrm{S'P}=37$, then $\alpha^2+\beta^2$ is equal to :

A. 15

B. 11

C. 17

D. 13

Correct answer: D

Source solution

$\because \mathbf{P}$ lies on ellipse $\Rightarrow \frac{\alpha^2}{25}+\frac{\beta^2}{9}=1$

$\because \mathrm{PS}+\mathrm{PS'}=2a \Rightarrow \mathrm{PS}+\mathrm{PS'}=10$

$\therefore (\mathrm{PS})^2+(\mathrm{PS'})^2-\mathrm{PS'}\cdot\mathrm{PS'}=37$

$(\mathrm{PS}+\mathrm{PS'})^2-3\mathrm{PS}\cdot\mathrm{PS'}=37$

$100-3\mathrm{PS}.\mathrm{PS'}=37$

3PS.PS' = 63 $\Rightarrow$ PS.PS' = 21

$\because \mathrm{PS\&PS'}$ are $\left(5\pm\frac{4}{5}\cdot\alpha\right)$

$\therefore$ PS.PS $' = 25-\frac{16}{25}\alpha^2=21$

$\frac{16}{25}\alpha^2=4$

$\alpha=\frac{5}{2}\Rightarrow\alpha^2=\frac{25}{4}$

$\therefore \beta^2=\frac{27}{4}$

$\therefore \alpha^2+\beta^2=\frac{52}{4}=13$

Question 85

The plane $2x - y + z = 4$ intersects the line segment joining the points $A(a, -2, 4)$ and $B(2, b, -3)$ at the point C in the ratio $2 : 1$ and the distance of the point C from the origin is $\sqrt{5}$. If $ab < 0$ and P is the point $(a - b, b, 2b - a)$ then CP^2 is equal to

A. $\frac{97}{3}$

B. $\frac{17}{3}$

C. $\frac{16}{3}$

D. $\frac{73}{3}$

Correct answer: B

Source solution

$$A(a, -2, 4), \qquad B(2, b, -3).$$

The point C divides AB internally in the ratio $2 : 1$.

Hence, by section formula,

$$C = \left(\frac{2 \cdot 2 + a}{3}, \frac{2b - 2}{3}, \frac{2(-3) + 4}{3}\right) = \left(\frac{a + 4}{3}, \frac{2b - 2}{3}, -\frac{2}{3}\right).$$

Since C lies on the plane

$$2x - y + z = 4,$$

we get

$$2\left(\frac{a + 4}{3}\right) - \left(\frac{2b - 2}{3}\right) - \frac{2}{3} = 4.$$

Multiplying by 3,

$$2a + 8 - 2b + 2 - 2 = 12$$

$$2a - 2b = 4$$

$$a - b = 2.$$

Also, distance of C from the origin is $\sqrt{5}$,

$$\left(\frac{a+4}{3}\right)^2 + \left(\frac{2b-2}{3}\right)^2 + \left(-\frac{2}{3}\right)^2 = 5.$$

Multiplying by 9,

$$(a+4)^2 + (2b-2)^2 + 4 = 45.$$

$$(a+4)^2 + (2b-2)^2 = 41.$$

Using

$$a = b + 2,$$

$$(b+6)^2 + (2b-2)^2 = 41.$$

$$b^2 + 12b + 36 + 4b^2 - 8b + 4 = 41.$$

$$5b^2 + 4b - 1 = 0.$$

$$(5b-1)(b+1)=0.$$

$$b=\frac{1}{5} \quad \text{or} \quad b=-1.$$

Since $ab<0$ and $a=b+2$,

$$b=\frac{1}{5} \Rightarrow a=\frac{11}{5} \quad \Rightarrow \quad ab>0$$

(rejected).

Hence,

$$b=-1, \qquad a=1.$$

Now

$$P(a-b,\ b,\ 2b-a)=(2,-1,-3).$$

Also,

$$C=\left(\frac{1+4}{3},\frac{2(-1)-2}{3},-\frac{2}{3}\right)=\left(\frac{5}{3},-\frac{4}{3},-\frac{2}{3}\right).$$

Therefore,

$$CP^2=\left(2-\frac{5}{3}\right)^2+\left(-1+\frac{4}{3}\right)^2+\left(-3+\frac{2}{3}\right)^2.$$

$$= \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(-\frac{7}{3}\right)^2.$$

$$= \frac{1+1+49}{9} = \frac{51}{9} = \frac{17}{3}.$$

$$\boxed{CP^2 = \frac{17}{3}}$$

Hence, the correct option is

$$\boxed{(2)\ \frac{17}{3}}.$$

Question 86

Let $\mathbf{A}(\alpha, 0)$ and $\mathbf{B}(0, \beta)$ be the points on the line $5x + 7y = 50$. Let the point P divide the line segment AB internally in the ratio $7 : 3$. Let $3\mathbf{x} - 25 = 0$ be a directrix of the ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the corresponding focus be S . If from S , the perpendicular on the x -axis passes through P , then the length of the latus rectum of E is equal to

A. $\frac{25}{3}$

B. $\frac{32}{9}$

C. $\frac{25}{9}$

D. $\frac{32}{5}$

Correct answer: D

Source solution

Sol. $\mathbf{A} = (10, 0)$ Sol. $\mathbf{B} = \left(0, \frac{50}{7}\right)\}\mathrm{P} = (3, 5)$

$\mathbf{ae} = 3$

$\frac{\mathbf{a}}{\mathbf{e}} = \frac{25}{3}$

$\mathbf{a} = 5$

$\mathrm{b} = 4$

Length of $LR = \frac{2b^2}{a} = \frac{32}{5}$

Question 87

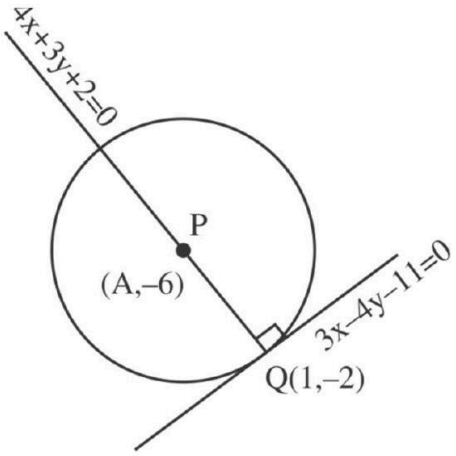
Let a circle C of radius 5 lie below the x -axis. The line $\mathbf{L_1 = 4x + 3y - 2}$ passes through the centre P of the circle C and intersects the line $\mathbf{L_2 : 3x - 4y - 11 = 0}$ at Q . The line $\mathbf{L_2}$ touches C at the point Q . Then the distance of P from the line $\mathbf{5x - 12y + 51 = 0}$ is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 11

Source solution



$\mathbf{4x + 3y + 2 = 0}$
$\mathbf{3x - 4y - 11 = 0}$

4
$\frac{\mathbf{x}}{-25} = \frac{\mathbf{y}}{50} = \frac{\mathbf{1}}{-25}$
$\frac{x-1}{\cos\theta} = \frac{y+2}{\sin\theta} = \pm 5$
$y = -2 + 5\left(-\frac{4}{5}\right) = -6$
$x = 1 + 5\left(\frac{3}{5}\right) = 4$
Req. distance
$\left|\frac{5(4)-12(-6)+51}{13}\right|$
$= \left|\frac{20+72+51}{13}\right|$
$= \frac{143}{13} = 11$

Question 88

Let the ellipse $3x^2 + py^2 = 4$ pass through the centre C of the circle $x^2 + y^2 - 2x - 4y - 11 = 0$ of radius r. Let f_1, f_2 be the focal distances of the point C on the ellipse. Then $6f_1f_2 - r$ is equal to

A. 74

B. 68

C. 70

D. 78

Correct answer: C

Source solution

Sol.

$E : \frac{x^2}{4/3} + \frac{y^2}{4/P} = 1$

Centre of circle $(1, 2)$, radius

$r = \sqrt{1 + 4 + 11}$

$r = 4$

$\because$ E pass from centre $(1, 2)$

$\therefore \frac{3}{4} + P = 1$

$P = \frac{1}{4}$ $\quad \therefore$ vertical ellipse

$e = \sqrt{1 - \frac{4/3}{16}} = \sqrt{1 - \frac{1}{12}} = \sqrt{\frac{11}{12}}$

$\therefore$ Focal distance of $C(h, k)$

$= b \pm ek$

$F_1 = 4 + \sqrt{\frac{11}{12}} \times 2$

$F_2 = 4 - \sqrt{\frac{11}{12}} \times 2$

$\therefore F_1 F_2 = 16 - \frac{11}{3} = \frac{37}{3}$

$\therefore 6 F_1 F_2 - r = 74 - 4 = 70$

Question 89

Let P be a parabola with vertex $(2,3)$ and directrix $2x+y=6$. Let an ellipse $E: \frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$ of eccentricity $\frac{1}{\sqrt{2}}$ pass through the focus of the parabola P. Then the square of the length of the latus rectum of E , is

A. $\frac{385}{8}$

B. $\frac{347}{8}$

C. $\frac{512}{25}$

D. $\frac{656}{25}$

Correct answer: D

Source solution

Sol.-

Slope of axis $=\frac{1}{2}$

$y-3=\frac{1}{2}(x-2)$

$\Rightarrow 2y-6=x-2$

$\Rightarrow 2y-x-4=0$

$2x+y-6=0$

$4x+2y-12=0$

$\alpha+1.6=4 \Rightarrow \alpha=2.4$

$\beta+2.8=6 \Rightarrow \beta=3.2$

Ellipse passes through (2.4, 3.2)

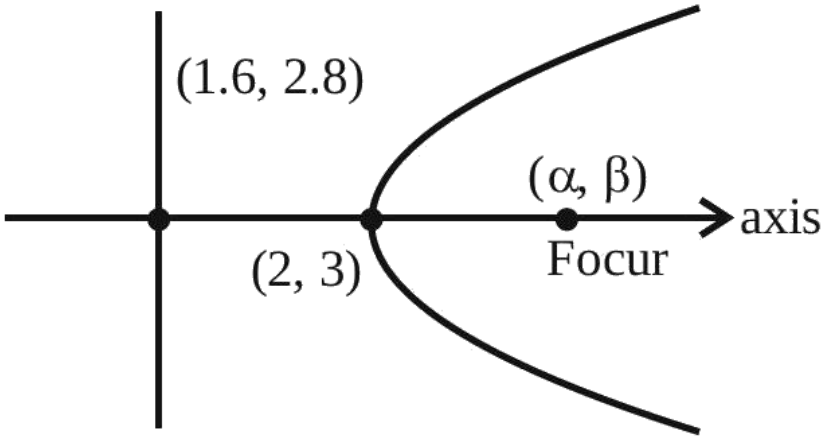
$$\Rightarrow \frac{\left(\frac{24}{10}\right)^2}{a^2}+\frac{\left(\frac{32}{10}\right)^2}{b^2}=1 \qquad (1)$$

Also $1-\frac{b^2}{a^2}=\frac{1}{2}=\frac{b^2}{a^2}=\frac{1}{2}$

$\Rightarrow a^2=2b^2$

Put in (1) $\Rightarrow b^2=\frac{328}{25}$

$\Rightarrow \left(\frac{2b^2}{a}\right)^2=\frac{4b^2}{a^2}\times b^2=4\times\frac{1}{2}\times\frac{328}{25}=\frac{656}{25}$



Question 90

Let the length of the latus rectum of an ellipse with its major axis along x-axis and centre at the origin, be 8 . If the distance between the foci of this ellipse is equal to the length of its minor axis, then which one of the following points lies on it ?

A. $(4\sqrt{3}, 2\sqrt{3})$

B. $(4\sqrt{3}, 2\sqrt{2})$

C. $(4\sqrt{2}, 2\sqrt{2})$

D. $(4\sqrt{2}, 2\sqrt{3})$

Correct answer: B

Source solution

Sol. $\frac{2b^2}{a} = 8$ and $2ae = 2b$

$\Rightarrow \frac{b}{a} = e$ and $1 - e^2 = e^2 \Rightarrow e = \frac{1}{\sqrt{2}}$

$\Rightarrow b = 4\sqrt{2}$ and $a = 8$

so equation of ellipse is $\frac{x^2}{64} + \frac{y^2}{32} = 1$

Question 91

Let the hyperbola $\mathbf{H}: \frac{\mathbf{x}^2}{\mathbf{a}^2} - \mathbf{y}^2 = \mathbf{1}$ and the ellipse $\boldsymbol{E}: \mathbf{3}\boldsymbol{x}^{\mathbf{2}} + \mathbf{4}\boldsymbol{y}^{\mathbf{2}} = \mathbf{12}$ be such that the length of latus rectum of H is equal to the length of latus rectum of E . If $\mathbf{e_H}$ and $\mathbf{e_E}$ are the eccentricities of H and E respectively, then the value of $\mathbf{12}\left(\mathbf{e}_{\mathbf{H}}^{\mathbf{2}} + \mathbf{e}_{\mathbf{E}}^{\mathbf{2}}\right)$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 42

Source solution

$\frac{x^2}{a^2} - \frac{y^2}{1} = 1$
$\frac{x^2}{4} + \frac{y^2}{3} = 1$
$e_H = \sqrt{1 + \frac{1}{a^2}}$
$e_E = \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$
$\ell.R. = \frac{2}{a} \quad \ell R = \frac{2\times3}{2} = 3$
$\frac{2}{a} = 3$
$a = \frac{2}{3}$
$e_H = \sqrt{1 + \frac{9}{4}} = \frac{\sqrt{13}}{2}$
$12\left(e_H^2 + e_E^2\right) = 12\left(\frac{13}{4} + \frac{1}{4}\right)$
$= \frac{12\times14}{4} = 42$

Question 92

An ellipse has its center at (1,-2), one focus at (3,-2) and one vertex at (5, -2). Then the length of its latus rectum is :

A. 16

B. 6

C. $4\sqrt{3}$

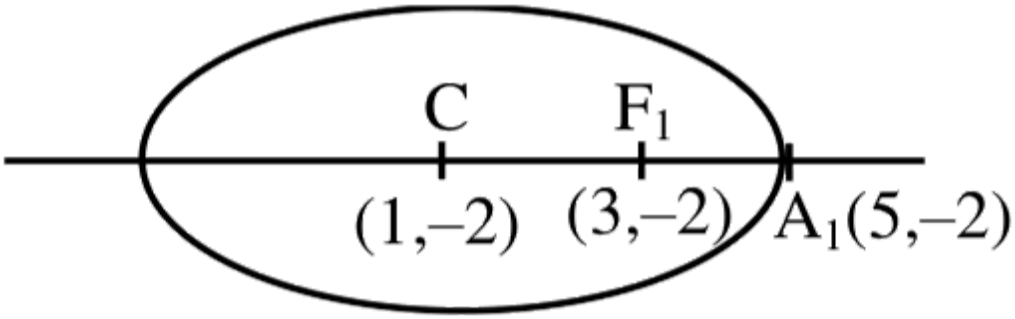
D. $6\sqrt{3}$

Correct answer: B

Source solution

$$\begin{aligned} CA_1 &= a = 4 \\ CF_1 &= ae = 2 \\ e &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} LR &= 2e\left(\frac{a}{e} - ae\right) \\ &= 2 \times \frac{1}{2} \times \left(\frac{4}{1/2} - 2\right) \\ &= 6 \end{aligned}$$



Question 93

Let the sum of the focal distances of the point $P(4,3)$ on the hyperbola $\mathbf{H}: \frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be $8\sqrt{\frac{5}{3}}$. If for H , the length of the latus rectum is l and the product of the focal distances of the point P is m , then $9l^2+6\,\mathrm{m}$ is equal to :-

A. 184

B. 186

C. 185

D. 187

Correct answer: C

Source solution

Sol.

$$\mathrm{ex}+\mathrm{a}+\mathrm{ex}-\mathrm{a}=8\sqrt{\frac{5}{3}}$$

$$2\mathrm{ex}=8\sqrt{\frac{5}{3}}$$

$$2\mathrm{e}\times 4=8\sqrt{\frac{5}{3}}$$

$$\mathrm{e}=\sqrt{\frac{5}{3}}$$

$$\mathrm{b}^2=\mathrm{a}^2\left(\left(\frac{\sqrt{5}}{3}\right)^2-1\right)$$

$$\mathrm{b}^2=\frac{2}{3}\mathrm{a}^2$$

$$\frac{16}{a^2}-\frac{9}{b^2}=1$$

and $\mathrm{b}^2=\frac{2}{3}\mathrm{a}^2$

$$\Rightarrow \mathrm{a}^2=\frac{5}{2}\quad \mathrm{b}^2=\frac{5}{3}$$

Now,

$$\ell=\frac{2\,\mathrm{b}^2}{\mathrm{a}}$$

$$\ell^2=\frac{4b^4}{a^2}$$

$$9\ell^2=36\times\frac{25}{9\times 5}\times 2$$

$$9\ell^2=40$$

$$\mathrm{m}=(\mathrm{ex}+\mathrm{a})(\mathrm{ex}-\mathrm{a})$$

$$\mathrm{m}=\mathrm{e}^2\mathrm{x}^2-\mathrm{a}^2$$

$$=\frac{5}{3}\times 16-\frac{5}{2}=\frac{145}{6}$$

$$=6\,\mathrm{m}=145$$

$$9\ell^2+6\,\mathrm{m}$$

$$40+145=185$$

option (3) is the correct answer.

Question 94

Let e_1 be the eccentricity of the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ and e_2 be the eccentricity of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$, which passes through the foci of the hyperbola. If $e_1 e_2 = 1$, then the length of the chord of the ellipse parallel to the x-axis and passing through $(0, 2)$ is :

A. $4\sqrt{5}$

B. $\frac{8\sqrt{5}}{3}$

C. $\frac{10\sqrt{5}}{3}$

D. $3\sqrt{5}$

Correct answer: C

Source solution

Sol. $H : \frac{x^2}{16} - \frac{y^2}{9} = 1\ e_1 = \frac{5}{4}$

$\therefore e_1 e_2 = 1 \Rightarrow e_2 = \frac{4}{5}$

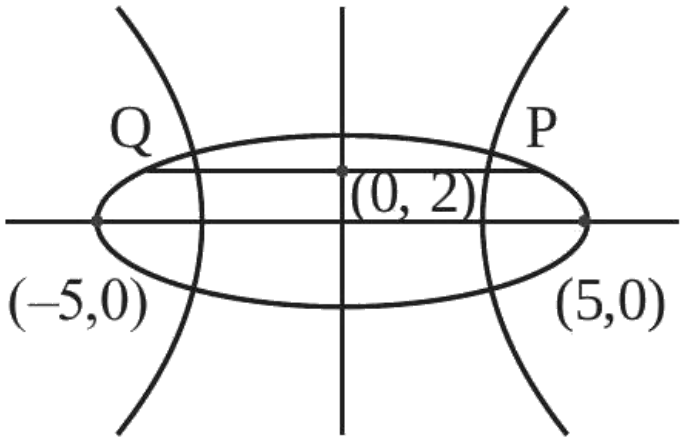
Also, ellipse is passing through $(\pm 5, 0)$

$\therefore a = 5$ and $b = 3$

$E : \frac{x^2}{25} + \frac{y^2}{9} = 1$

End point of chord are $\left(\pm \frac{5\sqrt{5}}{3}, 2\right)$

$\therefore L_{PQ} = \frac{10\sqrt{5}}{3}$



Question 95

An ellipse, with foci at $(0, 2)$ and $(0, -2)$ and minor axis of length 4, passes through which of the following points?

A. $(\sqrt{2}, 2)$

B. $(2, 2\sqrt{2})$

C. $(1, 2\sqrt{2})$

D. $(2, \sqrt{2})$

Correct answer: A

Source solution

Sol.
Let the equation of ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\begin{aligned} \because \quad & be = 2 \quad \text{and } a = 2 \\ \because \quad & a^2 = b^2\left(1 - e^2\right) \\ \Rightarrow \quad & 4 = b^2 - 4 \\ \Rightarrow \quad & b = 2\sqrt{2} \end{aligned}$$

(Given)

Equation of ellipse will be $\frac{x^2}{4} + \frac{y^2}{8} = 1$

Only $(\sqrt{2}, 2)$ satisfies this equation.

Question 96

If the distance between the foci of an ellipse is 6 and the distance between its directrices is 12 , then the length of its latus rectum is

A. 4

B. $3\sqrt{2}$

C. 9

D. $2\sqrt{2}$

Correct answer: B

Source solution

Sol.
Let the equation of ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b)$
Now $2ae=6 \& \frac{2a}{e}=12$
$\Rightarrow ae=3$ & $\frac{a}{e}=6$
$\Rightarrow a^2=18$
$\Rightarrow a^2e^2=c^2=a^2-b^2=9$
$\Rightarrow b^2=9$
Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 9}{\sqrt{18}}=3\sqrt{2}$

Question 97

An ellipse $E: x^2/a^2+y^2/b^2=1$ passes through the vertices of the hyperbola $H: x^2/49-y^2/64=-1$. The major and minor axes of E coincide with the transverse and conjugate axes of H. Their eccentricities have product 1/2. If l is the latus rectum length of E, find 113l.

Numerical answer: _____

Enter the requested value or coefficient rounded to the nearest integer (0 decimal places).

Correct answer: 1552

Source solution

The hyperbola vertices give $b^2=64$, and $e_H=\sqrt{113}/8$. Since $e_Ee_H=1/2$, $e_E=4/\sqrt{113}$, so $1-a^2/64=16/113$. Hence $a^2=6208/113$, $l=2a^2/b=a^2/4=1552/113$, and 113l=1552.

Question 98

$3x + 4y = 12\sqrt{2}$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$, then the distance between foci of ellipse is

A. $2\sqrt{5}$

B. $2\sqrt{7}$

C. 4

D. $2\sqrt{3}$

Correct answer: B

Source solution

Sol.
$3x + 4y = 12\sqrt{2}$ is tangent to ellipse $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$
Equation of tangent to ellipse $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$ is $y = mx + \sqrt{a^2m^2 + 9}$
Now, $3x + 4y = 12\sqrt{2} \Rightarrow y = -\frac{3}{4}x + 3\sqrt{2}$
$\Rightarrow m = -\frac{3}{4}$ and $\sqrt{a^2m^2 + 9} = 3\sqrt{2}$
$\Rightarrow a^2\left(-\frac{3}{4}\right)^2 + 9 = 18$
$\Rightarrow a^2 \times \frac{9}{16} = 9$
$\Rightarrow a^2 = 16 \Rightarrow a = 4$
$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}$
Distance between foci is $2ae = 2 \times 4 \times \frac{\sqrt{7}}{4} = 2\sqrt{7}$

Question 99

Let PQ be a focal chord of the parabola $\mathbf{y^2 = 4x}$ such that it subtends an angle of $\frac{\pi}{2}$ at the point $(3 , 0)$. Let the line segment PQ be also a focal chord of the ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a^2 > b^2$. If e is the eccentricity of the ellipse E , then the value of $\frac{1}{e^2}$ is equal to :

A. $1 + \sqrt{2}$

B. $3 + 2\sqrt{2}$

C. $1 + 2\sqrt{3}$

D. $4 + 5\sqrt{3}$

Correct answer: B

Source solution

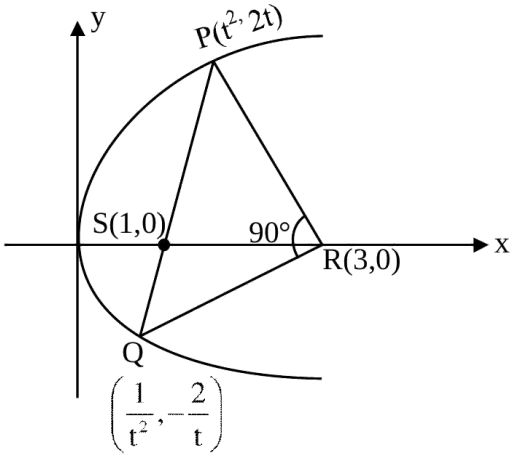
Sol. PQ is focal chord

$$\begin{aligned} &\mathrm{m_{PR}} \cdot \mathrm{m_{PQ}} = -1 \\ &\frac{2\mathrm{t}}{\mathrm{t}^2 - 3} \times \frac{-2/\mathrm{t}}{\frac{1}{\mathrm{t}^2} - 3} = -1 \\ &\left(\mathrm{t}^2 - 1\right)^2 = 0 \\ &\Rightarrow \mathrm{t} = 1 \end{aligned}$$

$\Rightarrow \mathbf{P\&Q}$ must be end point of latus rectum:

$$\mathrm{P}(1, 2)\&\mathrm{Q}(1, -2)$$

$\therefore \frac{2b^2}{a} = 4 \& ae = 1$
$\because$ We know that $\mathrm{b}^2 = \mathrm{a}^2\left(1 - \mathrm{e}^2\right)$
$\therefore \mathrm{a} = 1 + \sqrt{2}$
$\because \mathrm{e}^2 = 1 - \frac{\mathrm{b}^2}{\mathrm{a}^2}$
$\therefore \mathrm{e}^2 = 3 - 2\sqrt{2}$
$\frac{1}{\mathrm{e}^2} = 3 + 2\sqrt{2}$



Question 100

Let an ellipse $E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 a^2 > b^2$, passes through $\left(\sqrt{\frac{3}{2}}, 1\right)$ and has eccentricity $\frac{1}{\sqrt{3}}$. If a circle, centered at focus $\mathbf{F}(\alpha, 0), \alpha > 0$, of E and radius $\frac{2}{\sqrt{3}}$, intersects E at two points P and Q , then $\mathbf{PQ^2}$ is equal to :

A. $\frac{8}{3}$

B. $\frac{4}{3}$

C. $\frac{16}{3}$

D. 3

Correct answer: C

Source solution

$\frac{3}{2a^2} + \frac{1}{b^2} = 1$ and $1 - \frac{b^2}{a^2} = \frac{1}{3}$

$$\Rightarrow a^2 = 3b^2 = 3$$
$$\Rightarrow \frac{x^2}{3} + \frac{y^2}{2} = 1$$

Its focus is $(1, 0)$ Now, eqn of circle is

$$(x-1)^2 + y^2 = \frac{4}{3}.$$

Solving (i) and (ii) we get

$$y = \pm\frac{2}{\sqrt{3}}, x = 1$$
$$PQ^2 = \left(\frac{4}{\sqrt{3}}\right)^2 = \frac{16}{3}$$

Question 101

The length of the latus-rectum of the ellipse, whose foci are $(2, 5)$ and $(2, -3)$ and eccentricity is $\frac{4}{5}$, is

A. $\frac{6}{5}$

B. $\frac{50}{3}$

C. $\frac{10}{3}$

D. $\frac{18}{5}$

Correct answer: D

Source solution

Sol. $2be = 8$

$be = 4$

$F_1 \circ (2, 5)$

$F_2 \circ (2, -3)$

$b\left(\frac{4}{5}\right) = 4 \Rightarrow b = 5$

$\because c^2 = b^2 - a^2$

$16 = 25 - a^2 \Rightarrow a = 3$

L.R. $= \frac{2a^2}{b} = \frac{18}{5}$

Option (4)is the correct answer.

Question 102

The length of the latus rectum and the directrices of a hyperbola with eccentricity e are 9 and $x = \pm 4/\sqrt{13}$, respectively. Let the line $y - \sqrt{3}x + \sqrt{3} = 0$ touch this hyperbola at (x_0, y_0). If m is the product of the focal distances of (x_0, y_0), then $4e^2 + m$ is equal to ___.

Numerical answer: _____

This question was incorrect in the original exam paper and ALLEN marked it BONUS. This corrected practice version uses the directrices x = +/-4/sqrt(13), explicitly corrected by ALLEN, instead of the printed +/-4/sqrt(3). Enter an integer (0 decimal places).

Correct answer: 61

Source solution

Use the corrected standard hyperbola $x^2/a^2 - y^2/b^2 = 1$. The latus rectum gives $2b^2/a = 9$. Tangency of $y = \sqrt{3}x - \sqrt{3}$ gives $3 = 3a^2 - b^2$. Thus $a = 2$, $b^2 = 9$, and $e^2 = 1 + b^2/a^2 = 13/4$. The corrected directrices are $x = \pm a/e = \pm 4/\sqrt{13}$. The tangent contact is $(4, 3\sqrt{3})$. Hence $m = (ex_0 - a)(ex_0 + a) = 48$, so $4e^2 + m = 13 + 48 = 61$. The original ALLEN BONUS designation is retained in the instructions; this answer applies to the disclosed corrected practice version.

Question 103

Let the hyperbola $\mathbf{H}: \frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ pass through the point $(2\sqrt{2},-2\sqrt{2})$. A parabola is drawn whose focus is same as the focus of H with positive abscissa and the directrix of the parabola passes through the other focus of H . If the length of the latus rectum of the parabola is e times the length of the latus rectum of H , where e is the eccentricity of H , then which of the following points lies on the parabola?

A. $(2\sqrt{3},3\sqrt{2})$

B. $(3\sqrt{3},-6\sqrt{2})$

C. $(\sqrt{3},-\sqrt{6})$

D. $(3\sqrt{6},6\sqrt{2})$

Correct answer: B

Source solution

Sol. $\mathbf{H}: \frac{x^2}{a^2}-\frac{y^2}{b^2}=1$
Foci : S (ae, 0), $S'(-ae,0)$
Foot of directrix of parabola is ($-ae,0$)
Focus of parabola is (ae, 0)
Now, semi latus rectum of parabola $=|SS'|=2ae$
Given, $4ae=e\left(\frac{2b^2}{a}\right)$

$$\Rightarrow b^2=2a^2 \tag{1}$$

Given, $(2\sqrt{2},-2\sqrt{2})$ lies on H

$$\Rightarrow \frac{1}{a^2}-\frac{1}{b^2}=\frac{1}{8} \tag{2}$$

From (1) and (2)
$a^2=4, b^2=8$
$\because b^2=a^2\left(e^2-1\right)$

$\therefore e = \sqrt{3}$

$\Rightarrow$ Equation of parabola is $y^2 = 8\sqrt{3}x$

Question 104

If the line $x - 2y = 12$ is tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point $\left(3, \frac{-9}{2}\right)$, then the length of the latus rectum of the ellipse is :

A. 5

B. $8\sqrt{3}$

C. $12\sqrt{2}$

D. 9

Correct answer: D

Source solution

Sol.

Equation of tangent at $\left(3, -\frac{9}{2}\right)$ to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{3x}{a^2} - \frac{y9}{2b^2} = 1$ which is equivalent to $x - 2y = 12$

$\frac{3}{a^2} = \frac{-9}{2b^2 \cdot (-2)} = \frac{1}{12}$ (On comparing)

$a^2 = 3 \times 12$ and $b^2 = \frac{9 \times 12}{4}$

$$a = 6 \Rightarrow b = 3\sqrt{3}$$

So latus rectum $= \frac{2b^2}{a} = \frac{2 \times 27}{6} = 9$

Question 105

Let $\mathbf{H}: \frac{-x^2}{a^2}+\frac{y^2}{b^2}=1$ be the hyperbola, whose eccentricity is $\sqrt{3}$ and the length of the latus rectum is $4\sqrt{3}$. Suppose the point $(\alpha, 6), \alpha>0$ lies on H . If β is the product of the focal distances of the point $(\alpha, 6)$, then $\alpha^2+\beta$ is equal to :

A. 170

B. 171

C. 169

D. 172

Correct answer: B

Source solution

Sol.

$\mathbf{H}: \frac{y^2}{b^2}-\frac{x^2}{a^2}=1, e=\sqrt{3}$

$$e=\sqrt{1+\frac{a^2}{b^2}}=\sqrt{3} \Rightarrow \frac{a^2}{b^2}=2 \quad a^2=2 b^2$$

length of L.R. $=\frac{2a^2}{b}=4\sqrt{3}$

$$a=\sqrt{6}$$

$P(\alpha, 6)$ lie on $\frac{y^2}{3}-\frac{x^2}{6}=1$

$$12-\frac{\alpha^2}{6}=1 \Rightarrow \alpha^2=66$$

Foci $=(0, \pm \text{ be })=(0,3) \& (0,-3)$

Let $d_1 \& d_2$ be focal distances of $P(\alpha, 6)$

$d_1=\sqrt{\alpha^2+(6+be)^2}, d_2=\sqrt{\alpha^2+(6-be)^2}$

$d_1=\sqrt{66+81}, d_2=\sqrt{66+9}$

$\beta = \mathrm{d}_1\,\mathrm{d}_2 = \sqrt{147 \times 75} = 105$
$\alpha^2 + \beta = 66 + 105 = 171$

Question 106

In an ellipse, with centre at the origin, if the difference of the lengths of major axis and minor axis is 10 and one of the foci is at $(0, 5\sqrt{3})$, then the length of its latus rectum is :

A. 5

B. 6

C. 8

D. 10

Correct answer: A

Source solution

Sol.

$\because$ Focus is $(0, 5\sqrt{3}) \Rightarrow |b| > |a|$

Let $\mathbf{b > a > 0}$

foci ($\mathbf{0, \pm}$ be)

$a^2 = b^2 - b^2e^2 \quad \Rightarrow b^2e^2 = b^2 - a^2$

$be = \sqrt{b^2 - a^2}$

$$\Rightarrow \mathbf{b^2 - a^2 = 75} \qquad \textbf{(i)}$$

$2b - 2a = 10$

$$\Rightarrow \mathbf{b - a = 5} \qquad \textbf{(ii)}$$

From (i) and (ii)

$$b + a = 15 \qquad \textbf{(iii)}$$

$\Rightarrow b=10, a=5$

Length of $L \cdot R \cdot=\frac{2a^2}{b}=\frac{50}{10}=5$

Question 107

If the length of the latus rectum of the ellipse $x^2+4y^2+2x+8y-\lambda=0$ is 4 , and l is the length of its major axis, then $\lambda+l$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 75

Source solution

$\lambda+\ell=75$
$x^2+4y^2+2x+8y-\lambda=0$
$\frac{(x+1)^2}{\lambda+5}+\frac{(y+1)^2}{\frac{\lambda+5}{4}}=1$
$\because \frac{2b^2}{a}=4$
$\frac{2(\lambda+5)}{4}=4(\sqrt{\lambda+5})$
$\Rightarrow \lambda=59$
$\lambda \neq -5$
$l=2a=2\sqrt{\lambda+5}=2\sqrt{65}=16$
$\Rightarrow \lambda+\ell=59+16=75$

Question 108

Let $\mathrm{P}(10, 2\sqrt{15})$ be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, whose foci are S and S'. If the length of its latus rectum is 8, then the square of the area of $\triangle \mathrm{PSS}'$ is equal to :

A. 4200

B. 900

C. 1462

D. 2700

Correct answer: D

Source solution

$\mathrm{P}(10, 2\sqrt{15})$ lies on $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$\therefore \frac{100}{a^2} - \frac{60}{b^2} = 1$$

$\because$ length of latus rectum $= 8$

$$\frac{2 \cdot b^2}{a} = 8 \Rightarrow \frac{b^2}{a} = 4$$

From (1) & (2)

$$\frac{100}{a^2} - \frac{60}{4a} = 1$$
$$400 - 60a = 4a^2$$
$$4a^2 + 60a - 400 = 0$$
$$a^2 + 15a - 100 = 0$$
$$a = 5 \& -20 \text{ (rejected)}$$
$$\Rightarrow b = \sqrt{20}$$

$\therefore$ Hyperbola is $\frac{x^2}{25} - \frac{y^2}{20} = 1$

$\therefore$ Focal length $S_1 S_2 = 2ae = 2.5 \cdot \left(\sqrt{1 + \frac{4}{5}}\right) = 6\sqrt{5}$

$\therefore$ Area of $\triangle \mathrm{PS_1 S_2} = \frac{1}{2}.6\sqrt{5}.2\sqrt{15} = 30\sqrt{3} = A$

$$\therefore A^2 = 2700$$

Question 109

Let one focus of the hyperbola $\mathbf{H}: \frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be at $(\sqrt{10}, 0)$ and the corresponding directrix be $\mathbf{x}=\frac{9}{\sqrt{10}}$. If e and l respectively are the eccentricity and the length of the latus rectum of H , then $9\left(e^2+l\right)$ is equal to:

A. 14

B. 15

C. 16

D. 12

Correct answer: C

Source solution

Sol. $ae=\sqrt{10}$ and $\frac{a}{e}=\frac{9}{10}$

$\Rightarrow a^2=9$ and $e=\frac{\sqrt{10}}{3}$

Now $(ae)^2=a^2+b^2$

$10=9+b^2 \Rightarrow b^2=1$

$\ell=\frac{2b^2}{a}=\frac{2(1)}{3}$

$\Rightarrow \quad 9\left(e^2+\ell\right)$

$=9\left(\frac{10}{9}+\frac{2}{3}\right)$

$=10+6$

$=16$

Question 110

If S and S' are the foci of the ellipse $\frac{x^2}{18}+\frac{y^2}{9}=1$ and P be a point on the ellipse, then $\min(SP.S'P)+\max(SP'S'P)$ is equal to :

A. $3(1+\sqrt{2})$

B. $3(6+\sqrt{2})$

C. 9

D. 27

Correct answer: D

Source solution

Sol.

$PS+PS'=2\times 3\sqrt{2}$

$b^2=a^2\left(1-e^2\right)\Rightarrow 9=18\left(1-e^2\right)$

$\Rightarrow e=\frac{1}{\sqrt{2}}$

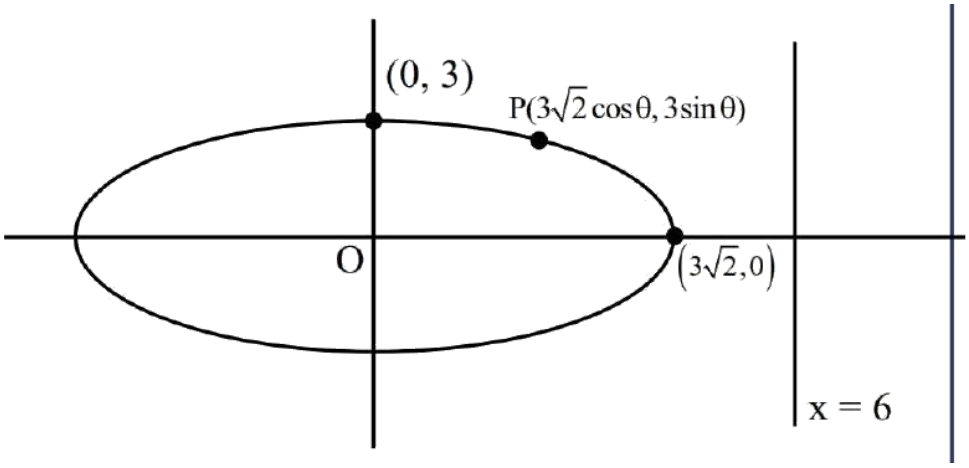
Directrix $x=\frac{a}{e}=\frac{3\sqrt{2}}{\frac{1}{\sqrt{2}}}=6$

$PS\cdot PS'=\left|\frac{1}{\sqrt{2}}(3\sqrt{2}\cos\theta-6)\frac{1}{\sqrt{2}}(3\sqrt{2}\cos\theta+6)\right|$

$=\frac{1}{2}\left|18\cos^2\theta-36\right|$

$(PS\cdot PS)_{max}=18;(PS\cdot PS)_{min}=9$

sum $=27$



Question 111

Let P be a point on the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$. Let the line passing through P and parallel to y -axis meet the circle $x^2+y^2=9$ at point Q such that P and Q are on the same side of the x -axis. Then, the eccentricity of the locus of the point R on PQ such that $\mathbf{PR : RQ = 4 : 3}$ as P moves on the ellipse, is :

A. $\frac{11}{19}$

B. $\frac{13}{21}$

C. $\frac{\sqrt{139}}{23}$

D. $\frac{\sqrt{13}}{7}$

Correct answer: D

Source solution

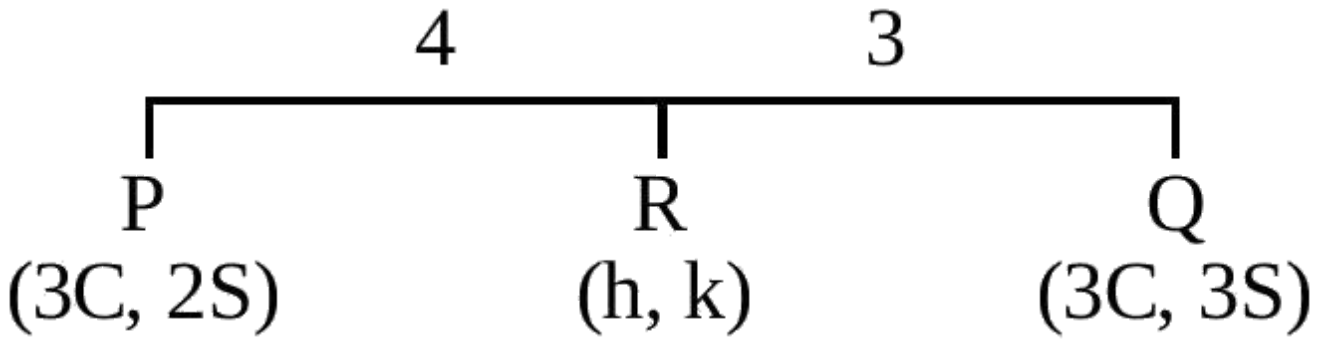
Sol.

$\mathbf{h}=3\cos\theta$;

$\mathbf{k}=\frac{18}{7}\sin\theta$

$\therefore$ locus $=\frac{x^2}{9}+\frac{49y^2}{324}=1$

$\mathbf{e}=\sqrt{1-\frac{324}{49\times9}}=\frac{\sqrt{117}}{21}=\frac{\sqrt{13}}{7}$



Question 112

If the foci of a hyperbola are same as that of the ellipse $\frac{x^2}{9}+\frac{y^2}{25}=1$ and the eccentricity of the hyperbola is $\frac{15}{8}$ times the eccentricity of the ellipse, then the smaller focal distance of the point $\left(\sqrt{2}, \frac{14}{3}\sqrt{\frac{2}{5}}\right)$ on the hyperbola, is equal to

A. $7\sqrt{\frac{2}{5}}-\frac{8}{3}$

B. $14\sqrt{\frac{2}{5}}-\frac{4}{3}$

C. $14\sqrt{\frac{2}{5}}-\frac{16}{3}$

D. $7\sqrt{\frac{2}{5}}+\frac{8}{3}$

Correct answer: A

Source solution

Sol. $\frac{x^2}{9}+\frac{y^2}{25}=1$

$\mathrm{a}=3,\ \mathrm{b}=5$

$\mathrm{e}=\sqrt{1-\frac{9}{25}}=\frac{4}{5}\ \therefore \text{foci}=(0,\pm\mathrm{be})=(0,\pm4)$

$$\therefore e_H=\frac{4}{5}\times\frac{15}{8}=\frac{3}{2}$$

Let equation hyperbola

$\frac{x^2}{A^2}-\frac{y^2}{B^2}=-1$

$\therefore \mathrm{B}\cdot\mathrm{e_H}=4 \quad \therefore\ \mathrm{B}=\frac{8}{3}$

$\therefore \mathrm{A}^2=\mathrm{B}^2\left(\mathrm{e_H^2}-1\right)=\frac{64}{9}\left(\frac{9}{4}-1\right) \quad \therefore \mathrm{A}^2=\frac{80}{9}$

$\therefore \frac{x^2}{\frac{80}{9}}-\frac{y^2}{\frac{64}{9}}=-1$

Directrix : $y=\pm\frac{B}{e_H}=\pm\frac{16}{9}$

$\mathrm{PS}=\mathrm{e}\cdot\mathrm{PM}=\frac{3}{2}\left|\frac{14}{3}\cdot\sqrt{\frac{2}{5}}-\frac{16}{9}\right|$

$=7\sqrt{\frac{2}{5}}-\frac{8}{3}$

Question 113

Let the tangents at the points P and Q on the ellipse $\frac{x^2}{2}+\frac{y^2}{4}=1$ meet at the point $\mathbf{R}(\sqrt{2}, 2\sqrt{2}-2)$. If S is the focus of the ellipse on its negative major axis, then $\mathbf{SP}^2+\mathbf{SQ}^2$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 13

Source solution

Ellipse is
$\frac{x^2}{2}+\frac{y^2}{4}=1; e=\frac{1}{\sqrt{2}}; S\equiv(0,-\sqrt{2})$
Chord of contact is
$\frac{x}{\sqrt{2}}+\frac{(2\sqrt{2}-2)y}{4}=1$
$\Rightarrow \frac{x}{\sqrt{2}}=1-\frac{(\sqrt{2}-1)y}{2}$ solving with ellipse
$\Rightarrow \mathbf{y}=0,\sqrt{2} \quad \therefore \mathbf{x}=\sqrt{2},1$
$\mathbf{P}\equiv(1,\sqrt{2})\mathbf{Q}\equiv(\sqrt{2},0)$
$\therefore (\mathbf{SP})^2+(\mathbf{SQ})^2=13$

Question 114

Let the eccentricity of the hyperbola $\mathbf{H}: \frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be $\sqrt{\frac{5}{2}}$ and length of its latus rectum be $6\sqrt{2}$, If $y=2x+c$ is a tangent to the hyperbola H , then the value of c^2 is equal to

A. 18

B. 20

C. 24

D. 32

Correct answer: B

Source solution

Sol. $y=mx \pm \sqrt{a^2 m^2-b^2}$

$m=2, c^2=a^2 m^2-b^2$

$c^2=4a^2-b^2$

$e^2=1+\frac{b^2}{a^2}$

$\frac{5}{2}=1+\frac{b^2}{a^2}$

$\frac{3}{2}=\frac{b^2}{a^2} \Rightarrow b^2=\frac{3a^2}{2}$

$\frac{2b^2}{a}=6\sqrt{2}$

$\frac{2}{a} \times \frac{3a^2}{2}=6\sqrt{2}$

$3a=6\sqrt{2}$

$a=2\sqrt{2}$

$b^2=\frac{3}{2} \times 8=12$

$b=2\sqrt{3}$

$\therefore c^2=4 \times 8-12$

$c^2=20$

Question 115

Let (h, k) lie on the circle $C : x^2 + y^2 = 4$ and the point $(2h + 1, 3k + 2)$ lie on an ellipse with eccentricity e. Then the value of $\frac{5}{e^2}$ is equal to

_____

.

Numerical answer: _____

Enter an integer. Use the units and scaling requested in the question.

Correct answer: 9

Source solution

Let $P \equiv (2\cos\theta, 2\sin\theta)$

$$\therefore \text{coordinates of } Q = (4\cos\theta + 1, 6\sin\theta + 3)$$

$$\therefore \text{ locus of } Q \text{ is } \left(\frac{x-1}{4}\right)^2 + \left(\frac{y-3}{6}\right)^2 = 1$$

$$\therefore e^2 = 1 - \frac{16}{36} = \frac{5}{9}$$

$$\therefore \frac{5}{e^2} = 9$$

Question 116

For the hyperbola $\mathbf{H}: \mathbf{x}^2 - \mathbf{y}^2 = \mathbf{1}$ and the ellipse $\mathbf{E}: \frac{\mathbf{x}^2}{\mathbf{a}^2} + \frac{\mathbf{y}^2}{\mathbf{b}^2} = \mathbf{1}, \mathbf{a} > \mathbf{b} > \mathbf{0}$, let the

(1) eccentricity of E be reciprocal of the eccentricity of H , and

(2) the line $y = \sqrt{\frac{5}{2}}x + K$ be a common tangent of E and H .

Then $4\left(a^2 + b^2\right)$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 3

Source solution

$e_E = \sqrt{1 - \frac{b^2}{a^2}}, e_H = \sqrt{2}$

If $\Rightarrow \mathbf{e_E} = \frac{1}{\mathbf{e_H}}$

$\Rightarrow \frac{\mathbf{a^2 - b^2}}{\mathbf{a^2}} = \frac{1}{2}$

$2\mathbf{a}^{2-2\,\mathbf{b}}2 = \mathbf{a}^2$

$a^2 = 2b^2$

and $\mathbf{y} = \sqrt{\frac{5}{2}}\mathbf{x} + \mathbf{k}$ is tangent to ellipse then

$\mathbf{K}^2 = \mathbf{a}^2 \times \frac{5}{2} + \mathbf{b}^2 = \frac{3}{2}$

$6\,\mathbf{b}^2 = \frac{3}{2} \Rightarrow \mathbf{b}^2 = \frac{1}{4}$ and $\mathbf{a}^2 = \frac{1}{2}$

$\therefore 4 \cdot \left(a^2 + b^2\right) = 3$

Question 117

Let the product of the focal distances of the point $\left(\sqrt{3}, \frac{1}{2}\right)$ on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,(a>b)$, be $\frac{7}{4}$. Then the absolute difference of the eccentricities of two such ellipses is

A. $\frac{3-2\sqrt{2}}{3\sqrt{2}}$

B. $\frac{1-\sqrt{3}}{\sqrt{2}}$

C. $\frac{3-2\sqrt{2}}{2\sqrt{3}}$

D. $\frac{1-2\sqrt{2}}{\sqrt{3}}$

Correct answer: C

Source solution

Sol. Product of focal distances $=(a+ex_1)(a-ex_1)$

$=a^2-e^2x_1^2=a^2-e^2(3)$

$=a^2-3e^2=\frac{7}{4} \Rightarrow a^2=\frac{7}{4}+3e^2$

$\Rightarrow 4a^2=7+12e^2$

& $\left(\sqrt{3}, \frac{1}{2}\right)$ lines on $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$\therefore \frac{3}{a^2}+\frac{1}{4b^2}=1$

$\frac{3}{a^2}+\frac{1}{4(a^2)(1-e^2)}=1$

$12\left(1-e^2\right)+1=4a^2\left(1-e^2\right)$

$13-12e^2=\left(7+12e^2\right)\left(1-e^2\right)$

$\Rightarrow 13-12e^2=7-7e^2+12e^2-12e^4$

$\Rightarrow 12e^4-17e^2+6=0$

$\therefore \quad e^2=\frac{17 \pm \sqrt{289-288}}{24}=\frac{17 \pm 1}{24}=\frac{3}{4} \& \frac{2}{3}$

$\therefore e=\frac{\sqrt{3}}{2} \& \sqrt{\frac{2}{3}}$

$\therefore$ difference $=\frac{\sqrt{3}}{2}-\sqrt{\frac{2}{3}}=\frac{3-2\sqrt{2}}{2\sqrt{3}}$

Option (3)}

Question 118

If the length of the minor axis of an ellipse is equal to one fourth of the distance between the foci, then the eccentricity of the ellipse is :

A. $\frac{4}{\sqrt{17}}$

B. $\frac{\sqrt{3}}{16}$

C. $\frac{3}{\sqrt{19}}$

D. $\frac{\sqrt{5}}{7}$

Correct answer: A

Source solution

Sol. $2b = \frac{1}{4}(2ae)$

$\frac{b}{a} = \frac{e}{4}$

$e = \sqrt{1 - \frac{b^2}{a^2}}$

$e = \sqrt{1 - \frac{e^2}{16}}$

$e^2\left(1 + \frac{1}{16}\right) = 1$

$e = \frac{4}{\sqrt{17}}$

Question 119

Let the product of the focal distances of the point $\mathbf{P(4, 2\sqrt{3})}$ on the hyperbola $\mathbf{H : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$ be 32 . Let the length of the conjugate axis of H be p and the length of its latus rectum be q . Then $\mathbf{p^2 + q^2}$ is equal to .....

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 120

Source solution

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
P $(4, 2\sqrt{3})$
$\mathbf{PS_1 \cdot PS_2 = 32}$
$\mathbf{|PS_1 - PS_2| = 2a}$
$\mathbf{P(4, 2\sqrt{3})}$ lies on H
$\therefore \frac{16}{a^2} - \frac{12}{b^2} = 1$

$$16b^2 - 12a^2 = a^2b^2 \qquad (2)$$

$\mathbf{|PS_1 - PS_2|^2 = 4a^2}$
$\mathbf{{PS_1}^2 + {PS_2}^2 - 2PS_1.PS_2 = 4a^2}$
$(ae - 4)^2 + 12 + (ae + 4)^2 + 12 - 64 = 4a^2$
$\mathbf{2a^2e^2 - 8 = 4a^2}$
$a^2 + b^2 - 4 = 2a^2$
$b^2 - a^2 = 4$
(2) & (3) $\Rightarrow 16\left(a^2 + 4\right) - 12a^2 = a^2\left(a^2 + 4\right)$
$\Rightarrow \mathbf{16a^2 + 64 - 12a^2 = a^4 + 4a^2}$
$\Rightarrow \mathbf{a^4 = 64}$
$\Rightarrow \mathbf{a^2 = 8}$
$\therefore \mathbf{b^2 = 12}$
$\mathbf{p^2 + q^2 = 4\,b^2 + \frac{4\,b^4}{a^2}}$
$\mathbf{= 120}$

Question 120

Let $\mathbf{P}\left(\frac{2\sqrt{3}}{\sqrt{7}}, \frac{6}{\sqrt{7}}\right), \mathbf{Q}, \mathbf{R}$ and S be four points on the ellipse $9x^2 + 4y^2 = 36$. Let PQ and RS be mutually perpendicular and pass through the origin. If $\frac{1}{(\mathrm{PQ})^2} + \frac{1}{(\mathrm{RS})^2} = \frac{\mathrm{p}}{\mathrm{q}}$, where p and q are coprime, then $\mathbf{p} + \mathbf{q}$ is equal to

A. 143

B. 137

C. 157

D. 147

Correct answer: C

Source solution

Sol.

Let $\mathbf{R}(2\cos\theta, 3\sin\theta)$

as $\mathbf{OP} \perp \mathbf{OR}$

so $\frac{3\sin\theta}{2\cos\theta} \times \frac{\frac{6}{\sqrt{7}}}{\frac{2\sqrt{3}}{\sqrt{7}}} = -1$

$\Rightarrow \tan\theta = \frac{-2}{3\sqrt{3}}$

$\Rightarrow \mathbf{R}\left(\frac{-6\sqrt{3}}{\sqrt{31}}, \frac{6}{\sqrt{31}}\right)$ or $\mathbf{R}\left(\frac{6\sqrt{3}}{\sqrt{31}}, \frac{-6}{\sqrt{31}}\right)$

Now $= \frac{1}{(\mathrm{PQ})^2} + \frac{1}{(\mathrm{RS})^2} = \frac{1}{4}\left(\frac{1}{(\mathrm{OP})^2} + \frac{1}{(\mathrm{OR})^2}\right)$

$= \frac{1}{4}\left(\frac{1}{\frac{48}{7}} + \frac{1}{\frac{144}{31}}\right) = \frac{1}{4}\left(\frac{7}{48} + \frac{31}{144}\right)$

$= \frac{13}{144}$

$\Rightarrow \mathrm{p} + \mathrm{q} = 157$

Question 121

Let the ellipse $\mathbf{E}: \frac{x^2}{144}+\frac{y^2}{169}=1$ and the hyperbola $\mathbf{H}: \frac{x^2}{16}-\frac{y^2}{\lambda^2}=-1$ have the same foci. If e and L, respectively denote the eccentricity and the length of the latus rectum of H, then the value of $24(e+L)$ is :

A. 296

B. 126

C. 148

D. 67

Correct answer: A

Source solution

Equation of hyperbola : $\frac{y^2}{\lambda^2}-\frac{x^2}{16}=1$

Equation of ellipse : $\frac{x^2}{144}+\frac{y^2}{169}=1$

$$e'=\sqrt{1-\frac{144}{169}}=\frac{5}{13}$$

focus $\Rightarrow (0,5)$

$$\Rightarrow \lambda\sqrt{1+\frac{16}{\lambda^2}}=5$$
$$\Rightarrow \lambda^2+16=25$$
$$\lambda=3$$

Eccentricity of hyperbola $=\sqrt{1+\frac{16}{\lambda^2}}=\frac{5}{3}$

Length of latus rectum of hyperbola $=\frac{2(16)}{3}=\frac{32}{3}$

$$24(e+\ell)=24\left[\frac{5}{3}+\frac{32}{3}\right]=8\times 37=296$$

Question 122

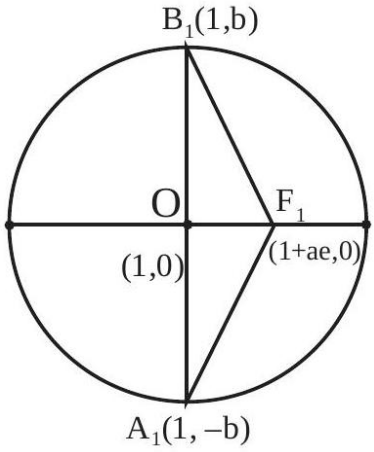
Let an ellipse with centre $(1, 0)$ and latus rectum of length $\frac{1}{2}$ have its major axis along x-axis. If its minor axis subtends an angle $60°$ at the foci, then the square of the sum of the lengths of its minor and major axes is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 9

Source solution



L.R. $= \frac{2b^2}{a} = \frac{1}{2}$

$$4b^2 = a \quad \text{(i)}$$

Ellipse $\frac{(x-1)^2}{a^2} + \frac{y^2}{b^2} = 1$

$m_{B_2F_1} = \frac{1}{\sqrt{3}}$

$\frac{b}{ae} = \frac{1}{\sqrt{3}}$

$3b^2 = a^2e^2 = a^2 - b^2$

$$4b^2 = a^2 \quad \text{(ii)}$$

From (i) and (ii)

$\mathbf{a = a^2}$

$\therefore \mathbf{a = 1}$

$\mathbf{b^2 = \frac{1}{4}}$

$((2a) + (2b))^2 = 9$

Question 123

For some $\theta \in \left(0, \frac{\pi}{2}\right)$, let the eccentricity and the length of the latus rectum of the hyperbola $x^2 - y^2 \sec^2\theta = 8$ be e_1 and ℓ_1, respectively, and let the eccentricity and the length of the latus rectum of the ellipse $x^2 \sec^2\theta + y^2 = 6$ be e_2 and ℓ_2, respectively. If $e_1^2 = e_2^2\left(\sec^2\theta + 1\right)$, then $\left(\frac{\ell_1\ell_2}{e_1e_2}\right)\tan^2\theta$ is equal to_____.

Numerical answer: _____

Enter an integer. Use the units and scaling requested in the question.

Correct answer: 8

Source solution

$\frac{x^2}{8} - \frac{y^2}{8\cos^2\theta} = 1, e_1 = \sqrt{1 + \frac{8\cos^2\theta}{8}}$

$$\ell_1 = \frac{2b^2}{a} = \frac{2 \cdot \left(8\cos^2\theta\right)}{2\sqrt{2}}$$

$$\frac{x^6}{6} + \frac{y^2}{6\cos^2\theta} = 1; e_2 = \sqrt{1 - \frac{6\cos^2\theta}{6}} = \sin\theta$$

$$\ell_2 = \frac{2b^2}{a} = \frac{2 \cdot 6\cos^2\theta}{\sqrt{6}}$$

$$e_1^2 = e_2^2\left(1 + \sec^2\theta\right)$$

$$1 + \cos^2\theta = \sin^2\theta\left(1 + \frac{1}{\cos^2\theta}\right)$$

$$1 + \cos^2\theta = \sin^2\theta + \tan^2\theta$$

Solving we get $\theta = \frac{\pi}{4}$

$$\ell_1 = 2\sqrt{2}$$

$$e_1 = \sqrt{\frac{3}{2}}$$

$$\ell_2 = \sqrt{6}$$

$$e_2 = \frac{1}{\sqrt{2}}$$

$$\left(\frac{\ell_1\ell_2}{e_1e_2}\right)\tan^2\theta = 8 \quad \text{(By putting values)}$$

Question 124

Let the foci of a hyperbola H coincide with the foci of the ellipse $E: \frac{(x-1)^2}{100}+\frac{(y-1)^2}{75}=1$ and the eccentricity of the hyperbola H be the reciprocal of the eccentricity of the ellipse E. If the length of the transverse axis of H is α and the length of its conjugate axis is β, then $3\alpha^2+2\beta^2$ is equal to :

A. 242

B. 225

C. 237

D. 205

Correct answer: B

Source solution

Sol.

$$e_1=\sqrt{1-\frac{75}{100}}=\frac{5}{10}=\frac{1}{2}$$
$$e_2=2$$
$$F_1(6,1), F_2(-4,1)$$
$$2ae_2=10 \Rightarrow a=\frac{5}{2} \Rightarrow 2a=5$$
$$\Rightarrow \alpha=5$$
$$4=1+\frac{b^2}{a^2} \Rightarrow b^2=3a^2$$
$$b=\sqrt{3}\times\frac{5}{2}$$
$$\beta=5\sqrt{3}$$
$$3\alpha^2+2\beta^2=3\times25+2\times25\times3$$
$$=225$$

Question 125

For some $\theta \in \left(0, \frac{\pi}{2}\right)$, if the eccentricity of the hyperbola, $x^2 - y^2 \sec^2\theta = 10$ is $\sqrt{5}$ times the eccentricity of the ellipse, $x^2 \sec^2\theta + y^2 = 5$, then the length of the latus rectum of the ellipse, is:

A. $\frac{4\sqrt{5}}{3}$

B. $\frac{2\sqrt{5}}{3}$

C. $2\sqrt{6}$

D. $\sqrt{30}$

Correct answer: A

Source solution

Sol.\
$\mathbf{H : x^2 - y^2 \sec^2\theta = 10}$
E : $x^2 \sec^2\theta + y^2 = 5$
$\sqrt{1 + \frac{10\cos^2\theta}{10}} = \sqrt{5}\sqrt{1 - \frac{5\cos^2\theta}{5}}$
$1 + \cos^2\theta = 5 - 5\cos^2\theta$
$6\cos^2\theta = 4$
$\cos\theta = \pm\sqrt{\frac{2}{3}}$
$\mathbf{I(LR)}$ of ellipse $= \frac{2.5\cos^2\theta}{\sqrt{5}}$
$= 2\sqrt{5} \cdot \frac{2}{3} = \frac{4\sqrt{5}}{3}$

Question 126

Let the foci of a hyperbola be $(1, 14)$ and $(1, -12)$. If it passes through the point $(1, 6)$, then the length of its latus-rectum is :

A. $\frac{25}{6}$

B. $\frac{24}{5}$

C. $\frac{288}{5}$

D. $\frac{144}{5}$

Correct answer: C

Source solution

Sol.

$\mathrm{be} = 13,\ \mathrm{b} = 5$

$a^2 = b^2\left(e^2 - 1\right)$

$= b^2e^2 - b^2$

$= 169 - 25 = 144$

$\ell(\mathrm{LR}) = \frac{2\mathrm{a}^2}{\mathrm{b}} = \frac{2\times144}{5} = \frac{288}{5}$

Question 127

The length of the latus rectum of a parabola, whose vertex and focus are on the positive x -axis at a distance R and $S(>R)$ respectively from the origin, is :

A. $4(S+R)$

B. $2(S-R)$

C. $4(S-R)$

D. $2(S+R)$

Correct answer: C

Source solution

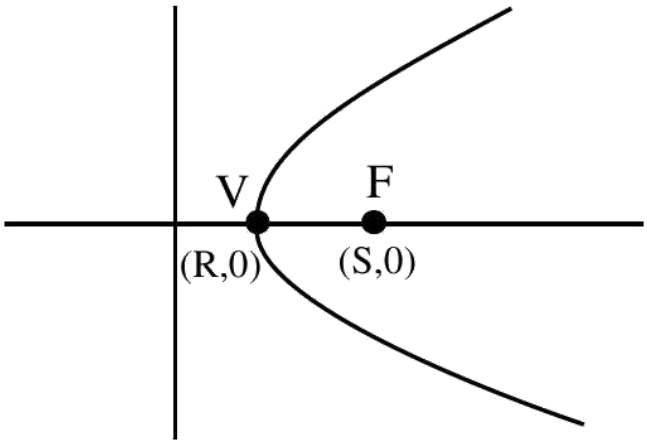
Sol.

$V \rightarrow$ Vertex

$F \rightarrow$ focus

$VF = S - R$

So latus rectum $= 4(S-R)$



Question 128

Let A be the focus of the parabola $y^2 = 8x$. Let the line $y = mx + c$ intersect the parabola at two distinct points B and C. If the centroid of the triangle ABC is $\left(\frac{7}{3}, \frac{4}{3}\right)$, then $(BC)^2$ is equal to :

A. 41

B. 80

C. 89

D. 32

Correct answer: B

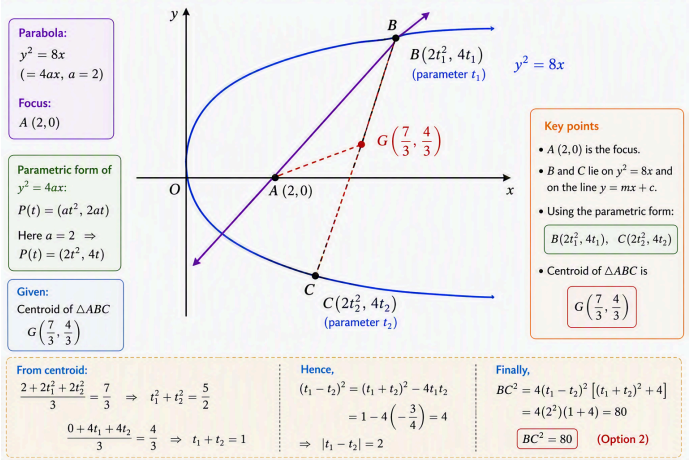
Source solution

Coordinates of centroid of triangle ABC are

$$\frac{2}{3}\left(t_1^2 + t_2^2 + 1\right) = \frac{7}{3} \Rightarrow t_1^2 + t_2^2 = \frac{5}{2}$$
$$\frac{4}{3}(t_1 + t_2) = \frac{4}{3} \Rightarrow t_1 + t_2 = 1$$

$$(t_1 + t_2)^2 = t_1^2 + t_2^2 + 2t_1t_2 \Rightarrow t_1t_2 = \frac{-3}{4}$$

$$(t_1 - t_2)^2 = (t_1 + t_2)^2 - 4t_1t_2 = 4$$
$$(BC)^2 = 4\left(t_1^2 - t_2^2\right)^2 + 16(t_1 - t_2)^2$$
$$\Rightarrow (BC)^2 = 80$$



Limits and Derivatives

Questions 129-130 / 2 selected / 0.7% practice weight / Consolidate

Question 129

If the domain of the function
$f(x)=\log_e\left(4x^2+11x+6\right)+\sin^{-1}$
$(4x+3)+\cos^{-1}\left(\frac{10x+6}{3}\right)$ is $(\alpha,\beta]$,
Then $36|\alpha+\beta|$ is equal to :

A. 63

B. 45

C. 72

D. 54

Correct answer: B

Source solution

Sol. $f(x)=\ln\left(4x^2+11x+6\right)+\sin^{-1}(4x+3)$
$+\cos^{-1}\left(\frac{10x+6}{3}\right)$
(i) $4x^2+11x+6>0$
$4x^2+8x+3x+6>0$
$(4x+3)(x+2)>0$
$x\in(-\infty,-2)\cup\left(-\frac{3}{4},\infty\right)$
(ii) $4x+3\in[-1,1]$
$x\in[-1,-1/2]$
(iii) $\frac{10x+6}{3}\in[-1,1]$
$x\in\left[-\frac{9}{10},-\frac{3}{10}\right]$
$x\in\left(-\frac{3}{4},-\frac{1}{2}\right]\quad\alpha=-\frac{3}{4},\beta=-\frac{1}{2}$
$\alpha+\beta=-\frac{5}{4}$
$36|\alpha+\beta|=45$

Question 130

Suppose f is a function satisfying $f(x+y)=f(x)+f(y)$ for all $x, y \in \mathbb{N}$ and $f(1)=\frac{1}{5}$. If $\sum_{n=1}^{m} \frac{f(n)}{n(n+1)(n+2)}=\frac{1}{12}$, then m is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 10

Source solution

10

$f(x+y)=f(x)+f(y) \forall x, y \in N$ and $f(1)=\frac{1}{5}$

for $x=y=1$

$f(2)=f(1)+f(1)=2f(1)$

$f(3)=f(2+1)=f(2)+f(1)=3f(1)$

In General

$f(n)=nf(1)=\frac{n}{5}$

$\sum_{n=1}^{m} \frac{f(n)}{n(n+1)(n+2)}=\frac{1}{12}$

$\Rightarrow \sum_{n=1}^{m} \frac{n}{5n(n+1)(n+2)}=\frac{1}{12}$

$\Rightarrow \frac{1}{5} \sum_{n=1}^{m} \frac{1}{(n+1)(n+2)}=\frac{1}{12}$

$\Rightarrow \sum_{n=1}^{m}\left(\frac{1}{n+1}-\frac{1}{n+2}\right)=\frac{5}{12}$

$\Rightarrow\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\ldots\ldots\ldots+\left(\frac{1}{m+1}-\frac{1}{m+2}\right)=\frac{5}{12}$

$\Rightarrow \frac{1}{2}-\frac{1}{m+2}=\frac{5}{12}$

$\Rightarrow \frac{1}{m+2}=\frac{1}{2}-\frac{5}{12}=\frac{1}{12}$

$\Rightarrow m=10$

Statistics

Questions 131-146 / 16 selected / 5.3% practice weight / Next

Question 131

The mean of 10 numbers
$7 \times 8, 10 \times 10, 13 \times 12, 16 \times 14, \ldots$. is .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 398

Source solution

$$7 \times 8, 10 \times 10, 13 \times 12, 16 \times 14 \ldots\ldots$$
$$T_n = (3n+4)(2n+6) = 2(3n+4)(n+3)$$
$$= 2\left(3n^2+13n+12\right) = 6n^2+26n+24$$
$$S_{10} = \sum_{n=1}^{10} T_n = 6\sum_{n=1}^{10} n^2 + 26\sum_{n=1}^{10} n + 24\sum_{n=1}^{10} 1$$
$$= \frac{6(10\times11\times21)}{6} + 26 \times \frac{10\times11}{2} + 24 \times 10$$
$$= 10 \times 11(21+13) + 240$$
$$= 3980$$
$$\text{Mean} = \frac{S_{10}}{10} = \frac{3980}{10} = 398$$

Question 132

The mean and standard deviation of the marks of 10 students were found to be 50 and 12 respectively. Later, it was observed that two marks 20 and 25 were wrongly read as 45 and 50 respectively. Then the correct variance is .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 269

Source solution

$\bar{x} = 50$

$\sum x_i = 500$

$\sum x_{i_{\text{correct}}} = 500 + 20 + 25 - 45 - 50 = 450$

$\sigma^2 = 144$

$\frac{\sum x_i^2}{10} - (50)^2 = 144$

$$\sum x_{\text{icorrect}}^2 = \left(144 + (50)^2\right) \times 10 - (45)^2 - (50)^2 + (20)^2 + (25)^2$$

$$= 22940$$

$$\text{Correct variance } = \frac{\sum \left(x_{\text{icorrect}}\right)^2}{10} - \left(\frac{\sum x_{\text{icorrect}}}{10}\right)^2$$

$$= 2294 - (45)^2$$

$$= 2294 - 2025 = 269$$

Question 133

The mean and variance of a set of 15 numbers are 12 and 14 respectively. The mean and variance of another set of 15 numbers are 14 and σ^2 respectively. If the variance of all the 30 numbers in the two sets is 13 , then σ^2 is equal to

A. 9

B. 12

C. 11

D. 10

Correct answer: D

Source solution

Sol. Combine var. $= \frac{n_1\sigma^2+n_2\sigma^2}{n_1+n_2} + \frac{n_1n_2(m_1-m_2)^2}{(n_1+n_2)^2}$

$13 = \frac{15.14+15.\sigma^2}{30} + \frac{15.15(12-14)^2}{30\times30}$

$13 = \frac{14+\sigma^2}{2} + \frac{4}{4}$

$\sigma^2 = 10$

Question 134

From a lot of 10 items, which include 3 defective items, a sample of 5 items is drawn at random. Let the random variable X denote the number of defective items in the sample. If the variance of X is σ^2, then $96\sigma^2$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 56

Source solution

$\mathbf{X} =$ denotes number of defective

x	0	1	2	3
$\mathbf{P(x)}$	$\frac{7}{15}$	$\frac{5}{12}$	$\frac{5}{12}$	$\frac{1}{12}$
$\mathbf{x_1}^2$	0	1	4	9
$\mathbf{P_i x_1}^2$	0	$\frac{5}{12}$	$\frac{20}{12}$	$\frac{9}{12}$
$\mathbf{p_i x_i}$	0	$\frac{5}{12}$	$\frac{10}{12}$	$\frac{3}{12}$

$\mu = \Sigma \mathbf{p_i x_i} = \frac{18}{12}$
$\Sigma \mathbf{p_i x_1^2} = \frac{34}{12}$
$\sigma^2 = \Sigma \mathbf{p_i x_1^2} - (\mu)^2$
$= \frac{34}{12} - \left(\frac{18}{12}\right)^2 = \frac{17}{6} - \frac{9}{4}$
$\frac{34-27}{12} = \frac{7}{12}$
$96\sigma^2 = 96 \times \frac{7}{12} = 56$

Question 135

Let the mean of 6 observation $\mathbf{1, 2, 4, 5, x}$ and y be 5 and their variance be 10 . Then their mean deviation about the mean is equal to

A. $\frac{10}{3}$

B. $\frac{7}{3}$

C. 3

D. $\frac{8}{3}$

Correct answer: D

Source solution

Sol. $x + y = 18 \quad \because \text{mean} = 5$

$$10 = \frac{1 + 4 + 16 + 25 + \mathrm{x}^2 + \mathrm{y}^2}{6} - 25 \qquad \text{(i)}$$

$$x^2 + y^2 = 164 \qquad \text{(ii)}$$

By solving (i) and (ii)

$\mathbf{x = 8, y = 10}$

M.D. $(\bar{x}) = \frac{\sum|x_i - \bar{x}|}{6} = \frac{8}{3}$

Question 136

Let the mean and variance of 12 observations be $\frac{9}{2}$ and 4 respectively. Later on, it was observed that two observations were considered as 9 and 10 instead of 7 and 14 respectively. If the correct variance is $\frac{\mathbf{m}}{\mathbf{n}}$, where m and n are co-prime, then $\mathbf{m}+\mathbf{n}$ is equal to

A. 316

B. 314

C. 317

D. 315

Correct answer: C

Source solution

$$\textbf{Given:} \qquad n = 12, \quad \bar{x} = \frac{9}{2}, \quad \sigma^2 = 4.$$

Using

$$\sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2,$$

we get

$$4 = \frac{\sum x_i^2}{12} - \left(\frac{9}{2}\right)^2.$$

Hence,

$$\frac{\sum x_i^2}{12} = 4 + \frac{81}{4} = \frac{97}{4}.$$

Therefore,

$$\sum x_i^2 = 12 \cdot \frac{97}{4} = 291.$$

Also,

$$\sum x_i = 12 \cdot \frac{9}{2} = 54.$$

The observations 9 and 10 were taken instead of the correct values 7 and 14.

Thus,

$$\text{Correct sum} = 54 - (9 + 10) + (7 + 14) = 56.$$

Hence the correct mean is

$$\bar{x}_c = \frac{56}{12} = \frac{14}{3}.$$

Similarly,

$$\text{Correct } \sum x_i^2 = 291 - (9^2 + 10^2) + (7^2 + 14^2).$$

$$= 291 - (81 + 100) + (49 + 196) = 355.$$

Now,

$$\sigma_c^2 = \frac{355}{12} - \left(\frac{14}{3}\right)^2.$$

$$= \frac{355}{12} - \frac{196}{9}.$$

Taking LCM $= 36$,

$$\sigma_c^2 = \frac{1065 - 784}{36} = \frac{281}{36}.$$

Thus,

$$\sigma_c^2 = \frac{281}{36}.$$

Therefore,

$$m = 281, \qquad n = 36.$$

Hence,

$$m + n = 281 + 36 = 317.$$

$$\boxed{317}$$

Hence, the correct option is

$$\boxed{(3)\ 317}.$$

Question 137

Let sets A and B have 5 elements each. Let the mean of the elements in sets A and B be 5 and 8 respectively and the variance of the elements in sets A and B be 12 and 20 respectively A new set C of 10 elements is formed by subtracting 3 from each element of A and adding 2 to each element of B. Then the sum of the mean and variance of the elements of C is ..... .

A. 32

B. 38

C. 40

D. 36

Correct answer: B

Source solution

Sol. $\omega \mathrm{A}=\left\{\mathrm{a}_1, \mathrm{a}_2, \mathrm{a}_3, \mathrm{a}_4, \mathrm{a}_5\right\}$

$$\mathrm{B}=\left\{\mathrm{b}_1,\ \mathrm{b}_2,\ \mathrm{b}_3,\ \mathrm{b}_4,\ \mathrm{b}_5\right\}$$

Given, $\sum_{\mathrm{i}=1}^5 \mathrm{ai}=25, \sum_{\mathrm{i}=1}^5 \mathrm{bi}=40$

$\frac{\sum_{\mathrm{i}=1}^5 \mathrm{a}_{\mathrm{i}}^2}{5}-\left(\frac{\sum_{\mathrm{i}=1}^5 \mathrm{a}_{\mathrm{i}}}{5}\right)^2=12, \frac{\sum_{\mathrm{i}=1}^5 \mathrm{b}_{\mathrm{i}}^2}{5}-\left(\frac{\sum_{\mathrm{i}=1}^5 \mathrm{b}_{\mathrm{i}}}{5}\right)^2=20$

$\Rightarrow \quad \sum_{\mathrm{i}=1}^5 \mathrm{a}_{\mathrm{i}}^2=185 \quad , \quad \sum_{\mathrm{i}=1}^5 \mathrm{b}_{\mathrm{i}}^2=420$

Now, $\mathrm{C}=\left\{\mathrm{C}_1, \mathrm{C}_2, \ldots . . \mathrm{C}_{10}\right\}$

s.f. $\mathrm{C}_{\mathrm{i}}=\mathrm{a}_{\mathrm{i}}=3$ or irstive $\mathrm{b}_{\mathrm{i}}+2 \quad$ elements Last five elements

$\therefore \quad$ Mean of $\mathrm{C}, \overline{\mathrm{C}}=\frac{(\sum \mathrm{a}_{\mathrm{i}}-15)+(\sum \mathrm{b}_{\mathrm{i}}+10)}{10}$

$\overline{\mathrm{C}}=\frac{10+50}{10}=6$

$\therefore \quad \sigma^2=\frac{\sum_{i=1}^{10} C_i^2}{10}=(\bar{C})^2$

$=\frac{\sum\left(a_i-3\right)^2+\sum\left(b_i+2\right)^2}{10}-(6)^2$

$=\frac{\sum \mathrm{a}_{\mathrm{i}}{}^2+\sum \mathrm{b}_{\mathrm{i}}{}^2-6 \sum \mathrm{a}_{\mathrm{i}}+4 \sum \mathrm{b}_{\mathrm{i}}+65}{10}-36$

$=\frac{185+420-150+160+65}{10}-36$

$=32$

$\therefore \quad$ Mean + Variance $=\overline{\mathrm{C}}+\sigma^2=6+32=38$

Question 138

Let the six numbers $a_1, a_2, a_3, a_4, a_5, a_6$, be in A.P. and $a_1 + a_3 = 10$. If the mean of these six numbers is $\frac{19}{2}$ and their variance is σ^2, then $8\sigma^2$ is equal to :

A. 210

B. 220

C. 200

D. 105

Correct answer: A

Source solution

Sol. 1}

$$a + (a + 2d) = 10 \Rightarrow a + d = 5 \quad (1)$$

Mean $\Rightarrow \frac{\frac{6}{2}[2a+5d]}{6} = \frac{19}{2}$

$$2a + 5d = 19 \quad (2)$$

from (1) and (2)

$3d = 9 \Rightarrow d = 3; a = 2$

$\therefore \sigma^2 = \frac{\sum x_i^2}{n} - (\overline{x})^2$

$$= \frac{2^2 + 5^2 + 8^2 + 11^2 + 14^2 + 17^2}{6} - \left(\frac{19}{2}\right)^2$$

$$= \frac{699}{6} - \frac{361}{4}$$

$$= \frac{233}{2} - \frac{361}{4}$$

$$8\sigma^2 = 932 - 722 = 210$$

Question 139

The mean and variance of the marks obtained by the students in a test are 10 and 4 respectively. Later, the marks of one of the students is increased from 8 to 12 . If the new mean of the marks is 10.2, then their new variance is equal to :

A. 2,24

B. (1) 3.96

C. (2) 4.08

D. (3) 4.04

Correct answer: C

Source solution

Sol.

Let number of students be n.

Initial mean $= 10, \quad$ variance $= 4$

$$\Rightarrow \sum x = 10n$$

$$\text{Using } \sigma^2 = \frac{\sum x^2}{n} - (\text{mean})^2$$

$$4 = \frac{\sum x^2}{n} - 100 \Rightarrow \frac{\sum x^2}{n} = 104 \Rightarrow \sum x^2 = 104n$$

One value changes from 8 to 12.

$$\text{New sum } = 10n - 8 + 12 = 10n + 4$$

Given new mean $= 10.2$

$$\frac{10n+4}{n} = 10.2 \Rightarrow 10 + \frac{4}{n} = 10.2 \Rightarrow \frac{4}{n} = 0.2 \Rightarrow n = 20$$

$\text{New } \sum x = 10.2 \times 20 = 204$

$$\text{New } \sum x^2 = 104n - 8^2 + 12^2 = 104 \cdot 20 - 64 + 144$$

$$= 2080 - 64 + 144 = 2160$$

$$\text{नई variance } = \frac{2160}{20} - (10.2)^2$$

$$= 108 - 104.04 = 3.96$$

$$\boxed{\text{New variance} = 3.96}$$

Question 140

Let the mean and standard deviation of marks of class A of 100 students be respectively 40 and α (> 0), and the mean and standard deviation of marks of class B of n students be respectively 55 and $30-\alpha$. If the mean and variance of the marks of the combined class of $100+n$ studants are respectively 50 and 350 , then the sum of variances of classes A and B is :

A. 650

B. 450

C. 900

D. 500

Correct answer: D

Source solution

Sol. 4

$$
\begin{aligned}
&m_A = 40 S.d_A = \alpha > 0 n_A = 100 \\
&m_B = 55. S.d_B = 30-\alpha n_B = n \\
&m_{AVB} = 50 \quad \text{Variance}_{AVB} = 350 n_{AVB} = 100+n \\
&A = \{x_1, \ldots\ldots, x_{100}\} B = \{y_1 \ldots., y_n\} \\
&\sum x_i = 4000 \\
&\sum y_i = 55n \\
&\sum (x_i + y_i) = 50(100+n) \\
&4000 + 55n = 5000 + 50n
\end{aligned}
$$

Using formula of standard deviation

$$
\begin{aligned}
&5n = 1000 n = 200 \\
&\alpha^2 = \frac{\sum {x_i}^2}{100} - (40)^2 \quad \Big| \quad (30-\alpha)^2 \frac{\sum y_i^2}{200} - (55)^2 \\
&\sum {x_i}^2 = 100\left(1000+\alpha^2\right) \\
&\sum {y_i}^2 = 200\left((55)^2 + (30-\alpha)^2\right) \\
&350 = \frac{\sum ({x_i}^2 + {y_i}^2)}{300} - (50)^2 \\
&\sum x_i^2 + \sum y_i^2 = \left((50)^2 + 350\right)300 \\
&160000 + 100\alpha^2 + 200(55)^2 + 200(30-\alpha)^2 \\
&(50)^2 300 + 350 \times 300 \\
&1600 + \alpha^2 + 6050 + 2(30-\alpha)^2 = 7500 + 1050 \\
&\alpha^2 + 1800 - 120\alpha + 2\alpha^2 - 900 = 0 \\
&3\alpha^2 - 120\alpha + 900 = 0
\end{aligned}
$$

$\alpha^2 - 40\alpha + 300 = 0$
$(\alpha - 10)(\alpha - 30) = 0$
$\alpha = 10$ or $\alpha = 30$
if $\alpha = 10\,\mathrm{VarA} = 100 \& \mathrm{VarB} = 400$
$\mathrm{Var_A} + \mathrm{V_{arB}} = 500$

Question 141

Let $9 = x_1 < x_2 < \ldots\ldots < x_7 \ldots\ldots, x_7$ be in an A.P. with common difference d . If the standard deviation of $x_1 \cdot x_2 \ldots., x_7$ is 4 and mean is $\overline{x}$, then $\overline{x} + x_6$ is equal to :

A. $2\left(9 + \frac{8}{\sqrt{7}}\right)$

B. $18\left(1 + \frac{1}{\sqrt{3}}\right)$

C. 25

D. 34

Correct answer: D

Source solution

Sol. 4
Mean $\Rightarrow \bar{x} = \frac{\sum_{i=1}^{7} x_i}{7} = \frac{\frac{7}{2}[2a+6d]}{7} = a + 3d = x_4$
Variance $= \frac{\sum_{i=1}^{7} (x_i - \bar{x})^2}{7} = (4)^2 \Rightarrow \frac{\sum_{i=1}^{7} (x_i - x_4)^2}{7} = 16$
$\Rightarrow \frac{(3d)^2 + (2d)^2 + d^2 + 0 + d^2 + (2d)^2 + (3d)^2}{7} = 16$
$= 4d^2 = 16 \Rightarrow d = 2$
$\Rightarrow \bar{x} = 9 + 3(2) = 15$
$\& x_0 = a + 5d = 9 + 5(2) = 19 \Rightarrow \bar{x} + x_0 = 34$

Question 142

Let $X = \{11, 12, 13, \ldots, 40, 41\}$ and $Y = \{61, 62, 63, \ldots, 90, 91\}$ be the two sets of observations. If $\bar{x}$ and $\bar{y}$ are their respective means and σ^2 is the variance of all the observations in $X \cup Y$, then $\left|\bar{x} + \bar{y} - \sigma^2\right|$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 603

Source solution

603

$\bar{x} = \frac{11+12+\ldots\ldots+41}{31} \qquad \bar{y} = \frac{61+62+63+\ldots+91}{31}$

$= \frac{\frac{31}{2}(11+41)}{31} = \frac{52}{2} = 26 \quad = \frac{\frac{31}{2}(61+91)}{31} = \frac{152}{2} = 76$

$\sigma^2 = \frac{\sum x_i^2 + \sum y_i^2}{31+31} - \bar{x}^2$

$= \frac{\left(\sum_{n=1}^{41} n^2 - \sum_{n=1}^{10} n^2\right) + \left(\sum_{n=1}^{91} n^2 - \sum_{n=1}^{60} n^2\right)}{62} - \left(\frac{31\times26+76\times31}{62}\right)^2$

$\Rightarrow \frac{\frac{41\times42\times83}{6} - \frac{10\times11\times21}{6} + \frac{91\times92\times183}{6} - \frac{60\times61\times121}{6}}{62} - (51)^2$

$\Rightarrow \frac{7(41\times83-55)+61(91\times46-1210)}{62}$

$\Rightarrow \frac{7(3403-55)+61(4186-1210)}{62}$

$\Rightarrow \frac{7\times3348+61\times2976}{62} - 2601$

$\Rightarrow 3306 - 2601 = 705 \Rightarrow \sigma^2 = 705$

$\therefore \left|\bar{x} + \bar{y} - \sigma^2\right| = |26 + 76 - 705| = 603$

Question 143

The mean and variance of 5 observations are 5 and 8 respectively. If 3 observations are $1, 3, 5$ then the sum of cubes of the remaining two observations is

A. 1216

B. 1072

C. 1456

D. 1792

Correct answer: B

Source solution

Sol. 2

Let remaining two observations are a and b

$5 = \frac{1+3+5+a+b}{5}$

$\Rightarrow a + b = 16 \ldots$ (i)

$8 = \frac{1^2+3^2+5^2+a^2+b^2}{5} - 25$

$\Rightarrow a^2 + b^2 = 130$

$$a^2 + b^2 = 130 \quad \text{(ii)}$$

$(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow 256 = 130 + 2ab$

$ab = 63$

$a^3 + b^3 = (a + b)^3 - 3ab(a + b)$

$= (16)^3 - 3(63)(16)$

= 4096-3024

$\Rightarrow a^3 + b^3 = 1072$

Question 144

If the variance of the frequency distribution

x_i	2	3	4	5	6	7	8
Frequency f_i	3	6	16	α	9	5	6

is 3 , then α is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 5

Source solution

5

x_i	f_i	$d_i = x_i - 5$	$(f_i d_i)^2$	$f_i d_i$
2	3	-3	27	-9
3	6	-2	24	-12
4	16	-1	16	-16
5	α	0	0	0
6	9	1	9	9
7	5	2	20	10
8	6	3	54	18

$$\sigma^2 = \frac{\sum f_i d_i^2}{\sum f_i} - \left(\frac{\sum f_i d_i}{\sum f_i}\right)^2$$
$$= \frac{150}{45+\alpha} - 0 = 3$$
$$\Rightarrow 150 = 135 + 3\alpha$$
$$\Rightarrow 3\alpha = 15$$
$$\Rightarrow \alpha = 5$$

Question 145

For 20 observations mean and variance is given as 10 and 4 , later it was observed that by mistake 9 was taken in place of 11 , then the correct variance is

A. 3.98

B. 3.99

C. 4.01

D. 4.02

Correct answer: B

Source solution

Sol.
Mean $= 10 \Rightarrow \frac{\sum x_i}{20} = 10 \Rightarrow \sum x_i = 200$
Variance $= 4 \Rightarrow \frac{\sum x_i^2}{20} - 100 = 4 \Rightarrow \sum {x_i}^2 = 2080$
New mean $= \frac{200-9+11}{20} = \frac{202}{20} = 10.1$
New variance $= \frac{2080-81+121}{20} - (10.1)^2$

$$= 106 - 102.01$$
$$= 3.99$$

Question 146

The mean of five observations is 5 and their variance is 9.20 . If three of the given five observations are 1,3 and 8 , then a ratio of other two observations is :

A. $4:9$

B. $6:7$

C. $5:8$

D. $10:3$

Correct answer: A

Source solution

Sol. Let two observations are $\mathbf{x_1 \& x_2}$
mean $= \frac{\sum x_i}{5} = 5 \Rightarrow 1+3+8+x_1+x_2 = 25$

$$\Rightarrow x_1 + x_2 = 13 \quad (1)$$

$\text{variance}\left(\sigma^2\right) = \frac{\sum x_i^2}{5} - 25 = 9.20$
$\Rightarrow \quad \sum x_i^2 = 171$

$$\Rightarrow x_1^2 + x_2^2 = 97 \quad (2)$$

by (1) & (2)
$(x_1+x_2)^2 - 2x_1x_2 = 97$
or $x_1x_2 = 36$
$\therefore x_1 : x_2 = 4:9$

Probability

Questions 147-148 / 2 selected / 0.7% practice weight / Consolidate

Question 147

Three rotten apples are mixed accidently with seven good apples and four apples are drawn one by one without replacement. Let the random variable X denote the number of rotten apples. If μ and σ^2 represent mean and variance of X, respectively, then $10\left(\mu^2+\sigma^2\right)$ is equal to

A. 250

B. 25

C. 30

D. 20

Correct answer: D

Source solution

$$\text{Let } X = \text{number of rotten apples drawn.}$$

There are

$$N = 10$$

apples in total, of which

$$K = 3$$

are rotten and

$$n = 4$$

are drawn without replacement.

Hence,

$$X \sim \text{Hypergeometric}(N = 10,\ K = 3,\ n = 4).$$

For a hypergeometric distribution,

$$\mu = E(X) = \frac{nK}{N}.$$

Therefore,

$$\mu = \frac{4 \cdot 3}{10} = \frac{6}{5}.$$

Also,

$$\sigma^2 = \frac{nK(N-K)(N-n)}{N^2(N-1)}.$$

Substituting the given values,

$$\begin{aligned} \sigma^2 &= \frac{4 \cdot 3 \cdot 7 \cdot 6}{10^2 \cdot 9} \\ &= \frac{504}{900} \\ &= \frac{14}{25}. \end{aligned}$$

Now,

$$\mu^2 = \left(\frac{6}{5}\right)^2 = \frac{36}{25}.$$

Hence,

$$\begin{aligned}\mu^2 + \sigma^2 &= \frac{36}{25} + \frac{14}{25} \\ &= \frac{50}{25} \\ &= 2.\end{aligned}$$

Therefore,

$$10(\mu^2 + \sigma^2) = 10 \times 2 = 20.$$

$$\boxed{20}$$

Hence, the correct answer is

$$\boxed{20}.$$

Question 148

In a binomial distribution $\mathbf{B(n, p)}$, the sum and the product of the mean and the variance are 5 and 6 respectively, then $6(n+p-q)$ is equal to

A. 52

B. 50

C. 51

D. 53

Correct answer: A

Source solution

Sol. 1}
Given
$\mathbf{np + npq = 5}$

$$\Rightarrow \mathbf{np(1+q) = 5} \qquad \mathbf{(i)}$$

and (np) (npq) = 6

$$\Rightarrow \mathbf{n^2p^2q = 6} \qquad \mathbf{(ii)}$$

(i) $\overset{2}{\div}$ (ii)
$\frac{(1+9)^2}{9} = \frac{25}{6}$
$\Rightarrow 6q^2 - 13q + 6 = 0$
$\Rightarrow \mathbf{q} = \frac{2}{3}, \frac{3}{2}$ (reject)
$\mathbf{P} = 1 - \frac{2}{3} = \frac{1}{3}$
$\frac{n}{3}\left(1 + \frac{2}{3}\right) = 5$
$\Rightarrow \mathbf{n} = 9$
$6(n+p-q) = 52$

Inverse Trigonometric Functions

Questions 149-149 / 1 selected / 0.3% practice weight / Consolidate

Question 149

Let $S=\left\{x \in\left(-\frac{\pi}{2}, \frac{\pi}{2}\right): 9^{1-\tan^2 x}+9^{\tan^2 x}=10\right\}$ and $\beta=\sum_{x \in S} \tan^2\left(\frac{x}{3}\right)$, then $\frac{1}{6}(\beta-14)^2$ is equal to

A. 32

B. 8

C. 64

D. 16

Correct answer: A

Source solution

Sol. Let $9^{\tan^2 x}=P$

$\frac{9}{P}+P=10$

$P^2-10P+9=0$

$(P-9)(P-1)=0$

$P=1,9$

$9^{\tan^2 x}=1, 9^{\tan^2 x}=9$

$\tan^2 x=0, \tan^2 x=1$

$x=0, \pm\frac{\pi}{4} \quad \therefore x \in\left(-\frac{\pi}{2}, \frac{p}{2}\right)$

$\beta=\tan^2(0)+\tan^2\left(+\frac{\pi}{12}\right)+\tan^2\left(-\frac{\pi}{12}\right)$

$=0+2(\tan 15^\circ)^2$

$2(2-\sqrt{3})^2$

$2(7-4\sqrt{3})$

Than $\frac{1}{6}(14-8\sqrt{3}-14)^2=32$

Matrices

Questions 150-152 / 3 selected / 1.0% practice weight / Consolidate

Question 150

Let A be a matrix of order 3×3 and $|\mathbf{A}| = 5$. If $|2\,\mathbf{adj}(3\,\mathbf{A}\,\mathbf{adj}(2\,\mathbf{A}))| = 2^{\alpha} \cdot 3^{\beta} \cdot 5^{\gamma} \alpha, \beta, \gamma \in \mathbf{N}$ then $\alpha + \beta + \gamma$ is equal to

A. 25

B. 26

C. 27

D. 28

Correct answer: C

Source solution

Sol. $|\,\mathbf{2}$ adj (3A adj(2A))|

$\mathbf{2^3}.|\,\mathbf{3\,A}$ adj(2A)| $^{\mathbf{2}}$

$2^3 \cdot \left(3^3\right)^2 \cdot |\,\mathbf{A}|^2 \cdot |\,\mathbf{adj}(2\,\mathbf{A})|^2$

$2^3.3^6.|\mathbf{A}|^2.\left(|2\,\mathbf{A}|^2\right)^2$

$2^3 \cdot 3^6 \cdot |\,\mathbf{A}|^2\left[\left(2^3\right)^2 \cdot |\,\mathbf{A}|^2\right]^2$

$2^3.3^6.|\mathbf{A}|^2.2^{12}.|\mathbf{A}|^4$

$2^{15} \cdot 3^6 \cdot |\,\mathbf{A}|^6$

$2^{15}.3^6.5^6 = 2^{\alpha}.3^{\beta}.5^{\gamma}$

$\alpha = 15, \quad \beta = 6, \quad \gamma = 6$

$\alpha + \beta + \gamma = 27$

Option (3)

Question 151

If system of equations $x+y+z=6, x+2y+3z=10, 3x+2y+\lambda z=\mu$ has more than 2 solutions, then $(\mu-\lambda^2)$ is .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 13

Source solution

Solution:

The system of equations has more than 2 solutions

$\therefore D=D_3=0$

$$\Rightarrow \begin{vmatrix}1 & 1 & 1\\ 1 & 2 & 3\\ 3 & 2 & \lambda\end{vmatrix}=0 \Rightarrow 2\lambda-6-\lambda+9+2-6=0$$

$\Rightarrow \lambda=1$

$$\begin{vmatrix}1 & 1 & 6\\ 1 & 2 & 10\\ 3 & 2 & \mu\end{vmatrix}=0 \Rightarrow 2\mu-20-\mu+30-24=0$$

$\Rightarrow \mu=14$

So, $\mu-\lambda^2=13$

Question 152

Let $|\mathbf{A}| = 6$, where A is a 3×3 matrix. If $\left|\operatorname{adj}\left(3 \operatorname{adj}\left(\mathbf{A}^2 . \operatorname{adj}(2\ \mathbf{A})\right)\right)\right| = 2^{\mathrm{m}} . 3^{\mathrm{n}}, \mathrm{m}, \mathrm{n} \in \mathbf{N}$, then m + n is equal to

_____

.

Numerical answer: _____

Enter an integer. Use the units and scaling requested in the question.

Correct answer: 62

Source solution

$$\operatorname{adj} 2\ \mathbf{A} = 2^2 \operatorname{adj}\ \mathbf{A} \quad \because \operatorname{adj} \mathrm{kA} = \mathrm{k}^{\mathrm{n}-1}(\operatorname{adj}\ \mathbf{A})$$
$$= 4 \operatorname{adj}\ \mathbf{A}$$

Now $\mathbf{A}^2(\operatorname{adj} 2\ \mathbf{A}) = 4\ \mathbf{A}(\operatorname{adj}\ \mathbf{A})$
$$= 4\ \mathbf{A}|\ \mathbf{A}|\mathrm{I}_3$$
$$= 24\ \mathbf{A}$$

Now 3 adj $\left(\mathbf{A}^2(\operatorname{adj} 2\ \mathbf{A})\right) = 3 \operatorname{adj}(24\ \mathbf{A})$
$$= 3 . (24)^2 \operatorname{adj}\ \mathbf{A}$$

Now $\left|\operatorname{adj}\left(3 \operatorname{adj}\left(\mathbf{A}^2(\operatorname{adj} 2\mathrm{A})\right)\right)\right|$
$$= \left|\operatorname{adj}\left(3 \cdot (24)^2 \operatorname{adj}\ \mathbf{A}\right)\right|$$
$$= \left|\left(3 \cdot (24)^2\right)^2 (\operatorname{adj} \operatorname{adj}\ \mathbf{A})\right|$$
$$= \left|3^6 \cdot 2^{12} \operatorname{adj} \operatorname{adj}\ \mathbf{A}\right|$$
$$= \left(3^6 \cdot 2^{12}\right)^3 |\operatorname{adj} \operatorname{adj}\ \mathbf{A}|$$
$$= 3^{18} \cdot 2^{36} \cdot (\ \mathbf{A})^4$$
$$3^{22} \cdot 2^{40}$$
$$\therefore\ \mathrm{m} + \mathrm{n} = 62$$

Determinants

Questions 153-159 / 7 selected / 2.3% practice weight / Next

Question 153

Let $\lambda, \mu \in \mathbf{R}$. If the system of equations

$3x + 5y + \lambda z = 3$

$7x + 11y - 9z = 2$

$97x + 155y - 189z = \mu$

has infinitely many solutions, then $\mu + 2\lambda$ is equal to :

A. 25

B. 24

C. 27

D. 22

Correct answer: A

Source solution

Sol. $3x + 5y + \lambda z = 3$

$7x + 11y - 9z = 2$

$97x + 155y - 189z = \mu$

$93x + 155y + 31\lambda z = 93$

$97x + 155y - 189z = \mu$

$\frac{\quad\quad\quad\quad -+-}{-4x+(31\lambda+189)z=93-\mu}$

$1085x + 1705y - 1395z = 310$

$1067x + 1705y - 2079z = 11\mu$

$\frac{\quad\quad - \quad + \quad -}{18x+684z=310-11\mu}$

$-36x + 9(31\lambda + 189)z = 9(93 - \mu)$

$36x + 1368z = 2(310 - 11\mu)$

$(279\lambda + 3069)z = 1457 - 31\mu$

for infinite solutions -

$\lambda = \frac{-3069}{279} = \frac{-341}{31}$

$\mu = \frac{1457}{31}$

$\mu + 2\lambda = \frac{1457-682}{31} = \frac{775}{31} = 25$

Question 154

If the system of equations

$2x + 7y + \lambda z = 3$

$3x + 2y + 5z = 4$

$x + \mu y + 32z = -1$

has infinitely many solutions, then ($\lambda - \mu$) is equal to :

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 38

Source solution

$$D = D_1 = D_2 = D_3 = 0$$

$$D_3 = \begin{vmatrix} 2 & 7 & 3 \\ 3 & 2 & 4 \\ 1 & \mu & -1 \end{vmatrix} = 0 \Rightarrow \mu = -39$$

$$D = \begin{vmatrix} 2 & 7 & \lambda \\ 3 & 2 & 5 \\ 1 & -39 & 32 \end{vmatrix} = 0 \Rightarrow \lambda = -1$$

$$\lambda - \mu = 38$$

Question 155

The values of $\mathbf{m, n}$, for which the system of equations
$\mathbf{x+y+z=4}$,
$2x+5y+5z=17$,
$\mathbf{x+2y+mz=n}$
has infinitely many solutions, satisfy the equation :

A. $m^2+n^2-m-n=46$

B. $m^2+n^2+m+n=64$

C. $m^2+n^2+mn=68$

D. $m^2+n^2-mn=39$

Correct answer: D

Source solution

Sol. $\mathbf{D=|1\quad 1\quad 12\quad 5\quad 51\quad 2\quad m|=0\Rightarrow\ m=2}$
$\mathbf{D_3=|1\quad 1\quad 42\quad 5\quad 171\quad 2\quad n|=0\Rightarrow n=7}$

Question 156

If the system of equations

$11x + y + \lambda z = -5$

$2x + 3y + 5z = 3$

$8x - 19y - 39z = \mu$

has infinitely many solutions, then $\lambda^4 - \mu$ is equal to :

A. 49

B. 45

C. 47

D. 51

Correct answer: C

Source solution

Sol. $11x + y + \lambda z = -5$

$2x + 3y + 5z = 3$

$8x - 19y - 39z = \mu$

for infinite sol.

$D = |11 \quad 1 \quad \lambda 2 \quad 3 \quad 58 \quad -19 \quad -39| = 0$

$\Rightarrow 11(-117 + 95) - 1(-78 - 40) + \lambda(-38 - 24)$

$\Rightarrow 11(-22) + 118 - \lambda(62) = 0$

$\Rightarrow 62\lambda = 118 - 242$

$\Rightarrow \lambda = \frac{-124}{62} = -2$

$D_1 = |-5 \quad 1 \quad -23 \quad 3 \quad 5\mu \quad -19 \quad -39| = 0$

$\Rightarrow -5(-117 + 95) - 1(-117 - 5\mu) - 2(-57 - 3\mu) = 0$

$\Rightarrow -5(-22) + 117 + 5\mu + 114 + 6\mu = 0$

$\Rightarrow 11\mu = -110 - 231 = -341$

$\Rightarrow \mu = -31$

$\lambda^4 - \mu = (-2)^4 - (-31) = 16 + 31 = 47$

Question 157

Let A be a non-singular matrix of order 3. If $\det(3\,\mathrm{adj}(2\,\mathrm{adj}((\det\ \mathrm{A})\mathrm{A}))) = 3^{-13}\cdot 2^{-10}$ and $\det(3\,\mathrm{adj}(2A)) = 2^m\cdot 3^n$, then $|3m+2n|$ is equal to

.

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 14

Source solution

$$|3\,\mathrm{adj}(2\,\mathrm{adj}(|\,\mathrm{A}|\mathrm{A}))| = |\,3\,\mathrm{adj}\,(2|\,\mathrm{A}|^2\,\mathrm{adj}(\,\mathrm{A})\,|$$
$$= |3.2^2|\mathrm{A}|^4\,\mathrm{adj}\,(\mathrm{adj}(\,\mathrm{A})\,| = 2^6 3^3|\mathrm{A}|^{12}|\,\mathrm{A}|^4$$
$$= 2^6 3^3|\,\mathrm{A}|^{16} = 2^{-10}3^{-13}$$
$$\Rightarrow |\mathrm{A}|^{16} = 2^{-16}3^{-16} \Rightarrow |\,\mathrm{A}| = 2^{-1}3^{-1}$$

Now $|3\,\mathrm{adj}(2\,\mathrm{A})| = |3.2^2\,\mathrm{adj}(\,\mathrm{A})|$
$$= 2^6 3^3|\,\mathrm{A}|^2 = 2^{-m}3^{-n}$$
$$\Rightarrow 2^6 3^3 2^{-2}3^{-2} = 2^{-m}3^{-n}$$
$$\Rightarrow 2^{-m}3^{-n} = 2^4 3^1$$
$$\Rightarrow m = -4, n = -1$$
$$\Rightarrow |3\,m + 2n| = |-12-2| = 14$$

Question 158

If A is a square matrix of order 3 such that $\det(\mathrm{A}) = 3$ and
$\det\left(\operatorname{adj}\left(-4\operatorname{adj}\left(-3\operatorname{adj}\left(3\operatorname{adj}\left((2\ \mathrm{A})^{-1}\right)\right)\right)\right)\right) = 2^{\mathrm{m}}3^{\mathrm{n}}$, then $\mathrm{m}+\mid 2\mathrm{n}$ is equal to:

A. 3

B. 2

C. 4

D. 6

Correct answer: C

Source solution

Sol. $|\mathrm{A}| = 3$

$|\operatorname{adj}\left(-4\operatorname{adj}\left(-3\operatorname{adj}\left(3\operatorname{adj}\left((2A)^{-1}\right)\right)\right)\right)|$

$|-4\operatorname{adj}\left(-3\operatorname{adj}\left(3\operatorname{adj}(2\ \mathrm{A})^{-1}\right)\right|^2$

$4^6\left|\operatorname{adj}\left(-3\operatorname{adj}\left(3\operatorname{adj}(2A)^{-1}\right)\right)\right|^2$

$2^{12}\cdot 3^{12}\left|3\operatorname{adj}(2\ \mathrm{A})^{-1}\right|^8$

$2^{12}\cdot 3^{12}\cdot 3^{24}\left|\operatorname{adj}(2\ \mathrm{A})^{-1}\right|^8$

$2^{12}\cdot 3^{36}\left|(2\ \mathrm{A})^{-1}\right|^{16}$

$2^{12}\cdot 3^{36}\frac{1}{|2\ \mathrm{A}|^{16}}$

$2^{12}\cdot 3^{36}\frac{1}{2^{48}|\ \mathrm{A}|^{16}}$

$2^{12}\cdot 3^{36}\frac{1}{2^{48}.3^{16}}$

$\frac{3^{20}}{2^{36}} = 2^{-36}\cdot 3^{20}$

$\mathrm{m} = -36 \quad \mathrm{n} = 20$

$m + 2n = 4$

Question 159

Let the determinant of a square matrix A of order m be $\mathbf{m-n}$, where m and n satisfy $\mathbf{4\,m+n=22}$ and $\mathbf{17\,m+4n=93}$. If det $\mathbf{(n\,adj(adj(mA)))=3^a5^b6^c}$. then $\mathbf{a+b+c}$ is equal to:

A. 96

B. 101

C. 109

D. 84

Correct answer: A

Source solution

Sol. $|\mathbf{A}|=\mathbf{m-n}$

$4m+n=22$

$\mathbf{17\,m+4n=93}$

$\mathbf{m=5, n=2}$

$|\mathbf{A}|=\mathbf{3}$

$|\,\mathbf{2}\,\text{adj}\,(\mathbf{adj}\,\mathbf{5}\,\mathbf{A}))|=\mathbf{2^5}|\mathbf{5}\,\mathbf{A}|^{\mathbf{16}}$

$$=\mathbf{2^5\cdot 5^{80}}|\,\mathbf{A}|^{\mathbf{16}}$$
$$=\mathbf{2^5\cdot 5^{80}\cdot 3^{16}}$$
$$=\mathbf{3^{11}\cdot 5^{80}\cdot 6^5}$$

$a+b+c=96$

Application of Derivatives

Questions 160-164 / 5 selected / 1.7% practice weight / Consolidate

Question 160

Let $y = f(x)$ be a thrice differentiable function in $(-5, 5)$. Let the tangents to the curve $\mathbf{y = f(x)}$ at $(1, f(1))$ and $(3, f(3))$ make angles $\frac{\pi}{6}$ and $\frac{\pi}{4}$, respectively with positive x -axis. If $27\int_1^3 \left((f'(t))^2 + 1\right) f''(t)dt = \alpha + \beta\sqrt{3}$ where $\alpha,$ β are integers, then the value of $\alpha + \beta$ equals

A. -14

B. 26

C. -16

D. 36

Correct answer: B

Source solution

Sol. $y = f(x) \Rightarrow \frac{dy}{dx} = f'(x)$

$$\left.\frac{dy}{dx}\right)_{(1,f(1))} = f'(1) = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}} \Rightarrow f'(1) = \frac{1}{\sqrt{3}}$$

$$\left.\frac{dy}{dx}\right|_{(3,f(3))} = f'(3) = \tan\frac{\pi}{4} = 1 \Rightarrow f'(3) = 1$$

$$27\int_1^3 \left(\left(f'(t)\right)^2 + 1\right) f''(t)dt = \alpha + \beta\sqrt{3}$$

$$I = \int_1^3 \left(\left(f'(t)\right)^2 + 1\right) f''(t)dt$$

$$f'(t) = z \Rightarrow f'(t)dt = dz$$

$$z = f'(3) = 1$$

$$z = f'(1) = \frac{1}{\sqrt{3}}$$

$$I = \int_{1/\sqrt{3}}^{1} \left(z^2 + 1\right)dz = \left(\frac{z^3}{3} + z\right)_{1/\sqrt{3}}^{1}$$

$$= \left(\frac{1}{3} + 1\right) - \left(\frac{1}{3}\cdot\frac{1}{3\sqrt{3}} + \frac{1}{\sqrt{3}}\right)^{1}$$

$$= \frac{4}{3} - \frac{10}{9\sqrt{3}} = \frac{4}{3} - \frac{10}{27}\sqrt{3}$$

$$\alpha + \beta\sqrt{3} = 27\left(\frac{4}{3} - \frac{10}{27}\sqrt{3}\right) = 36 - 10\sqrt{3}$$

$$\alpha = 36, \beta = -10$$

$$\alpha + \beta = 36 - 10 = 26$$

\setcounter{enumi}{9}

Question 161

Let the area of a $\triangle PQR$ with vertices $P(5,4), Q(-2,4)$ and $R(a,b)$ be 35 square units. If its orthocenter and centroid are $O\left(2, \frac{14}{5}\right)$ and $C(c,d)$ respectively, then $c+2d$ is equal to

A. $\frac{7}{3}$

B. 3

C. 2

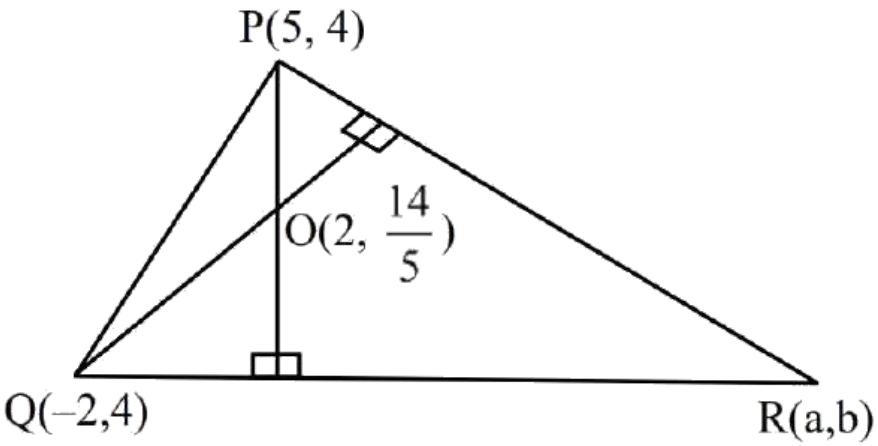
D. $\frac{8}{3}$

Correct answer: B

Source solution

Sol.

Equation of lines $QR = 5x + 2y + 2 = 0$
Equation of lines $PR = 10x - 3y - 38 = 0$
$\therefore$ Point $R(2,-6)$
Centroid $= \left(\frac{5-2+2}{3}, \frac{4+4-6}{3}\right)$
$= \left(\frac{5}{3}, \frac{2}{3}\right)$
$c+2d = \frac{5}{3} + \frac{4}{3} = 3$



Question 162

Let $a > 0$. If the function $f(x) = 6x^3 - 45ax^2 + 108a^2x + 1$ attains its local maximum and minimum values at the points $\mathbf{x_1}$ and $\mathbf{x_2}$ respectively such that $\mathbf{x_1x_2 = 54}$, then $\mathbf{a + x_1 + x_2}$ is equal to :-

A. 15

B. 18

C. 24

D. 13

Correct answer: B

Source solution

Sol. $f'(\mathbf{x}) = 18\mathbf{x}^2 - 90\mathbf{a}\mathbf{x} + 108\mathbf{a}^2 = 0$

$\mathbf{x = 2a \& x = 3a}$

$\mathbf{x_1 = 2a \quad x_2 = 3a}$

$\mathbf{x_1x_2 = 54}$

$\mathbf{6a^2 = 54}$

$\mathbf{a = 3}$

$\mathbf{a + x_1 + x_2}$

$\mathbf{3 + 2 \times 3 + 3 \times 3 = 18}$

option (2)

Question 163

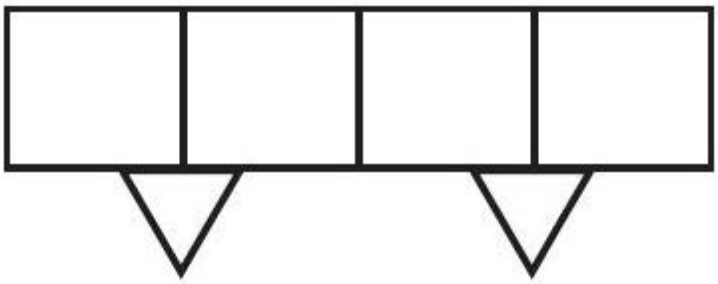
A person forgets his 4-digit ATM pin code. But he remembers that in the code all the digits are different, the greatest digit is 7 and the sum of the first two digits is equal to the sum of the last two digits. Then the maximum number of trials necessary to obtain the correct code is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 72

Source solution



Sum of first two digits

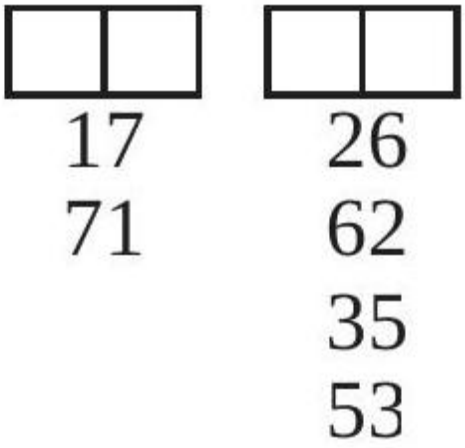
Sum of last two digits $= \alpha$

Case-I : $\alpha = 7$

$2 \times 12 = 24$ ways.

7	0		
0	7		
		1	6
		2	5
		3	4
		4	3
		5	2
		6	1

Case-II : $\alpha = 8$





2×8 ways

$= 16$ ways

Case-III : $\alpha = 9$

$\square$	
\cline { 1 - 2 } 27	
\cline { 1 - 1 } 72	36
	63
	45
	54

2×8 ways		45	
	54		

$= 16$ ways

Case-IV : $\alpha = 10$

$\square$	$\square$ 37 73

46
64

2×4 ways

8 ways

Case-V : $\alpha = 11$



47 56

74 65

2×4 ways

$= 8$ ways

Ans. $24 + 16 + 16 + 8 + 8 = 72$

Question 164

Let E be an ellipse whose axes are parallel to the co-ordinates axes, having its center at $(3, -4)$, one focus at $(4, -4)$ and one vertex at $(5, -4)$. If $mx - y = 4,\ m > 0$ is a tangent to the ellipse E, then the value of $5\ m^2$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 3

The source provides a brief answer only. A detailed worked solution is not available for this question.

Source solution

Integrals

Questions 165-165 / 1 selected / 0.3% practice weight / Consolidate

Question 165

Let $\mathbf{I(x)} = \int \frac{3dx}{(4x+6)\left(\sqrt{4x^2+8x+3}\right)}$ and $\mathbf{I(0)} = \frac{\sqrt{3}}{4} + \mathbf{20}$. If $\mathbf{I}\left(\frac{1}{2}\right) = \frac{a\sqrt{2}}{b} + \mathbf{c}$, where a, b, c $\in \mathbf{N}, \mathbf{gcd(a, b)} = \mathbf{1}$, then $\mathbf{a + b + c}$ is equal to :

A. 29

B. 28

C. 31

D. 30

Correct answer: C

Source solution

$$I(x) = \int \frac{3\,dx}{(4x+6)\sqrt{4(x+1)^2-1}}.$$

$$\text{Let } 4x+6 = \frac{1}{t}. \quad \Rightarrow \quad x = \frac{\frac{1}{t}-6}{4}, \qquad dx = -\frac{dt}{4t^2}, \qquad x+1 = \frac{1-2t}{4t}.$$

$$
\begin{aligned}
I(x) &= -\frac{3}{4}\int \frac{dt}{t\sqrt{\frac{(1-2t)^2}{4t^2}-1}} \\
&= -\frac{3}{4}\int \frac{2t\,dt}{t\sqrt{1-4t}} \\
&= -\frac{3}{2}\int \frac{dt}{\sqrt{1-4t}} \\
&= \frac{3}{4}\sqrt{1-4t}+C \\
&= \frac{3}{4}\sqrt{1-\frac{4}{4x+6}}+C \\
&= \frac{3}{4}\sqrt{\frac{4x+2}{4x+6}}+C.
\end{aligned}
$$

$$
I(0) = \frac{3}{4}\sqrt{\frac{2}{6}}+C = \frac{\sqrt{3}}{4}+C.
$$

Using the given initial condition, $C = 20$.

$$
\therefore \quad I(x) = \frac{3}{4}\sqrt{\frac{4x+2}{4x+6}}+20.
$$

$$
\begin{aligned}
I\left(\frac{1}{2}\right)
&= \frac{3}{4}\sqrt{\frac{4}{8}} + 20 \\
&= \frac{3}{4\sqrt{2}} + 20 \\
&= \frac{3\sqrt{2}}{8} + 20.
\end{aligned}
$$

$$
a = 3, \qquad b = 8, \qquad c = 20.
$$

$$
\boxed{a + b + c = 3 + 8 + 20 = 31}
$$

Application of Integrals

Question 166

Let $P_1 : y = 4x^2$ and $P_2 : y = x^2 + 27$ be two parabolas. If the area of the bounded region enclosed between $\mathbf{P_1}$ and $\mathbf{P_2}$ is six times the area of the bounded region enclosed between the line $\mathbf{y = \alpha x, \alpha > 0}$ and $\mathbf{P_1}$, then α is equal to :

A. 8

B. 15

C. 12

D. 6

Correct answer: C

Source solution

Area bounded between $\mathbf{P_1 \& P_2}$ is

$$\int_{-3}^{3} \left((x^2 + 27) - (4x^2)\right) dx$$

(P.O.I. of $\mathbf{P_1 \& P_2}$ is $\mathbf{x = \pm 3}$)

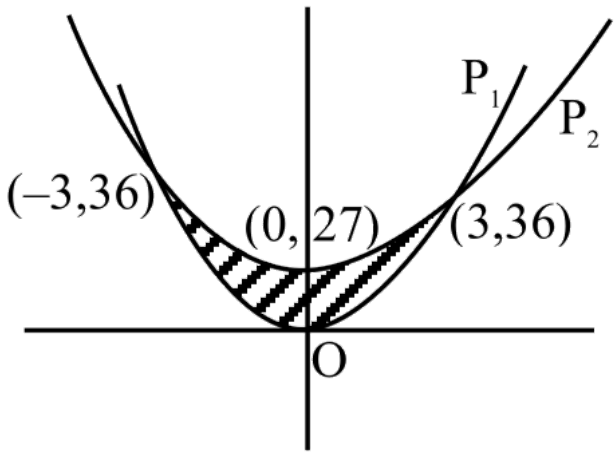
$$= 2\int_{0}^{3} (27 - 3x^2) dx = 2\left[27x - x^3\right]_0^3$$
$$= 2[81 - 27] = 108$$

$\therefore$ Area bounded between $\mathbf{P_1}$ & L is 18 sq. units
(Area between $\mathbf{x^2 = 4}$ ay & line $\mathbf{x = my}$) is $\frac{8a^2}{3\,m^3}$

$$\therefore \textbf{Area between } x^2 = \frac{y}{4} \& x = \frac{y}{\alpha} \textbf{ is}$$

$\frac{8 \cdot \left(\frac{1}{16}\right)^2}{3 \cdot \left(\frac{1}{\alpha}\right)^3} = 18$

$$\Rightarrow \frac{\frac{8}{16.16}}{\frac{3}{\alpha^3}} = 18 \Rightarrow \alpha^3 = 2^6.3^3$$
$$\Rightarrow \alpha = 12$$


P1
P2
(–3,36)
(0, 27)
(3,36)
O

Question 167

Let the area of the region $\{(x, y) : 2y \leq x^2 + 3, y + |x| \leq 3, y \geq |x - 1|\}$ be A . Then 6 A is equal to:

A. 16

B. 12

C. 18

D. 14

Correct answer: D

Source solution

Sol.

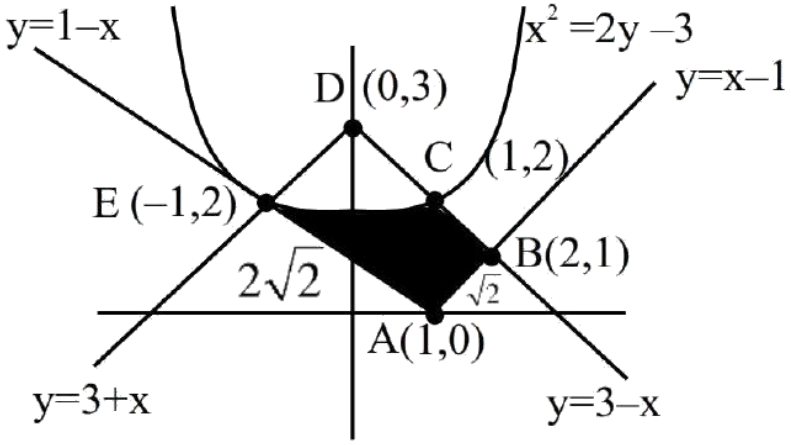
$\mathbf{A} \Rightarrow$ Rectangle ABDE - Area of region EDC

$A \Rightarrow 4 - 2\int_0^1 (3 - x) - \left(\frac{x^2+3}{2}\right) dx$

$A \Rightarrow 4 - 2\left\{3x - \frac{x^2}{2} - \frac{x^3}{6} - \frac{3}{2}x\right\}_0^1$

$\mathbf{A} \Rightarrow 4 - 2\left\{3 - \frac{1}{2} - \frac{1}{6} - \frac{3}{2}\right\} = \frac{7}{3}$

So $\mathbf{6\,A = 14}$



Question 168

The centre of a circle C is at the centre of the ellipse $\mathbf{E}: \frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$. Let C pass through the foci $\mathbf{F_1}$ and $\mathbf{F_2}$ of E such that the circle C and the ellipse E intersect at four points. Let P be one of these four points. If the area of the triangle $\mathbf{PF_1\ F_2}$ is 30 and the length of the major axis of E is 17 , then the distance between the foci of E is :

A. 26

B. 13

C. 12

D. $\frac{13}{2}$

Correct answer: B

Source solution

Sol.

$$\frac{1}{2}PF_1 \cdot PF_2 = 30$$
$$PF_1 + PF_2 = 17$$
$$PF_1 = 12 PF_2 = 5$$
$$F_1\ F_2 = 13$$

option (2) is the correct answer.

Question 169

On the ellipse $\frac{x^2}{8}+\frac{y^2}{4}=1$ let P be a point in the second quadrant such that the tangent at P to the ellipse is perpendicular to the line $x+2y=0$. Let S and S' be the foci of the ellipse and e be its eccentricity. If A is the area of the triangle SPS' then, the value of $\left(5-e^2\right).A$ is :

A. 6

B. 12

C. 14

D. 24

Correct answer: A

Source solution

Sol.

Equation of tangent : $y=2x+6$
at P
$\therefore P(-8/3,2/3)$
$e=\frac{1}{\sqrt{2}}$
$S\&S'=(-2,0)\&(2,0)$

Area of $\Delta SPS'=\frac{1}{2}\times 4\times\frac{2}{3}$
$A=\frac{4}{3}$
$\therefore \left(5-e^2\right)A=\left(5-\frac{1}{2}\right)\frac{4}{3}=6$

Question 170

Let the ellipse $E: x^2 + 9y^2 = 9$ intersect the positive x - and y -axes at the points A and B respectively Let the major axis of E be a diameter of the circle C. Let the line passing through A and B meet the circle C at the point P . If the area of the triangle which vertices $\mathbf{A, P}$ and the origin O is $\frac{m}{n}$, where m and n are coprime, then $\mathbf{m - n}$ is equal to

A. 18

B. 16

C. 17

D. 15

Correct answer: C

Source solution

Sol.

For line $\mathbf{ABx + 3y = 3}$ and circle is $\mathbf{x^2 + y^2 = 9}$
$(3 - 3y)^2 + y^2 = 9$
$\Rightarrow 10y^2 - 18y = 0$
$\Rightarrow y = 0, \frac{9}{5}$
$\therefore$ Area $= \frac{1}{2} \times 3 \times \frac{9}{5} = \frac{27}{10}$
$\mathbf{m - n = 17}$

Question 171

Let the ellipse, $E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$ and $\mathbf{E_2 : \frac{x^2}{A^2} + \frac{y^2}{B^2} = 1,\ A < B}$ have same eccentricity $\frac{1}{\sqrt{3}}$. Let the product of their lengths of latus rectums be $\frac{32}{\sqrt{3}}$, and the distance between the foci of $\mathbf{E_1}$ be 4. If $\mathbf{E_1}$ and $\mathbf{E_2}$ meet at $\mathbf{A, B, C}$ and D , then the area of the quadrilateral ABCD equals:

A. $6\sqrt{6}$

B. $\frac{18\sqrt{6}}{5}$

C. $\frac{12\sqrt{6}}{5}$

D. $\frac{24\sqrt{6}}{5}$

Correct answer: D

Source solution

Sol. $\mathbf{2ae = 4}$

$\mathbf{2a\left(\frac{1}{\sqrt{3}}\right) = 4}$

$\mathbf{\Rightarrow a = 2\sqrt{3}}$

$\mathbf{\Rightarrow 1 - \frac{b^2}{12} = \frac{1}{3} \Rightarrow b^2 = 8}$

Now $\mathbf{\frac{2b^2}{a} \cdot \frac{2A^2}{B} = \frac{32}{\sqrt{3}} \Rightarrow 2\left(\frac{8}{2\sqrt{3}}\right)\frac{2A^2}{B} = \frac{32}{\sqrt{3}}}$

$\mathbf{\Rightarrow A^2 = 2B}$

$\mathbf{1 - \frac{A^2}{B^2} = \frac{1}{3} \Rightarrow 1 - \frac{2B}{B^2} = \frac{1}{3} \Rightarrow B = 3}$

$\mathbf{\Rightarrow A^2 = 6}$

$$\frac{x^2}{12} + \frac{y^2}{8} = 1 \qquad (1)$$

$$\frac{x^2}{6} + \frac{y^2}{9} = 1 \qquad (2)$$

On solving (1) & (2) we get

$(x, y) \equiv \left(\frac{\sqrt{6}}{\sqrt{5}}, \frac{6}{\sqrt{5}}\right), \left(\frac{-\sqrt{6}}{\sqrt{5}}, \frac{6}{\sqrt{5}}\right), \left(\frac{\sqrt{6}}{\sqrt{5}}, \frac{-6}{\sqrt{5}}\right), \left(\frac{-\sqrt{6}}{\sqrt{5}}, \frac{-6}{\sqrt{5}}\right)$
The four points are vertices of rectangle and its area $= \frac{24\sqrt{6}}{5}$

Question 172

Let for two distinct values of p the lines $y = x + p$ touch the ellipse $E: \frac{x^2}{4^2} + \frac{y^2}{3^2} = 1$ at the points A and B . Let the line $y = x$ intersect E at the points C and D . Then the area of the quadrilateral ABCD is equal to

A. 36

B. 24

C. 48

D. 20

Correct answer: B

Source solution

Sol.
Point of contact are $\left(\frac{\mp a^2 m}{\sqrt{a^2 m^2 + b^2}}, \frac{\pm b^2}{\sqrt{a^2 m^2 + b^2}}\right)$
$A\left(\frac{-16}{5}, \frac{9}{5}\right) B\left(\frac{16}{5}, \frac{-9}{5}\right)$
Point D is $\left(\frac{12}{5}, \frac{12}{5}\right)$
Area of $ABD = \frac{1}{2}\left|-\frac{16}{5} \quad \frac{9}{5} \quad 1\frac{16}{5} \quad \frac{-9}{5} \quad 1\frac{12}{5} \quad \frac{12}{5} \quad 1\right|$
$= 12$

Area of ABCD is $= 24$

option (2) is the correct answer.

Question 173

Consider a hyperbola $\mathbf{H : x^2 - 2y^2 = 4}$. Let the tangent at a point $\mathbf{P(4, \sqrt{6})}$ meet the x -axis at Q and latus rectum at $\mathbf{R\,(x_1, y_1), x_1 > 0}$. If F is a focus of H which is nearer to the point P , then the area of $\mathbf{\triangle QFR}$ is equal to

A. $4\sqrt{6}$

B. $\sqrt{6} - 1$

C. $\frac{7}{\sqrt{6}} - 2$

D. $4\sqrt{6} - 1$

Correct answer: C

Source solution

Sol.

$\frac{x^2}{4} - \frac{y^2}{2} = 1$

$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{\frac{3}{2}}$

$\therefore$ Focus $F(ae, 0) \Rightarrow F(\sqrt{6}, 0)$

equation of tangent at P to the hyperbola is

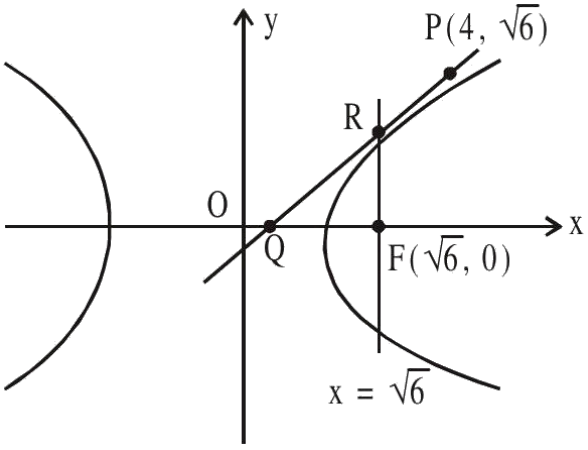
$2x - y\sqrt{6} = 2$

tangent meet x -axis at $Q(1, 0)$

& latus rectum $x = \sqrt{6}$ at $R\left(\sqrt{6}, \frac{2}{\sqrt{6}}(\sqrt{6} - 1)\right)$

$\therefore$ Area of $\Delta_{QFR} = \frac{1}{2}(\sqrt{6} - 1) \cdot \frac{2}{\sqrt{6}}(\sqrt{6} - 1)$

$= \frac{7}{\sqrt{6}} - 2$



Vector Algebra

Question 174

If the equation of the plane containing the line $x+2y+3z-4=0=2x+y-z+5$ and perpendicular to the plane $\vec{r}=(\hat{i}-\hat{j})+\lambda(\hat{i}+\hat{j}+\hat{k})+\mu(\hat{i}-2\hat{j}+3k)$ is $ax+by+cz=4$, then $(a-b+c)$ is equal to

A. 20

B. 24

C. 22

D. 18

Correct answer: C

Source solution

The given line is the intersection of the planes

$$x+2y+3z-4=0$$

and

$$2x+y-z+5=0.$$

The normals to these planes are

$$\vec{n}_1=(1,2,3), \qquad \vec{n}_2=(2,1,-1).$$

Hence the direction vector of the line is

$$\vec{d} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 1 & -1 \end{vmatrix} = (-5,\ 7,\ -3).$$

The given plane is

$$\vec{r} = (\hat{i} - \hat{j}) + \lambda(\hat{i} + \hat{j} + \hat{k}) + \mu(\hat{i} - 2\hat{j} + 3\hat{k}).$$

Its two direction vectors are

$$\vec{a} = (1, 1, 1), \qquad \vec{b} = (1, -2, 3).$$

Therefore, its normal vector is

$$\vec{n}_p = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & -2 & 3 \end{vmatrix} = (5,\ -2,\ -3).$$

Let the required plane have normal vector

$$\vec{N} = (a, b, c).$$

Since the required plane contains the given line,

$$\vec{N} \cdot \vec{d} = 0.$$

Thus,

$$-5a + 7b - 3c = 0.$$

Also, the required plane is perpendicular to the plane whose normal is $(5, -2, -3)$, so

$$\vec{N} \cdot (5, -2, -3) = 0,$$

i.e.,

$$5a - 2b - 3c = 0.$$

Solving

$$-5a + 7b - 3c = 0$$

and

$$5a - 2b - 3c = 0,$$

adding,

$$5b - 6c = 0$$

$$b = \frac{6c}{5}.$$

Substituting into

$$5a - 2b - 3c = 0,$$

$$5a - \frac{12c}{5} - 3c = 0$$

$$25a = 27c.$$

Taking $c = 25$,

$$a = 27, \qquad b = 30.$$

Hence,

$$\vec{N} = (27, 30, 25).$$

Therefore the required plane is

$$27x + 30y + 25z = 4.$$

So,

$$a - b + c = 27 - 30 + 25 = 22.$$

$$\boxed{22}$$

Hence, the correct option is

$$\boxed{\text{(3) } 22}.$$

Question 175

Let the plane $P : \vec{r} \cdot \vec{a} = d$ contain the line of intersection of two planes $\vec{r} \cdot (\hat{i} + 3\hat{j} - \hat{k}) = 6$ and $\vec{r} \cdot (-6\hat{i} + 5\hat{j} - \hat{k}) = 7$. If the plane P passes through the point $\left(2, 3, \frac{1}{2}\right)$, then the value of $\frac{|13\vec{a}|^2}{d^2}$ is equal to

A. 90

B. 93

C. 95

D. 97

Correct answer: B

Source solution

Sol.

Equation of plane passing through line of intersection of planes $P_1 : \vec{r}((\hat{i} + 3\hat{j} - \hat{k}) = 6$ and $P_2 : \vec{r} \cdot (-6\hat{i} + 5\hat{j} - \hat{k}) = 7$ is

$P_1 + \lambda P_2 = 0$

$(\vec{r} \cdot (\hat{i} + 3\hat{j} - \hat{k}) - 6) + \lambda(\vec{r} \cdot (-6\hat{i} + 5\hat{j} - \hat{k}) - 7) = 0$ and it passes through point $\left(2, 3, \frac{1}{2}\right)$

$\Rightarrow \left(2 + 9 - \frac{1}{2} - 6\right) + \lambda\left(-12 + 15 - \frac{1}{2} - 7\right) = 0$

$\Rightarrow \lambda = 1$

Equation of plane is $\vec{r} \cdot (-5\hat{i} + 8\hat{j} - 2\hat{k}) = 13$

$|\vec{a}|^2 = 25 + 64 + 4 = 93; d = 13$

Value of $\frac{|13\vec{a}|^2}{d^2} = 93$

Question 176

Let $\vec{a}=2\hat{i}+7\hat{j}-\hat{k}, \vec{b}=3\hat{i}+5\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}+2\hat{k}$
. Let $\vec{d}$ be a vector which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c}\cdot\vec{d}=12$. Then $(-\hat{i}+\hat{j}-\hat{k})\cdot(\vec{c}\times\vec{d})$ is equal to

A. 48

B. 42

C. 44

D. 24

Correct answer: C

Source solution

Sol. $\vec{a}=2\hat{i}+7\hat{j}-\hat{k}$

$\vec{b}=3\hat{i}+5\hat{k}$

$\vec{c}=\hat{i}-\hat{j}+2\hat{k}$

$\vec{d}=\lambda(\vec{a}\times\vec{b})=\lambda\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}2\\ 7 & -13 & 0\\ 5\end{vmatrix}$

$\vec{d}=\lambda(35\hat{i}-13\hat{j}-21\hat{k})$

$\lambda(35+13-42)=12$

$\lambda=2$

$\vec{d}=2(35\hat{i}-13\hat{j}-21\hat{k})$

$(\hat{i}+\hat{j}-\hat{k})(\vec{c}\times\vec{d})$

$=\begin{vmatrix}-1 & 1 & -11\\ -1 & 270 & -26\\ -42\end{vmatrix}=44$

Question 177

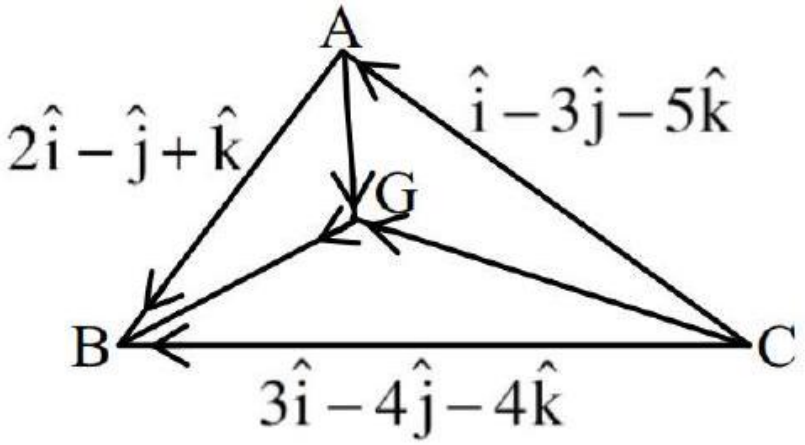
Let the three sides of a triangle ABC be given by the vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{j}-4\hat{k}$. Let G be the centroid of the triangle ABC . Then $6\left(|\overrightarrow{AG}|^2+|\overrightarrow{BG}|^2+|\overrightarrow{CG}|^2\right)$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 164

Source solution



By given data

$\overrightarrow{AB}+\overrightarrow{AC}=\overrightarrow{CB}$

Let pv of $\overrightarrow{A}$ are $\overrightarrow{O}$ then

$\overrightarrow{AB}=\overrightarrow{B}-\overrightarrow{A}$

i.e. pv of $\vec{B}-=2\hat{i}-\hat{j}+\hat{k}$

$\overrightarrow{CA}=\overrightarrow{A}-\overrightarrow{C}$

i.e. pv of $\vec{C}=-(\hat{i}-3\hat{j}-5\hat{k})$

Now pv of centroid

$(\overrightarrow{G})-=\frac{\overrightarrow{A}+\overrightarrow{B}+\overrightarrow{C}}{3}=\frac{\overrightarrow{0}+(2,-1,1)+(-1,3,5)}{3}$

$\overrightarrow{G}=\frac{1}{3}(\hat{i}+2\hat{j}+6\hat{k})$

Now $\overrightarrow{AG}=\frac{1}{3}(\hat{i}+2\hat{j}+6\hat{k})$

$\Rightarrow |\overrightarrow{AG}|^2 = \frac{1}{9} \times 41$

$\overrightarrow{BG} = \left(\frac{1}{3} - 2\right)\hat{i} + \left(\frac{2}{3} + 1\right)\hat{j} + (2-1)\hat{k}$

$\Rightarrow |\overrightarrow{BG}|^2 = \frac{59}{9}$

$\overrightarrow{CG} = \left(\frac{1}{3} + 1\right)\hat{i} + \left(\frac{2}{3} - 3\right)\hat{j} + (2-5)\hat{k}$

$\Rightarrow |\overrightarrow{CG}|^2 = \frac{146}{9}$

Now

$6\left[|\overrightarrow{AG}|^2 + |\overrightarrow{BG}|^2 + |\overrightarrow{CG}|^2\right] = 6 \times \left[\frac{41}{9} + \frac{59}{9} + \frac{146}{9}\right]$

$= 6 \times \frac{246}{9} = 164$

Question 178

Let $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \vec{b} = 2\hat{i} - 2\hat{j} - 2\hat{k}$ and $\vec{c} = -\hat{i} + 4\hat{j} + 3\hat{k}$. If $\vec{d}$ is a vector perpendicular to both $\vec{b}$ and $\vec{c}$ and $\vec{a} \cdot \vec{d} = 18$, Then $|\vec{a} \times \vec{d}|^2$ is equal to

A. 640

B. 760

C. 680

D. 720

Correct answer: D

Source solution

Sol. $a = \lambda(b \times c)$

$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & -2 \\ -1 & 4 & 3 \end{vmatrix} = 2\hat{i} - \hat{j} + 2\hat{k}$

$\vec{d} = \lambda(2\hat{i} - \hat{j} + 2\hat{k})$

$\vec{a} \cdot \vec{d} = 18$

$\lambda = 2$

So $\vec{d} = 2(2\hat{i} - \hat{j} + 2\hat{k})$

$\vec{d} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -2 & 4 \\ 2 & 3 & 4 \end{vmatrix} = -20\hat{i} - 8\hat{j} + 16\hat{k}$

$|\vec{d} \times \vec{a}|^2 = 720$

Question 179

Let $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}, \vec{b}=2\hat{i}+\hat{j}-\hat{k}, \vec{c}=\lambda\hat{i}+\hat{j}+\hat{k}$ and $\vec{v}=\vec{a}\times\vec{b}$. If $\vec{v}.\vec{c}=11$ and the length of the projection of $\vec{b}$ on $\vec{c}$ is p, then $9p^2$ is equal to :

A. 9

B. 6

C. 4

D. 12

Correct answer: D

Source solution

$\vec{a}=\hat{i}-2\hat{j}+3\hat{k}, \quad \vec{b}=2\hat{i}+\hat{j}-\hat{k}, \vec{c}=\lambda\hat{i}+\hat{j}+\hat{k}$, and $\vec{v}=\vec{a}\times\vec{b}$. If $\vec{v}.\vec{c}=11$

$$\vec{v}=(\vec{a}\times\vec{b})=(-\hat{i}+7\hat{j}+5\hat{k})$$
$$\vec{v}\cdot\vec{c}=11=(-\hat{i}+7\hat{j}+5\hat{k})\cdot(\lambda\hat{i}+\hat{j}+\hat{k})=11$$
$$\Rightarrow -\lambda+7+5=11$$
$$\Rightarrow \lambda=1$$

Length of projection of $\vec{b}$ on $\vec{c}=\vec{b}.\hat{c}$

$$\Rightarrow \left|(2\hat{i}+\hat{j}-\hat{k})\cdot\frac{(\hat{i}+\hat{j}+\hat{k})}{\sqrt{3}}\right|=\frac{2+1-1}{\sqrt{3}}=P=\frac{2}{\sqrt{3}}$$
$$\Rightarrow 9p^2=9\left(\frac{4}{3}\right)=12$$

Question 180

Let $\vec{\alpha} = 3\hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\vec{\beta} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$. If $\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$, then $\vec{\beta}_1 \times \vec{\beta}_2$ is equal to:

A. $\frac{1}{2}(3\hat{i} - 9\hat{j} + 5\hat{k})$

B. $\frac{1}{2}(-3\hat{i} + 9\hat{j} + 5\hat{k})$

C. $-3\hat{i} + 9\hat{j} + 5\hat{k}$

D. $3\hat{i} - 9\hat{j} - 5\hat{k}$

Correct answer: B

Source solution

Sol. $\vec{\beta} = \overrightarrow{\beta_1} - \overrightarrow{\beta_2}$

$$\therefore \quad \overrightarrow{\beta_2} \cdot \vec{\alpha} = 0 \qquad \text{(i)}$$

and Let $\overrightarrow{\beta_1} = \lambda\vec{\alpha}$

$\vec{\alpha} \cdot \vec{\beta} = \vec{\alpha} \cdot \overrightarrow{\beta_1} - \vec{\alpha} \cdot \overrightarrow{\beta_2}$

$\Rightarrow 5 = \lambda\alpha^2$

$\Rightarrow 5 = \lambda \times 10$

$\Rightarrow \lambda = \frac{1}{2}$

$\therefore \quad \overrightarrow{\beta_1} = \frac{\vec{\alpha}}{2}$

Cross product with $\overrightarrow{\beta_1}$ in equation (i)

$\Rightarrow \vec{\beta} \times \overrightarrow{\beta_1} = -\overrightarrow{\beta_2} \times \overrightarrow{\beta_1}$

$\Rightarrow \vec{\beta} \times \overrightarrow{\beta_1} = \overrightarrow{\beta_1} \times \overrightarrow{\beta_2} = \frac{(\vec{\beta} \times \vec{\alpha})}{2}$

$\Rightarrow \overrightarrow{\beta_1} \times \overrightarrow{\beta_2} = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k}2 \\ -1 & 33 & 1 \\ 0 & & \end{vmatrix}$

$$= \frac{1}{2}[-3\hat{i} - \hat{j}(-9) + \hat{k}(5)]$$
$$= \frac{1}{2}[-3\hat{i} + 9\hat{j} + 5\hat{k}]$$

Question 181

Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=3\hat{i}+2\hat{j}-\hat{k}, \vec{c}=\lambda\hat{j}+\mu\hat{k}$ and $\hat{d}$ be a unit vector such that $\vec{a}\times\hat{d}=\vec{b}\times\hat{d}$ and $\vec{c}.\hat{d}=1$, If $\vec{c}$ is perpendicular to $\vec{a}$, then $|3\lambda\hat{d}+\mu\vec{c}|^2$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 5

Source solution

$$\vec{a}\times\vec{d}-\vec{b}\times\vec{d}=0$$
$$(a-\vec{b})\times\vec{d}=0$$
$$\vec{d}=t(\vec{a}-\vec{b})$$
$$\vec{\mathrm{d}}=\mathrm{t}(-2\hat{\mathrm{i}}-\hat{\mathrm{j}}+2\hat{\mathrm{k}})$$
$$|\vec{\mathrm{d}}|=1$$
$$|t|=\tfrac{1}{3}$$
$$\vec{\mathrm{c}}\cdot\vec{\mathrm{a}}=0$$
$$\lambda+\mu=0$$
$$\mu=-\lambda$$
$$\vec{\mathrm{c}}=\lambda(\hat{\mathrm{j}}-\hat{\mathrm{k}}),\quad |\vec{\mathrm{c}}|^2=2\lambda^2$$
$$\vec{\mathrm{c}}\cdot\hat{\mathrm{d}}=1$$
$$\mathrm{t}(-2,-1,2)\cdot\lambda(0,1,-1)=1$$
$$\lambda\mathrm{t}=\tfrac{-1}{3}\Rightarrow\lambda^2=1$$
$$|3\lambda\,\hat{\mathrm{d}}+\mu\vec{\mathrm{c}}|^2=9\lambda^2|\,\hat{\mathrm{d}}|^2+\mu^2|\vec{\mathrm{c}}|^2+6\lambda\mu(\,\hat{\mathrm{d}}\cdot\vec{\mathrm{c}})$$
$$=3\lambda^2+2\lambda^4$$
$$=5$$

Question 182

Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + 3\hat{j} - 5\hat{k}$ and $\vec{c} = 3\hat{i} - \hat{j} + \lambda\hat{k}$ be three vectors. Let $\vec{r}$ be a unit vector along $\vec{b} + \vec{c}$. If $\vec{r}.\vec{a} = 3$, then 3λ is equal to :

A. 27

B. 25

C. 25

D. 21

Correct answer: B

Source solution

Sol. $\vec{r} = k(\vec{b} + \vec{c})$

$\vec{r} \cdot \vec{a} = 3$

$\vec{r}.\vec{a} = k(\vec{b}.\vec{a} + \vec{c}.\vec{a})$

$3 = k(2 + 6 - 15 + 3 - 2 + 3\lambda)$

$$3 = k(-6 + 3\lambda) \quad (1)$$

$\vec{r} = k(5\hat{i} + 2\hat{j} - (5 - \lambda)\hat{k})$

$$|\vec{r}| = k\sqrt{25 + 4 + 25 + \lambda^2 - 10\lambda} = 1 \quad (2)$$

$k = \frac{3}{-6+3\lambda} = \frac{1}{-2+\lambda}$ put in (2)

$4 + \lambda^2 - 4\lambda = 54 + \lambda^2 - 10\lambda$

$6\lambda = 50$

$3\lambda = 25$

Question 183

Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. If a vector $\vec{d}$ satisfies $\vec{d} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{d}.\vec{a} = 24$, then $|\vec{d}|^2$ is equal to

A. 413

B. 423

C. 323

D. 313

Correct answer: A

Source solution

Sol.:}

$\vec{d} \times \vec{b} = \vec{c} \times \vec{b}$

$\Rightarrow (\vec{d} - \vec{c}) \times \vec{b} = 0$

$\Rightarrow \vec{d} = \vec{c} + \lambda \vec{b}$

Also $\vec{d}.\vec{a} = 24$

$\Rightarrow (\vec{c} + \lambda \vec{b}) \cdot \vec{a} = 24$

$\lambda = \frac{24 - \vec{a} \cdot \vec{c}}{\vec{b} \cdot \vec{a}} = \frac{24-6}{9} = 2$

$\Rightarrow \vec{d} = \vec{c} + 2(\vec{b})$

$= 8\hat{i} - 5\hat{j} + 18\hat{k}$

$\Rightarrow |\vec{d}|^2 = 64 + 25 + 324 = 413$

Question 184

Let $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = 7\hat{i} + \hat{j} - 6\hat{k}$.

If $\vec{r} \times \vec{a} = \vec{r} \times \vec{b}, \vec{r} \cdot (\hat{i} + 2\hat{j} + \hat{k}) = -3$, then $\vec{r} \cdot (2\hat{i} - 3\hat{j} + \hat{k})$ is equal to :

A. 12

B. 8

C. 13

D. 10

Correct answer: A

Source solution

Sol. $\vec{r} \times \vec{a} - \vec{r} \times \vec{b} = 0$

$\Rightarrow \vec{r} \times (\vec{a} - \vec{b}) = 0$

$\Rightarrow \vec{r} = \lambda(\vec{a} - \vec{b})$

$\Rightarrow \vec{r} = \lambda(-5\hat{i} - 4\hat{j} + 10\hat{k})$

Also $\vec{r} \cdot (\hat{i} + 2\hat{j} + \hat{k}) = -3$

$\Rightarrow \lambda(-5 - 8 + 10) = -3$

$\lambda = 1$

Now $\vec{r} = -5\hat{i} - 4\hat{j} + 10\hat{k}$

$= \vec{r} \cdot (2\hat{i} - 3\hat{j} + \hat{k})$

$= -10 + 12 + 10 = 12$

Question 185

Let $\vec{a} = -\hat{i} + \hat{j} + 2\hat{k}, \vec{b} = \hat{i} - \hat{j} - 3\hat{k}, \vec{c} = \vec{a} \times \vec{b}$ and $\vec{d} = \vec{c} \times \vec{a}$. Then $(\vec{a} - \vec{b}) \cdot \vec{d}$ is equal to :

A. 4

B. -4

C. -2

D. 2

Correct answer: C

Source solution

$$\vec{d} = (\vec{a} \times \vec{b}) \times \vec{a}$$
$$\vec{d} = (a^2)\vec{b} - (\vec{a} \cdot \vec{b})\vec{a}$$
$$\vec{d} = 6\vec{b} + 8\vec{a}$$
$$(\vec{a} - \vec{b}) \cdot \vec{d} - = (\vec{a} - \vec{b}) \cdot (6\vec{b} + 8\vec{a})$$
$$= 8a^2 - 6b^2 - 2\vec{a} \cdot \vec{b}$$
$$= 48 - 66 + 16 = -2$$

Three Dimensional Geometry

Questions 186-299 / 114 selected / 38.0% practice weight / Focus first

Question 186

Let the line $\ell : x = \frac{1-y}{-2} = \frac{z-3}{\lambda}, \lambda \in \mathbb{R}$ meet the plane $\mathrm{P} : \mathrm{x} + 2\mathrm{y} + 3\mathrm{z} = 4$ at the point (α, β, γ). If the angle between the line ℓ and the plane P is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then $\alpha + 2\beta + 6\gamma$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 11

Source solution

$\ell : \mathrm{x} = \frac{\mathrm{y}-1}{2} = \frac{\mathrm{z}-3}{\lambda}, \lambda \in \mathbb{R}$
DR's of line $\ell(1, 2, \lambda)$
DR's of normal vector of plane $P : x + 2y + 3z = 4$ are (1, 2, 3)

Now, angle between line ℓ and plane P is given by
$\sin\theta = \left|\frac{1+4+3\lambda}{\sqrt{5+\lambda^2}\cdot\sqrt{14}}\right| = \frac{3}{\sqrt{14}}$ (given $\cos\theta = \sqrt{\frac{5}{14}}$)
$\Rightarrow \lambda = \frac{2}{3}$
Let variable point on line ℓ is $\left(t, 2t+1, \frac{2}{3}t+3\right)$
lies on plane P.
$\Rightarrow \mathrm{t} = -1$
$\Rightarrow \left(-1, -1, \frac{7}{3}\right) \equiv (\alpha, \beta, \gamma)$
$\Rightarrow \alpha + 2\beta + 6\gamma = 11$

Question 187

Let the image of the point $\left(\frac{5}{3}, \frac{5}{3}, \frac{8}{3}\right)$ in the plane $x - 2y + z - 2 = 0$ be P. If the distance of the point $Q(6, -2, \alpha), \alpha > 0$, from P is 13 , then α is equal to

.

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 15

Source solution

Image of point $\left(\frac{5}{3}, \frac{5}{3}, \frac{8}{3}\right)$

$\frac{x-\frac{5}{3}}{1} = \frac{y-\frac{5}{3}}{-2} = \frac{z-\frac{8}{3}}{1} = \frac{-2\left(1\times\frac{5}{3}+(-2)\times\frac{8}{3}+1\times\frac{8}{3}-2\right)}{1^2+2^2+1^2} = \frac{1}{3}$

$\therefore x = 2, y = 1, z = 3$

$13^2 = (6-2)^2 + (-2-1)^2 + (\alpha-3)^2$

$\Rightarrow (\alpha-3)^2 = 144 \Rightarrow \alpha = 15 (\because \alpha > 0)$

Question 188

Let the lines $l_1 : \frac{x+5}{3} = \frac{y+4}{1} = \frac{z-\alpha}{-2}$ and $l_2 : 3x+2y+z-2=0=x-3y+2z-13$ be coplanar. If the point $P(a,b,c)$ on l_1 is nearest to the point $Q(-4,-3,2)$, then $|a|+|b|+|c|$ is equal to

A. 12

B. 14

C. 10

D. 8

Correct answer: C

Source solution

Sol. $(3x+2y+z-2)+\mu(x-3y+2z-13)=0$

$3(3+\mu)+1\cdot(2-3\mu)-2(1+2\mu)=0$

$9-4\mu=0$

$\mu=\frac{9}{4}$

$4(-15-8+\alpha-2)+9(-5+12+2\alpha-13)=0$

$-100+4\alpha-54+18\alpha=0$

$\Rightarrow \alpha=7$

Let $P\equiv(3\lambda-5,\lambda-4,-2\lambda+7)$

Direction ratio of $PQ(3\lambda-1,\lambda-1,-2\lambda+5)$

But $PQ\perp \ell_1$

$\Rightarrow 3(3\lambda-1)+1\cdot(\lambda-1)-2(-2\lambda+5)=0$

$\Rightarrow \lambda=1$

$P(-2,-3,5)\Rightarrow |a|+|b|+|c|=10$

Question 189

Let the equation of plane passing through the line of intersection of the planes $x+2y+az=2$ and $x-y+z=3$ be $5x-11y+bz=6a-1$. For $c \in \mathbb{Z}$, if the distance of this plane from the point $(a,-c,c)$ is $\frac{2}{\sqrt{a}}$, then $\frac{a+b}{c}$ is equal to

A. -2

B. 2

C. -4

D. 4

Correct answer: C

Source solution

Sol.

$(x+2y+az-2)+\lambda(x-y+z-3)=0$

$\frac{1+\lambda}{5}=\frac{2-\lambda}{-11}=\frac{a+\lambda}{b}=\frac{2+3\lambda}{6a-1}$

$\lambda=-\frac{7}{2}, a=3,\ b=1$

$\frac{2}{\sqrt{a}}=\left|\frac{5a+11c+bc-6a+1}{\sqrt{25+121+1}}\right|$

$c=-1$

$\therefore \frac{a+b}{c}=\frac{3+1}{-1}=-4$

Question 190

If the equation of the plane passing through the line of intersection of the planes $2x - y + z = 3, 4x - 3y + 5z + 9 = 0$ and parallel to the line $\frac{x+1}{-2} = \frac{y+3}{4} = \frac{z-2}{5}$ is $ax + by + cz + 6 = 0$. then a $+\mathrm{b} + \mathrm{c}$ is equal to

A. 14

B. 12

C. 13

D. 15

Correct answer: A

Source solution

Sol. Equation of family of plane

$(2x - y + z - 3) + \lambda(4x - 3y + 5z + 9) = 0$

$x(2 + 4\lambda) - y(1 + 3\lambda) + z(1 + 5\lambda) - 3 + 9\lambda = 0$

Parallel to the line

$-2(2 + 4\lambda) - (1 + 3\lambda)4 + (1 + 5\lambda)5 = 0$

$5\lambda = 3$

$\lambda = \frac{3}{5}$

equation of plane

$11x - 7y + 10z + 6 = 0$

$a + b + c = 14$

Question 191

Let P be the point of intersection of the line $\frac{x+3}{3} = \frac{y+2}{1} = \frac{1-z}{2}$ and the plane $x + y + z = 2$.
If the distance of the point P from the plane $3x - 4y + 12z = 32$ is q, then q and $2q$ are the roots of the equation

A. $x^2 - 18x - 72 = 0$

B. $x^2 + 18x + 72 = 0$

C. $x^2 - 18x + 72 = 0$

D. $x^2 + 18x - 72 = 0$

Correct answer: C

Source solution

Sol. $P = (3\lambda - 3, \lambda - 2, 1 - 2\lambda)$

P lies on the plane, $x + y + z = 2$
$\Rightarrow \lambda = 3$
$\mathrm{P} = (6, 1, -5)$
$q = \left| \frac{18-4-60-32}{\sqrt{9+16+144}} \right| = \frac{78}{13} = 6$
$\mathrm{q} = 6, 2\mathrm{q} = 12$

Equation, $x^2 - 18x + 72 = 0$

Question 192

Let the lines $l_1 : \frac{x+5}{3} = \frac{y+4}{1} = \frac{z-\alpha}{-2}$ and $l_2 : 3x + 2y + z - 2 = 0 = x - 3y + 2z - 13$ be coplanar. If the point $P(a, b, c)$ on l_1 is nearest to the point $Q(-4, -3, 2)$, then $|a| + |b| + |c|$ is equal to

A. 12

B. 14

C. 10

D. 8

Correct answer: C

Source solution

Sol.

1. Start with the equation:

$$(3x + 2y + z - 2) + \mu(x - 3y + 2z - 13) = 0$$

2. Substitute and simplify:

$$3(3 + \mu) + 1(2 - 3\mu) - 2(1 + 2\mu) = 0$$

$$9 - 4\mu = 0$$

3. Solve for μ:

$$\mu = \frac{9}{4}$$

4. Next, consider the equation:

$$4(-15-8+\alpha-2)+9(-5+12+2\alpha-13)=0$$

5. Simplify:

$$-100+4\alpha-54+18\alpha=0$$

$$22\alpha-154=0$$

6. Solve for α:

$$\alpha=7$$

7. Define point P:

$$P\equiv(3\lambda-5,\lambda-4,-2\lambda+7)$$

8. Find the direction ratio of $\mathbf{PQ}$:

$$\mathbf{PQ}\equiv(3\lambda-1,\lambda-1,-2\lambda+5)$$

9. Since $\mathbf{PQ}\perp\ell_1$, set up the equation:

$$3(3\lambda-1)+1(\lambda-1)-2(-2\lambda+5)=0$$

10. Solve for λ:

$$\lambda=1$$

11. Finally, for point $P(-2, -3, 5)$:

$$|a| + |b| + |c| = 10$$

Question 193

Let P be the point of intersection of the line

$$\frac{x+3}{3}=\frac{y+2}{1}=\frac{1-z}{2}$$

and the plane

$$x+y+z=2.$$

If the distance of the point P from the plane

$$3x-4y+12z=32$$

is q, then q and $2q$ are the roots of the equation.

A. $x^2-18x-72=0$

B. $x^2+18x+72=0$

C. $x^2-18x+72=0$

D. $x^2+18x-72=0$

Correct answer: C

Source solution

$$\text{Let } P=(3\lambda-3,\lambda-2,1-2\lambda)$$

$$\text{Since } P \text{ lies on the plane } x+y+z=2:$$

$$(3\lambda-3)+(\lambda-2)+(1-2\lambda)=2$$

$$\Rightarrow 3\lambda - 3 + \lambda - 2 + 1 - 2\lambda = 2$$

$$\Rightarrow 2\lambda - 4 = 2$$

$$\Rightarrow 2\lambda = 6 \quad \Rightarrow \quad \lambda = 3$$

$$\text{Thus, } P = (6, 1, -5)$$

$$\text{Now, calculate } q:$$

$$q = \left| \frac{18 - 4 - 60 - 32}{\sqrt{9 + 16 + 144}} \right|$$

$$= \left| \frac{-78}{\sqrt{169}} \right| = \left| \frac{-78}{13} \right| = 6$$

$$\text{Therefore, } q = 6 \quad \text{and } 2q = 12$$

$$\text{The equation is } x^2 - 18x + 72 = 0$$

Question 194

If the line $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-1}{-1}$ intersects the plane $2x+3y-z+13=0$ at a point P and the plane $3x+y+4z=16$ at a point Q, then PQ is equal to:

A. $2\sqrt{14}$

B. 14

C. $2\sqrt{7}$

D. $\sqrt{14}$

Correct answer: A

Source solution

Sol. Let $\mathrm{P}(3\lambda+2, 2\lambda-1, -\lambda+1)$ and

$$\mathrm{Q}(3\mu+2, 2\mu-1, -\mu+1)$$

As P lies on $2x+3y-z+13=0$
$6\lambda+4+6\lambda-3+\lambda-1+13=0$
$\Rightarrow 13\lambda=-13$
$\Rightarrow \lambda=-1$
$\therefore \quad \mathrm{P}(-1,-3,2)$
Q lies on $3x+y+4z=16$
$9\mu+6+2\mu-1-4\mu+4=16$
$\Rightarrow 7\mu=7$
$\Rightarrow \mu=1$
Q is $(5,1,0)$
$PQ=\sqrt{36+16+4}=\sqrt{56}=2\sqrt{14}$

Question 195

Let the plane P contain the line $2x + y - z - 3 = 0 = 5x - 3y + 4z + 9$ and be parallel to the line $\frac{x+2}{2} = \frac{3-y}{-4} = \frac{z-7}{5}$. Then the distance of the point $A(8, -1, -19)$ from the plane P measured parallel to the line $\frac{x}{-3} = \frac{y-5}{4} = \frac{2-z}{-12}$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 26

Source solution

Plane $\mathbf{P} \equiv \mathbf{P_1} + \lambda \mathbf{P_2} = 0$

$(2x + y - z - 3) + \lambda(5x - 3y) + 4z + 9) = 0$

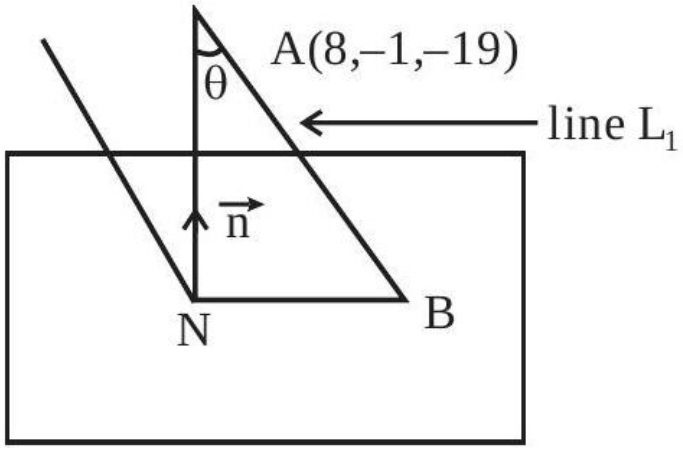
$(5\lambda + 2)x + (1 - 3\lambda)y + (4\lambda - 1)z + 9\lambda - 3 = 0$

$\vec{n}.\vec{b} = 0$ where $\vec{b}(2, 4, 5)$

$2(5\lambda + 2) + 4(1 - 3\lambda) + 5(4\lambda - 1) = 0$

$\lambda = -\frac{1}{6}$

Plane $7x + 9y - 10z - 27 = 0$



Equation of line AB is

$\frac{x-8}{-3} = \frac{y+1}{4} = \frac{z+19}{12} = \lambda$

Let $\mathbf{B} = (8 - 3\lambda, -1 + 4\lambda, -19 + 12\lambda)$ lies on plane P

$\therefore 7(8 - 3\lambda) + 9(4\lambda - 1) - 10(12\lambda - 19) = 27$

$\lambda = 2$

$\therefore$ Point $\mathbf{B} = (2, 7, 5)$

$\mathbf{AB} = \sqrt{6^2 + 8^2 + 24^2} = 26$

Question 196

Let P be the plane passing through the points $(5,3,0)$, $(13,3,-2)$ and $(1,6,2)$. For $\alpha \in \mathbb{N}$, if the distances of the points $\mathrm{A}(3,4,\alpha)$ and $\mathrm{B}(2,\alpha,a)$ from the plane P are 2 and 3 respectively, then the positive value of a is

A. 6

B. 4

C. 3

D. 5

Correct answer: B

Source solution

Sol.

1. Calculate the determinant to find the normal vector of the plane:

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & 0 & -2 \\ 4 & -3 & -2 \end{vmatrix} = \hat{i}(-6) + 8\hat{j} - 24\hat{k}$$

2. The normal vector of the plane is:

$$\mathbf{n} = 3\hat{i} - 4\hat{j} + 12\hat{k}$$

3. The equation of the plane is:

$$3x - 4y + 12z = 3$$

4. Calculate the distance from point $A(3,4,\alpha)$:

$$\left| \frac{9 - 16 + 12\alpha - 3}{13} \right| = 2$$

Simplifying gives:

$$\left|\frac{12\alpha - 10}{13}\right| = 2$$

This leads to two cases:
- Case 1: $\frac{12\alpha-10}{13} = 2$

$$12\alpha - 10 = 26 \implies 12\alpha = 36 \implies \alpha = 3$$

- Case 2: $\frac{12\alpha-10}{13} = -2$

$$12\alpha - 10 = -26 \implies 12\alpha = -16 \implies \alpha = -\frac{4}{3} \quad \text{(rejected)}$$

5. Calculate the distance from point $B(2, 3, a)$:

$$\left|\frac{6 - 12 + 12a - 3}{13}\right| = 3$$

Simplifying gives:

$$\left|\frac{12a - 9}{13}\right| = 3$$

This leads to two cases:
- Case 1: $\frac{12a-9}{13} = 3$

$$12a - 9 = 39 \implies 12a = 48 \implies a = 4$$

- Case 2: $\frac{12a-9}{13} = -3$

$$12a - 9 = -39 \implies 12a = -30 \implies a = -\frac{5}{2} \quad \text{(rejected)}$$

Final answers:

$$\alpha = 3, \quad a = 4$$

Question 197

If the line $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-1}{-1}$ intersects the plane $2x+3y-z+13=0$ at a point P and the plane $3x+y+4z=16$ at a point Q, then PQ is equal to:

A. $2\sqrt{14}$

B. 14

C. $2\sqrt{7}$

D. $\sqrt{14}$

Correct answer: A

Source solution

Sol. Let $\mathbf{P}(3\lambda+2, 2\lambda-1, -\lambda+1)$ and

$$\mathbf{Q}(3\mu+2, 2\mu-1, -\mu+1)$$

Since point P lies on the plane $2x+3y-z+13=0$, we substitute the coordinates of P:

$$2(3\lambda+2)+3(2\lambda-1)-(-\lambda+1)+13=0$$

Simplifying this:

$$6\lambda+4+6\lambda-3+\lambda-1+13=0$$

Combining like terms:

$$13\lambda+13=0$$

Solving for λ:

$$13\lambda=-13 \quad \Rightarrow \quad \lambda=-1$$

Thus,

$$\mathbf{P}(-1, -3, 2)$$

Next, point Q lies on the plane $3x + y + 4z = 16$. We substitute the coordinates of Q:

$$3(3\mu + 2) + (2\mu - 1) + 4(-\mu + 1) = 16$$

Simplifying this:

$$9\mu + 6 + 2\mu - 1 - 4\mu + 4 = 16$$

Combining like terms:

$$7\mu + 9 = 16$$

Solving for μ:

$$7\mu = 7 \quad \Rightarrow \quad \mu = 1$$

Thus,

$$\mathbf{Q}(5, 1, 0)$$

Now, we calculate the distance PQ:

$$PQ = \sqrt{(5 - (-1))^2 + (1 - (-3))^2 + (0 - 2)^2}$$

Calculating each term:

$$= \sqrt{(6)^2 + (4)^2 + (-2)^2} = \sqrt{36 + 16 + 4} = \sqrt{56} = 2\sqrt{14}$$

Final answer:

$$PQ = 2\sqrt{14}$$

Question 198

Let (α, β, γ) be the image of the point $\mathbf{P}(2, 3, 5)$ in the plane $2x + y - 3z = 6$. Then $\alpha + \beta + \gamma$ is equal to

A. 10

B. 5

C. 12

D. 9

Correct answer: A

Source solution

Sol.

We start with the equation:

$$\frac{\alpha - 2}{2} = \frac{\beta - 3}{1} = \frac{\gamma - 5}{-3} = -2$$

From this, we can derive the values of α, β, and γ.

1. For α:

$$\frac{\alpha - 2}{2} = -2 \implies \alpha - 2 = -4 \implies \alpha = -4 + 2 = -2$$

2. For β:

$$\frac{\beta - 3}{1} = -2 \implies \beta - 3 = -2 \implies \beta = -2 + 3 = 1$$

3. For γ:

$$\frac{\gamma - 5}{-3} = -2 \implies \gamma - 5 = 6 \implies \gamma = 6 + 5 = 11$$

Now we have:

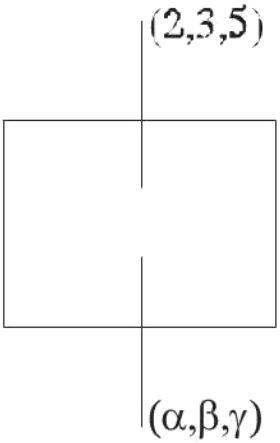
$$\alpha = -2, \quad \beta = 1, \quad \gamma = 11$$

Finally, we calculate:

$$\alpha + \beta + \gamma = -2 + 1 + 11 = 10$$

Thus, the final answer is:

$$\alpha + \beta + \gamma = 10$$



Question 199

If equation of the plane that contains the point ($-2, 3, 5$) and is perpendicular to each of the planes $2x + 4y + 5z = 8$ and $3x - 2y + 3z = 5$ is
$\alpha x + \beta y + \gamma z + 97 = 0$ then $\alpha + \beta + \gamma =$

A. 18

B. 17

C. 16

D. 15

Correct answer: D

Source solution

Sol. The equation of plane through ($-2, 3, 5$) is $a(x + 2) + b(y - 3) + c(z - 5) = 0$
it is perpendicular to $2x + 4y + 5z = 8 \& 3x - 2y + 3z = 5$
$\therefore \quad 2a + 4b + 5c = 0$

$$3a - 2b + 3c = 0$$

$\therefore \quad \frac{a}{\begin{vmatrix} 4 & 5 \\ -2 & 3 \end{vmatrix}} = \frac{-b}{\begin{vmatrix} 2 & 5 \\ 3 & 3 \end{vmatrix}} = \frac{c}{\begin{vmatrix} 2 & 4 \\ 3 & -2 \end{vmatrix}}$
$\Rightarrow \quad \frac{a}{22} = \frac{b}{9} = \frac{c}{-16}$
$\therefore$ Equation of Plane is
$22(x + 2) + 9(y - 3) - 16(z - 5) = 0$
$\Rightarrow \quad 22x + 9y - 16z + 97 = 0$

Comparing with $\alpha x + \beta y + \gamma x + 97 = 0$

We get $\alpha + \beta + \gamma = 22 + 9 - 16 = 15$

Question 200

Let the equation of plane passing through the line of intersection of the planes $x+2y+az=2$ and $x-y+z=3$ be $5x-11y+bz=6a-1$. For $c \in \mathbb{Z}$, if the distance of this plane from the point $(a,-c,c)$ is $\frac{2}{\sqrt{a}}$, then $\frac{a+b}{c}$ is equal to

A. -2

B. 2

C. -4

D. 4

Correct answer: C

Source solution

Sol.

1. Start with the equation:

$$(x+2y+az-2)+\lambda(x-y+z-3)=0$$

2. Set up the system of equations:

$$\frac{1+\lambda}{5}=\frac{2-\lambda}{-11}=\frac{a+\lambda}{b}=\frac{2+3\lambda}{6a-1}$$

3. Solve for λ, a, and b:

$$\lambda=-\frac{7}{2}, \quad a=3, \quad b=1$$

4. Substitute a into the equation:

$$\frac{2}{\sqrt{a}}=\left|\frac{5a+11c+bc-6a+1}{\sqrt{25+121+1}}\right|$$

5. Solve for c:

$$c = -1$$

6. Finally, calculate:

$$\frac{a+b}{c} = \frac{3+1}{-1} = -4$$

Question 201

Let the line passing through the points, $\mathbf{P(2,-1,2)}$ and $\mathbf{Q(5,3,4)}$ meet the plane $\mathbf{x-y+z=4}$ at the point R . Then the distance of the point R from the plane $\boldsymbol{x+2y+3z+2=0}$ measured parallel to the line $\frac{x-7}{2}=\frac{y+3}{2}=\frac{z-2}{1}$ is equal to

A. $\sqrt{31}$

B. $\sqrt{189}$

C. $\sqrt{61}$

D. 3

Correct answer: D

Source solution

Sol. Line : $\frac{x-5}{3}=\frac{y-3}{4}=\frac{z-4}{2}=\lambda$

$\mathbf{R}(3\lambda+5,4\lambda+3,2\lambda+4)$

$\therefore 3\lambda+5-4\lambda-3+2\lambda+4=4$

$\lambda+6=4 \therefore \lambda=-2$

$\therefore \mathbf{R} \equiv (-1,-5,0)$

Line : $\frac{x+1}{2}=\frac{y+5}{2}=\frac{z-0}{1}=\mu$

Point $T=(2\mu-1,2\mu-5,\mu)$

It lies on plane

$2\mu-1+2(2\mu-5)+3\mu+2=0$

$\mu=1$

$\therefore \mathbf{T}=(1,-3,1)$

$\therefore \mathbf{RT}=3$

Question 202

Let the point, on the line passing through the points $P(1, -2, 3)$ and $Q(5, -4, 7)$, farther from the origin and at a distance of 9 units from the point P, be (α, β, γ). Then $\alpha^2 + \beta^2 + \gamma^2$ is equal to :

A. 155

B. 150

C. 160

D. 165

Correct answer: A

Source solution

Sol.

The line PQ is given by the equations:

$$\frac{x-1}{4} = \frac{y+2}{-2} = \frac{z-3}{4}$$

We can express the points on the line in terms of a parameter t:

$$(x, y, z) = (4t + 1, -2t - 2, 4t + 3)$$

Next, we calculate the distance squared from the origin to the point on the line:

$$d^2 = (4t + 1)^2 + (-2t - 2)^2 + (4t + 3)^2$$

Expanding each term:

$$d^2 = (16t^2 + 8t + 1) + (4t^2 + 8t + 4) + (16t^2 + 24t + 9)$$

Combining like terms:

$$d^2 = 16t^2 + 4t^2 + 16t^2 + 8t + 8t + 24t + 1 + 4 + 9$$

$$d^2 = 36t^2 + 40t + 14$$

Setting $d^2 = 81$:

$$36t^2 + 40t + 14 = 81$$

Rearranging gives:

$$36t^2 + 40t - 67 = 0$$

Using the quadratic formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-40 \pm \sqrt{40^2 - 4 \cdot 36 \cdot (-67)}}{2 \cdot 36}$$

Calculating the discriminant:

$$b^2 - 4ac = 1600 + 9632 = 11232$$

Thus,

$$t = \frac{-40 \pm \sqrt{11232}}{72}$$

Calculating $\sqrt{11232}$:

$$\sqrt{11232} = 106$$

So,

$$t = \frac{-40 \pm 106}{72}$$

Calculating the two possible values for t:

1. $t = \frac{66}{72} = \frac{11}{12}$
2. $t = \frac{-146}{72} = -\frac{73}{36}$

Finding the corresponding points:

For $t = \frac{11}{12}$:

$$(4 \cdot \frac{11}{12} + 1, -2 \cdot \frac{11}{12} - 2, 4 \cdot \frac{11}{12} + 3) = (7, -5, 9)$$

For $t = -\frac{73}{36}$:

Calculating gives another point.

Finally, we find:

$$\alpha^2 + \beta^2 + \gamma^2 = 155$$

Thus, the answer is option (1).

Question 203

A circle with centre $(2, 3)$ and radius 4 intersects the line $x + y = 3$ at the points P and Q. If the tangents at P and Q intersect at the point $S(\alpha, \beta)$, then $4\alpha - 7\beta$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 11

Source solution

11

$(x-2)^2 + (y-3)^2 = 16$

$x^2 + y^2 - 4x - 6y - 3 = 0$

let the chord of contact w.r. to the point $S(\alpha, \beta)$ is

$\mathbf{T} = 0$

$\alpha x + \beta y - 2(x + \alpha) - 3(y + \beta) - 3 = 0$

$x(\alpha - 2) + y(\beta - 3) - 2\alpha - 3\beta - 3 = 0$

Comparison with $\mathbf{x + y = 3}$

$\frac{\alpha-2}{1} = \frac{\beta-3}{1} = \frac{-2\alpha-3\beta-3}{-3}$

$\alpha - 2 = \beta - 3 \quad -3\beta + 9 = -2\alpha - 3\beta - 3$

$\alpha - \beta = -1 \quad 2\alpha = -3 - 9$

$\beta = \alpha + 1 \quad \alpha = \frac{-12}{2}$

$= -6 + 1 = -5 \quad \alpha = -6$

$4\alpha - 7\beta$

$4(-6) - 7(-5)$

$\Rightarrow -24 + 35 = 11$

Question 204

Let the point, on the line passing through the points $\mathbf{P(1,-2,3)}$ and $\mathbf{Q(5,-4,7)}$, farther from the origin and at a distance of 9 units from the point P , be (α,β,γ). Then $\alpha^2+\beta^2+\gamma^2$ is equal to :

A. 155

B. 150

C. 160

D. 165

Correct answer: A

Source solution

Sol. PQ line
$\frac{x-1}{4}=\frac{y+2}{-2}=\frac{z-3}{4}$
pt $(4t+1,-2t-2,4t+3)$
distance $^2=16t^2+4t^2+16t^2=81$
$t=\pm\frac{3}{2}$
pt (7, -5, 9)
$\alpha^2+\beta^2+\gamma^2=155$
option (1)

Question 205

Let (α, β, γ) be the point $(8, 5, 7)$ in the line

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{5}.$$

Then $\alpha + \beta + \gamma$ is equal to

A. 16

B. 18

C. 14

D. 20

Correct answer: B

Source solution

$$\text{Given: } \overrightarrow{AM} \cdot (2\hat{i} + 3\hat{j} + 5\hat{k}) = 0$$

$$\text{Let } \overrightarrow{AM} = (2\lambda - 7)\hat{i} + (3\lambda - 6)\hat{j} + (5\lambda - 5)\hat{k}$$

$$\text{Setting up the dot product: } (2\lambda - 7)(2) + (3\lambda - 6)(3) + (5\lambda - 5)(5) = 0$$

$$\text{Expanding: } 4\lambda - 14 + 9\lambda - 18 + 25\lambda - 25 = 0$$

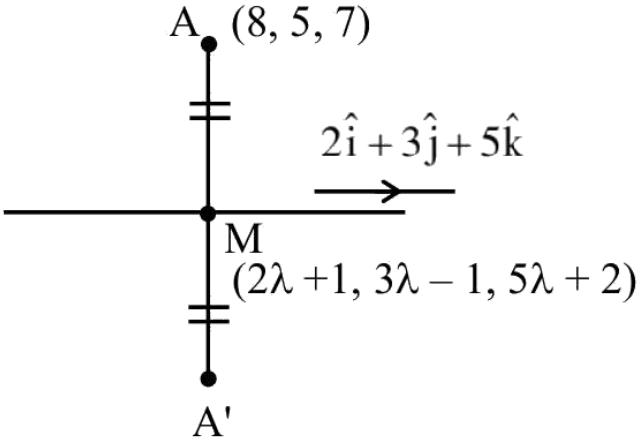
$$\text{Combining like terms: } (4\lambda + 9\lambda + 25\lambda) - 57 = 0$$

$$38\lambda = 57$$

$$\lambda = \frac{57}{38} = \frac{3}{2}$$

Coordinates of M : $M\left(4, \frac{7}{2}, \frac{19}{2}\right)$

Coordinates of A' : $A'(0, 2, 12)$



Question 206

Let d be the distance of the point of intersection of the lines $\frac{x+6}{3}=\frac{y}{2}=\frac{z+1}{1}$ and $\frac{x-7}{4}=\frac{y-9}{3}=\frac{z-4}{2}$ from the point $(7,8,9)$. Then d^2+6 is equal to :

A. 72

B. 69

C. 75

D. 78

Correct answer: C

Source solution

$$\text{Line } L_1: \quad \frac{x+6}{3}=\frac{y}{2}=\frac{z+1}{1}=\lambda$$

Hence,

$$x=3\lambda-6, \qquad y=2\lambda, \qquad z=\lambda-1.$$

Similarly,

$$\text{Line } L_2: \quad \frac{x-7}{4}=\frac{y-9}{3}=\frac{z-4}{2}=\mu$$

gives

$$x=4\mu+7, \qquad y=3\mu+9, \qquad z=2\mu+4.$$

Since the lines intersect,

$$3\lambda-6=4\mu+7,$$

$$2\lambda = 3\mu + 9,$$

$$\lambda - 1 = 2\mu + 4.$$

From the third equation,

$$\lambda = 2\mu + 5.$$

Substituting into the second equation,

$$2(2\mu + 5) = 3\mu + 9$$

$$4\mu + 10 = 3\mu + 9$$

$$\mu = -1.$$

Therefore,

$$\lambda = 2(-1) + 5 = 3.$$

Hence, the point of intersection is

$$P = (3 \cdot 3 - 6,\ 2 \cdot 3,\ 3 - 1) = (3, 6, 2).$$

Given point

$$Q = (7, 8, 9).$$

Therefore,

$$d = PQ.$$

So,

$$d^2 = (7 - 3)^2 + (8 - 6)^2 + (9 - 2)^2.$$

$$d^2 = 4^2 + 2^2 + 7^2.$$

$$d^2 = 16 + 4 + 49 = 69.$$

Hence,

$$d^2 + 6 = 69 + 6 = 75.$$

$$\boxed{75}$$

Hence, the correct option is

$$\boxed{\text{(3) } 75}.$$

Question 207

For real numbers α and $\beta \neq 0$, if the point of intersection of the straight lines $\frac{x-\alpha}{1} = \frac{y-1}{2} = \frac{z-1}{3}$ and $\frac{x-4}{\beta} = \frac{y-6}{3} = \frac{z-7}{3}$, lies on the plane $x + 2y - z = 8$, then $\alpha - \beta$ is equal to :

A. 5

B. 9

C. 3

D. 7

Correct answer: D

Source solution

\textbf{Given lines:}

$$\frac{x-\alpha}{1} = \frac{y-1}{2} = \frac{z-1}{3} = \lambda$$

and

$$\frac{x-4}{\beta} = \frac{y-6}{3} = \frac{z-7}{3} = \mu.$$

Since the lines intersect, let their point of intersection be $P(x, y, z)$.

From the first line,

$$x = \alpha + \lambda, \qquad y = 1 + 2\lambda, \qquad z = 1 + 3\lambda.$$

From the second line,

$$x = 4 + \beta\mu, \qquad y = 6 + 3\mu, \qquad z = 7 + 3\mu.$$

Equating the y-coordinates,

$$1+2\lambda = 6+3\mu. \tag{1}$$

Equating the z-coordinates,

$$1+3\lambda = 7+3\mu. \tag{2}$$

From (2),

$$\lambda - \mu = 2. \tag{3}$$

Substituting $\lambda = \mu + 2$ into (1),

$$1+2(\mu+2) = 6+3\mu,$$

$$5+2\mu = 6+3\mu,$$

$$\mu = -1.$$

Hence,

$$\lambda = 1.$$

Therefore the point of intersection is

$$P = (\alpha+1,\ 3,\ 4).$$

Since P lies on the plane

$$x + 2y - z = 8,$$

we have

$$(\alpha + 1) + 2(3) - 4 = 8.$$

$$\alpha + 3 = 8.$$

$$\alpha = 5.$$

Now equating the x-coordinates,

$$\alpha + \lambda = 4 + \beta\mu.$$

Substituting $\alpha = 5,\ \lambda = 1,\ \mu = -1$,

$$5 + 1 = 4 - \beta.$$

$$6 = 4 - \beta.$$

$$\beta = -2.$$

Hence,

$$\alpha - \beta = 5 - (-2) = 7.$$

$$\boxed{\alpha - \beta = 7}$$

$$\boxed{\text{Option (4)}}$$

Question 208

Let $\mathbf{P}(\alpha, \beta, \gamma)$ be the image of the point $\mathbf{Q}(1, 6, 4)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Then $2\alpha + \beta + \gamma$ is equal to
.

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 11

Source solution

\(\frac{\)

$\mathbf{Q}(1, 6, 4)$
$\mathbf{A}\left(\frac{17}{14}, \frac{48}{14}, \frac{79}{14}\right)$
$\frac{\mathrm{x}}{1} = \frac{\mathrm{y}-1}{2} = \frac{\mathrm{z}-2}{3}$
$\mathbf{P}(\alpha, \beta, \gamma) \quad \overrightarrow{\mathrm{b}} = \hat{\mathrm{i}} + 2\hat{\mathrm{j}} + 3\hat{\mathrm{k}}$

\}\{4\}

$\mathbf{A}(\mathrm{t}, 2\mathrm{t}+1, 3\mathrm{t}+2)$
$\overrightarrow{\mathbf{QA}} = (\mathrm{t}-1)\hat{\mathrm{i}} + (2\mathrm{t}-5)\hat{\mathrm{j}} + (3\mathrm{t}-2)\hat{\mathrm{k}}$
$\overrightarrow{\mathbf{QA}} \cdot \overrightarrow{\mathrm{b}} = 0$
$(\mathrm{t}-1) + 2(2\mathrm{t}-5) + 3(3\mathrm{t}-2) = 0$
$14\mathrm{t} = 17$
$\alpha = \frac{20}{14} \quad \beta = \frac{12}{14} \quad \gamma = \frac{102}{14}$
$2\alpha + \beta + \gamma = \frac{154}{14} = 11$

Question 209

Let $\mathrm{P}(\alpha, \beta, \gamma)$ be the image of the point $\mathrm{Q}(3, -3, 1)$ in the line $\frac{x-0}{1} = \frac{y-3}{1} = \frac{z-1}{-1}$ and R be the point $(2, 5, -1)$. If the area of the triangle PQR is λ and $\lambda^2 = 14\,\mathrm{K}$, then K is equal to:

A. 36

B. 72

C. 18

D. 81

Correct answer: D

Source solution

Given:

1. Calculate RQ:

$$\mathrm{RQ} = \sqrt{1 + 64 + 4} = \sqrt{69}$$

2. Define vectors:

$$\overrightarrow{\mathrm{RQ}} = \hat{\ell} - 8\hat{\mathrm{j}} + 2\hat{\mathrm{k}}$$

$$\overrightarrow{\mathrm{RS}} = \hat{\ell} + \hat{\mathrm{j}} - \hat{\mathrm{k}}$$

3. Calculate $\cos\theta$:

$$\cos\theta = \frac{\overrightarrow{\mathrm{RQ}} \cdot \overrightarrow{\mathrm{RS}}}{|\overrightarrow{\mathrm{RQ}}||\overrightarrow{\mathrm{RS}}|}$$

$$= \left| \frac{1 - 8 - 2}{\sqrt{69}\sqrt{3}} \right| = \frac{9}{3\sqrt{23}}$$

4. Simplify $\cos\theta$:

$$\cos\theta = \frac{3}{\sqrt{23}} = \frac{\mathrm{RS}}{\mathrm{RQ}} = \frac{\mathrm{RS}}{\sqrt{69}}$$

$$\Rightarrow \mathrm{RS} = 3\sqrt{3}$$

5. Calculate $\sin\theta$:

$$\sin\theta = \frac{\sqrt{14}}{\sqrt{23}} = \frac{\mathrm{QS}}{\sqrt{69}}$$

$$\Rightarrow \mathrm{QS} = \sqrt{42}$$

6. Calculate the area:

$$\text{Area} = \frac{1}{2} \cdot 2\mathrm{QS} \cdot \mathrm{RS} = \sqrt{42} \cdot 3\sqrt{3}$$

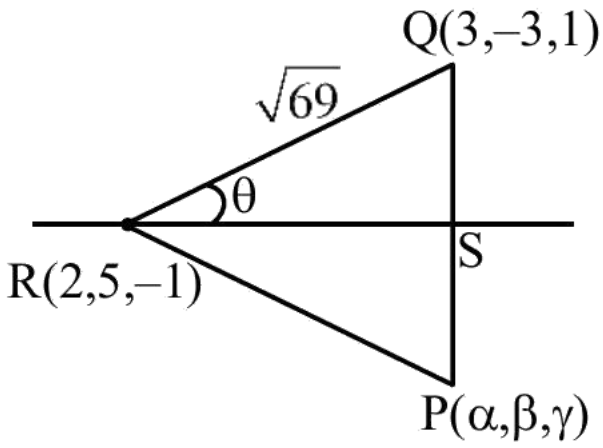
7. Calculate λ:

$$\lambda = 9\sqrt{14}$$

8. Calculate λ^2:

$$\lambda^2 = 81.14 = 14\mathrm{k}$$

$$\Rightarrow k = 81$$



Question 210

Let $\mathrm{P}(\alpha, \beta, \gamma)$ be the image of the point $\mathrm{Q}(3,-3,1)$ in the line $\frac{x-0}{1}=\frac{y-3}{1}=\frac{z-1}{-1}$ and R be the point ($2,5,-1$). If the area of the triangle PQR is λ and $\lambda^2=14 \mathrm{~K}$, then K is equal to:

A. 36

B. 72

C. 18

D. 81

Correct answer: D

Source solution

Sol.

$\mathrm{RQ}=\sqrt{1+64+4}=\sqrt{69}$

$\overrightarrow{\mathrm{RQ}}=\hat{\ell}-8 \hat{\mathrm{j}}+2 \hat{\mathrm{k}}$

$\overrightarrow{\mathrm{RS}}=\hat{\ell}+\hat{\mathrm{j}}-\hat{\mathrm{k}}$

$\cos \theta=\frac{\overrightarrow{\mathrm{RQ}} \cdot \overrightarrow{\mathrm{RS}}}{|\overrightarrow{\mathrm{RQ}}||\overrightarrow{\mathrm{RS}}|}=\left|\frac{1-8-2}{\sqrt{69} \sqrt{3}}\right|=\frac{9}{3 \sqrt{23}}$

$\cos \theta=\frac{3}{\sqrt{23}}=\frac{\mathrm{RS}}{\mathrm{RQ}}=\frac{\mathrm{RS}}{\sqrt{69}}$

$\mathrm{RS}=3 \sqrt{3}$

$\sin \theta=\frac{\sqrt{14}}{\sqrt{23}}=\frac{\mathrm{QS}}{\sqrt{69}}$

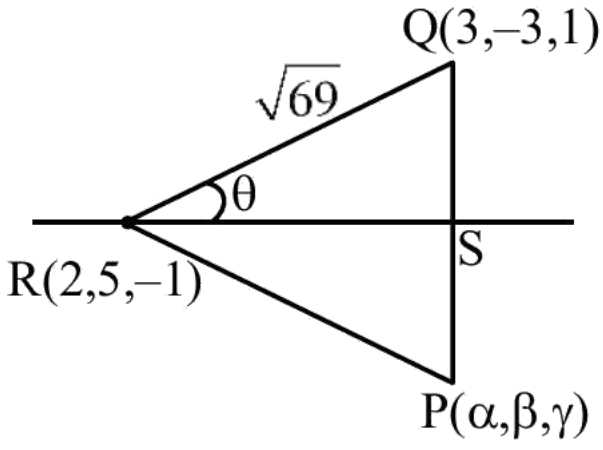
$\mathrm{QS}=\sqrt{42}$

area $=\frac{1}{2} \cdot 2 \mathrm{QS} \cdot \mathrm{RS}=\sqrt{42} \cdot 3 \sqrt{3}$

$\lambda=9 \sqrt{14}$

$\lambda^2=81.14=14 \mathrm{k}$

$\mathrm{k}=81$



Question 211

If the line $\frac{2-x}{3} = \frac{3y-2}{4\lambda+1} = 4 - z$ makes a right angle with the line $\frac{x+3}{3\mu} = \frac{1-2y}{6} = \frac{5-z}{7}$, then $4\lambda + 9\mu$ is equal to :

A. 13

B. 4

C. 5

D. 6

Correct answer: D

Source solution

Sol.

We start with the equations:

$$\frac{2-x}{3} = \frac{3y-2}{4\lambda+1} = 4 - z$$

$$\frac{x-2}{-3} = \frac{y-\frac{2}{3}}{\frac{4\lambda+1}{3}} = \frac{z-4}{-1}$$

From the second set of equations, we have:

$$\frac{x+3}{3\mu} = \frac{1-2y}{6} = \frac{5-z}{7}$$

This can be rewritten as:

$$\frac{x+3}{3\mu} = \frac{y-\frac{1}{2}}{-3} = \frac{z-5}{-7}$$

Using the right angle condition, we set up the equation:

$$(-3)(3\mu) + \left(\frac{4\lambda + 1}{3}\right)(-3) + (-1)(-7) = 0$$

Simplifying this gives:

$$-9\mu - 4\lambda - 1 + 7 = 0$$

Thus, we arrive at:

$$4\lambda + 9\mu = 6$$

Question 212

Let a line passing through the point $(4,1,0)$ intersect the line $L_1; \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ at the point $\mathrm{A} \quad (\alpha, \quad \beta, \quad \gamma)$ and the line $\mathrm{L_2 : x-6 = y = -z+4}$ at the point $\mathrm{B(a,b,c)}$.
Then $\begin{vmatrix} 1 & 0 & 1\alpha \\ \beta & \gamma a & b \\ c & & \end{vmatrix}$ is equal to

A. 8

B. 16

C. 12

D. 6

Correct answer: A

Source solution

Sol.}
$\mathrm{L_1 = \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = p}$
$\mathrm{L_2 = \frac{x-6}{1} = \frac{y}{1} = \frac{z-4}{-1} = q}$
$\mathrm{A(2p+1, 3p+2, 4p+3)}$
$\mathrm{B(q+6, q, 4-q)}$
D.R. of $\mathrm{PA = 2p-3, 3P+1, 4p+3}$
D.R. of $\mathrm{PB = q+2, q-1, 4-q}$
$\frac{2p-3}{q+2} = \frac{3p+1}{q-1} = \frac{4p+3}{4-q}$
$2pq - 2p - 3q + 3 = 3pq + 6p + q + 2$

$$\mathrm{pq + rp + 4q - 1 = 0} \quad (1)$$

$12p - 3pq + 4 - q = 4pq + 3q - 4p - 3$

$$7pq - 16p + 4q - 7 = 0 \quad (2)$$

$8p - 2pq - 12 + 3q = 4pq + 8p + 3q + 6$
$6pq = -18 \quad \therefore pq = -3$
$\mathrm{8p + 4q = 4 \quad \Rightarrow 2p + q = 1}$

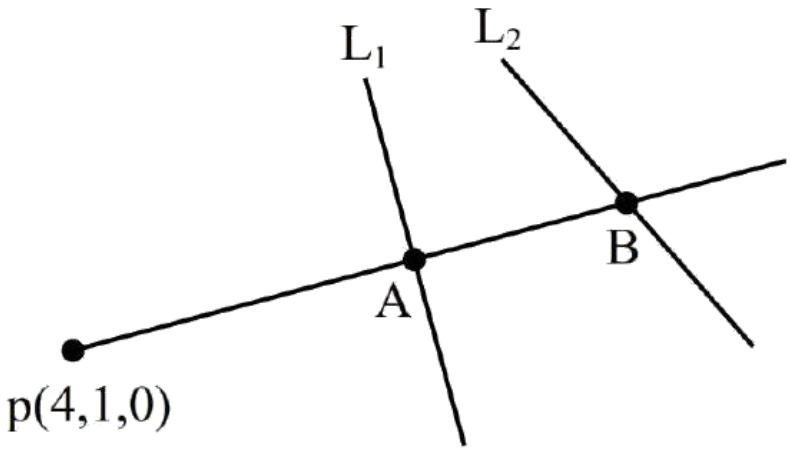
$-21-16p+4q-7 \quad \Rightarrow 4p-q=-7$

$16p-4q=-28 \quad \therefore p=-1, q=3$

$\mathrm{A}(-1,-1,-1) \quad \mathrm{B}(9,3,1)$

$|1 \quad 0 \quad 1-1 \quad -1 \quad -19 \quad 3 \quad 1| = |0 \quad -1 \quad 0-1 \quad -1 \quad -19 \quad 3 \quad 1| = 1(-1+9) = 8$

Option (1)



Question 213

The lines $\frac{x-2}{2}=\frac{y}{-2}=\frac{z-7}{16}$ and $\frac{x+3}{4}=\frac{y+2}{3}=\frac{z+2}{1}$ intersect at the point P. If the distance of P from the line $\frac{x+1}{2}=\frac{y-1}{3}=\frac{z-1}{1}$ is l, then $14l^2$ is equal to.

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 108

Source solution

$\frac{x-2}{1}=\frac{y}{-1}=\frac{z-7}{8}=\lambda$

$\frac{x+3}{4}=\frac{y+2}{3}=\frac{z+2}{1}=k$

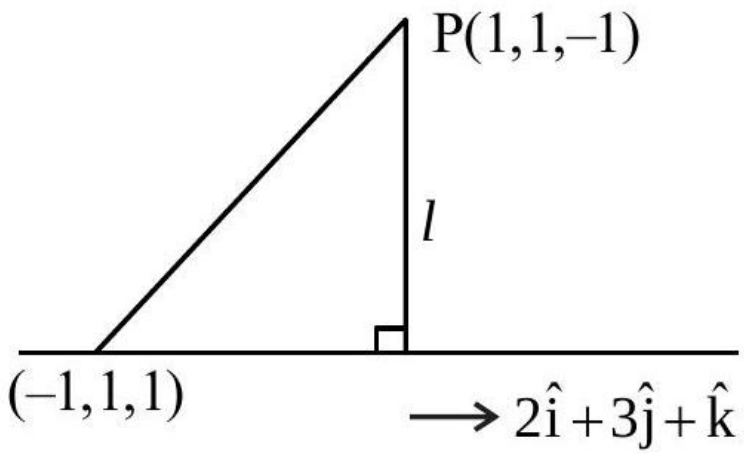
$\Rightarrow \lambda+2=4k-3$

$-\lambda=3k-2$

$\Rightarrow k=1, \lambda=-1$

$8\lambda+7=k-2$

$\therefore P=(1,1,-1)$



Projection of $2\hat{i}-2\hat{k}$ on $2\hat{i}+3\hat{j}+\hat{k}$ is

$=\frac{4-2}{\sqrt{4+9+1}}=\frac{2}{\sqrt{14}}$

$\therefore l^2=8-\frac{4}{14}=\frac{108}{14}$

$\Rightarrow 14l^2=108$

Question 214

Let a line passing through the point $(4,1,0)$ intersect the line $L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ at the point (α, β, γ) and the line $L_2 : x-6 = y = -z+4$ at the point (a,b,c). Then

$$\begin{vmatrix} 1 & 0 & 1 \\ \alpha & \beta & \gamma \\ a & b & c \end{vmatrix}$$

is equal to

A. 8

B. 16

C. 12

D. 6

Correct answer: A

Source solution

$$\text{Let } L_1 = \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = p$$

$$L_2 = \frac{x-6}{1} = \frac{y}{1} = \frac{z-4}{-1} = q$$

$$A(2p+1, 3p+2, 4p+3)$$

$$B(q+6, q, 4-q)$$

$$\text{Direction Ratios of } PA : (2p-3, 3p+1, 4p+3)$$

$$\text{Direction Ratios of } PB : (q+2, q-1, 4-q)$$

$$\frac{2p-3}{q+2} = \frac{3p+1}{q-1} = \frac{4p+3}{4-q}$$

$$2pq - 2p - 3q + 3 = 3pq + 6p + q + 2$$

$$pq + rp + 4q - 1 = 0 \tag{1}$$

$$12p - 3pq + 4 - q = 4pq + 3q - 4p - 3$$

$$7pq - 16p + 4q - 7 = 0 \tag{2}$$

$$8p - 2pq - 12 + 3q = 4pq + 8p + 3q + 6$$

$$6pq = -18 \quad \Rightarrow \quad pq = -3$$

$$8p + 4q = 4 \quad \Rightarrow \quad 2p + q = 1$$

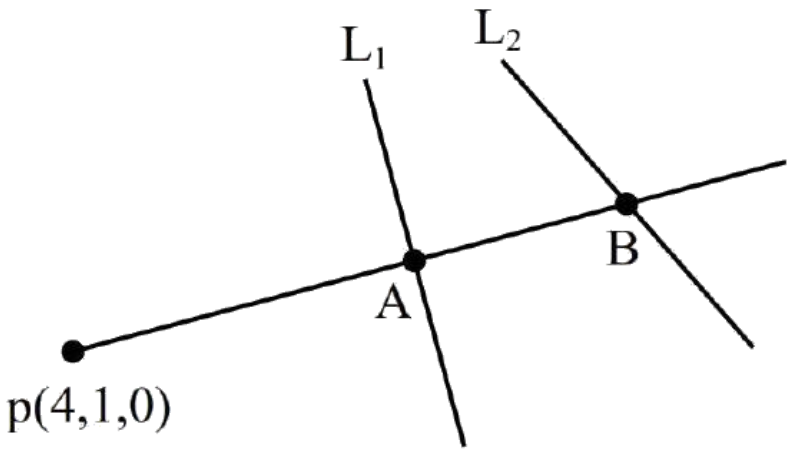
$$-21 - 16p + 4q - 7 \quad \Rightarrow \quad 4p - q = -7$$

$$16p - 4q = -28 \quad \Rightarrow \quad p = -1,\ q = 3$$

$$A(-1, -1, -1) \quad B(9, 3, 1)$$

$$\begin{vmatrix} 1 & 0 & 1 \\ -1 & -1 & -1 \\ 9 & 3 & 1 \end{vmatrix} = \begin{vmatrix} 0 & -1 & 0 \\ -1 & -1 & -1 \\ 9 & 3 & 1 \end{vmatrix} = 1(-1 + 9) = 8$$

Option (1)


L1
L2
B
A
p(4,1,0)

Question 215

Let L be a line obtained from the intersection of two planes $x + 2y + z = 6$ and $y + 2z = 4$. If point $P(\alpha, \beta, \gamma)$ is the foot of perpendicular from $(3, 2, 1)$ on L, then the value of $21(\alpha + \beta + \gamma)$ equals :

A. 142

B. 68

C. 136

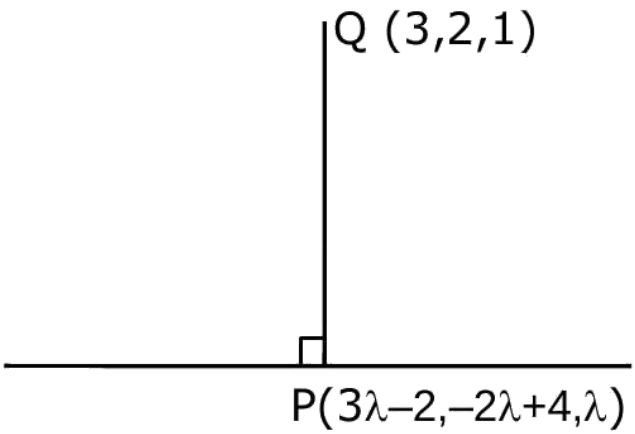
D. 102

Correct answer: D

Source solution

Sol. Dr's of line $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix} = 3\hat{i} - 2\hat{j} + \hat{k}$
Dr/s :- $(3, -2, 1)$
Points on the line $(-2, 4, 0)$
Equation of the line $\frac{x+2}{3} = \frac{y-4}{-2} = \frac{z}{1} = \lambda$

Dr's of PQ ; $3\lambda - 5, -2\lambda + 2. \lambda - 1$
Dr's of y lines are $(3, -2, 1)$
Since $\mathbf{PQ} \perp$ line
$3(3\lambda - 5) - 2(-2\lambda + 2) + 1(\lambda - 1) = 0$
$\lambda = \frac{10}{7}$
$P\left(\frac{16}{7}, \frac{8}{7}, \frac{10}{7}\right)$
$21(\alpha + \beta + \gamma) = 21\left(\frac{34}{7}\right) = 102$



Question 216

Consider the lines $\mathbf{L_1 : x-1=y-2=z}$ and $\mathbf{L_2 : x-2=y=z-1}$. Let the feet of the perpendiculars from the point $\mathbf{P(5,1,-3)}$ on the lines $\mathbf{L_1}$ and $\mathbf{L_2}$ be Q and R respectively. If the area of the triangle PQR is A , then $\mathbf{4\,A^2}$ is equal to :

A. 139

B. 147

C. 151

D. 143

Correct answer: B

Source solution

Sol. $\mathbf{L_1 : \frac{x-1}{1}=\frac{y-2}{1}=\frac{z-0}{2}}$

Let $\mathbf{Q(\lambda+1,\lambda+2,\lambda)}$

$\mathbf{\overrightarrow{PQ}=(\lambda-4,\lambda-1,\lambda+3)}$

$\mathbf{\overrightarrow{PQ}\cdot\vec{m}=0}$

$\mathbf{\Rightarrow \lambda-4+\lambda+1,\lambda+3=0}$

$\mathbf{\Rightarrow 3\lambda=0}$

$\mathbf{\lambda=0}$

$\mathbf{\Rightarrow Q(1,2,0)}$

$\mathbf{L_2 : \frac{x-2}{1}=\frac{y-0}{1}=\frac{z-1}{2}}$

Let $\mathbf{R(\mu+2,\mu,\mu+1)\overrightarrow{PR}=(\mu-3,\mu-1,\mu+4)}$

$\mathbf{\overrightarrow{PR}.\vec{n}=0}$

$\mathbf{\mu-3+\mu-1+\mu+4=0}$

$\mathbf{\neq \mu=0}$

R(2,0,1)

Area of $\mathbf{\triangle PQR(A)=\frac{1}{2}|\overrightarrow{PQ}\times\overrightarrow{PR}|}$

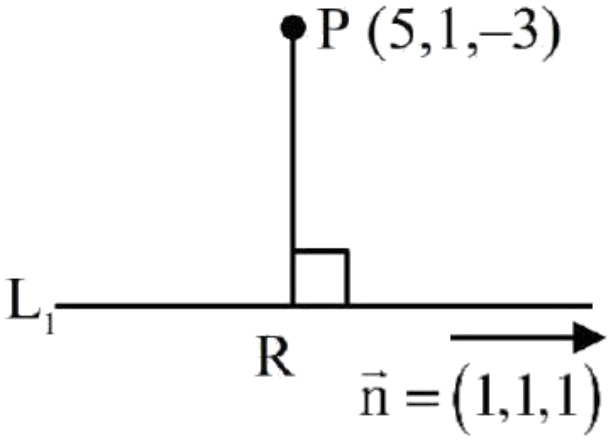
$\mathbf{A=\frac{1}{2}|(-4\hat{i}+\hat{j}+3\hat{k})\times(-3\hat{i}+\hat{j}+4\hat{k})|}$

$\mathbf{A=\frac{1}{2}|7(\hat{i}+\hat{j}+\hat{k})|}$

$\mathbf{\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -4 & 1 & 3\\ -3 & -1 & 4\end{vmatrix}}$

$\mathbf{=7\hat{i}+7\hat{j}+7\hat{k}}$

$\mathbf{4\,A^2=49\times3=147}$


P (5,1,–3)
L1
R
n = (1,1,1)

Question 217

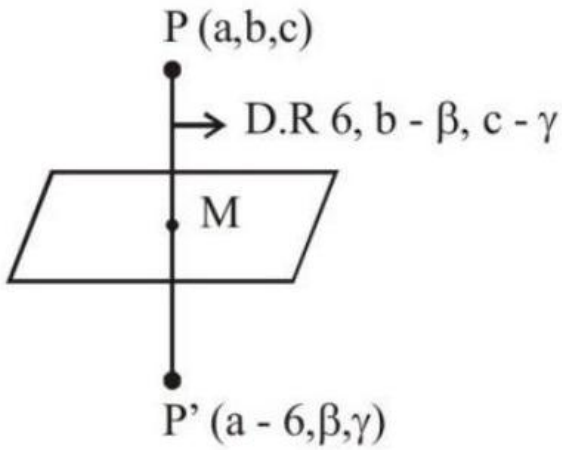
Let the mirror image of the point ($\mathbf{a, b, c}$) with respect to the plane $3x - 4y + 12z + 19 = 0$ be ($\mathbf{a - 6, \beta, \gamma}$). If $\mathbf{a + b + c = 5}$, then $7\beta - 9\gamma$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 137

Source solution



$\mathbf{M} = \left(a - 3, \frac{\beta + b}{2}, \frac{\gamma + c}{2}\right)$

Since M lies on $3x + 4y + 12z + 19 = 0$

$$\Rightarrow 6a - 4b + 12c - 4\beta + 12\gamma + 20 = 0 \quad (1)$$

Since PP' is parallel to normal of the plane then

$\frac{6}{3} = \frac{b - \beta}{-4} = \frac{c - \gamma}{12}$

$\Rightarrow \beta = b + 8, \quad \gamma = c - 24$

$a + b + c = 5 \Rightarrow a + \beta - 8 + \gamma + 24 = 5$

$\Rightarrow a = -\beta - \gamma - 11$

Now putting these values in (1) we get

$6(-\beta-\gamma-11)-4(\beta-8)+12(\gamma+24)-4\beta+12\gamma+20=0$
$\Rightarrow 7\beta-9\gamma=170-33=137$

Question 218

Let the line $L:\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z-3}{1}$ intersect the plane $2x+y+3z=16$ at the point P. Let the point Q be the foot of perpendicular from the point $R(1,-1,-3)$ on the line L. If α is the area of triangle PQR, then α^2 is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 180

Source solution

180
Point on line L is $(2\lambda+1,-\lambda-1,\lambda+3)$
If above point is intersection point of line L and plane then
$2(2\lambda+1)+(-\lambda-1)+3(\lambda+3)=16$
$\lambda=1$
Point $\mathrm{P}=(3,-2,4)$
Dr of $\mathrm{QR}=<2\lambda,-\lambda,\lambda+6>$
Dr of $\mathrm{L}=\langle 2,-1,1\rangle$
$4\lambda+\lambda+\lambda+6=0$
$\lambda=-1$
$\mathrm{Q}=(-1,0,2)$
$C^{P(3,-2,4)}_{(-1,0,2)}\mathrm{R}(1,-1,-3)$
$\overrightarrow{\mathrm{QR}}=2\hat{i}-\hat{j}-5\hat{k}$
$\overrightarrow{\mathrm{QP}}=4\hat{i}-2\hat{j}+2\hat{k}$
$\overrightarrow{\mathrm{QR}}\times\overrightarrow{\mathrm{QP}}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&-1&-5\\4&-2&2\end{vmatrix}=-12\hat{i}-24\hat{j}$
$\alpha=\frac{1}{2}\times\sqrt{144+576}$
$\alpha^2=\frac{720}{4}=180$
$\alpha^2=180$

Question 219

Let a line having direction ratio $1, -4, 2$ intersect the lines $\frac{x-7}{3} = \frac{y-1}{-1} = \frac{z+2}{1}$ and $\frac{x}{2} = \frac{y-7}{3} = \frac{z}{1}$ at the point A and B . Then $(AB)^2$ is equal to .

Numerical answer: _____

Enter the requested value or coefficient rounded to the nearest integer (0 decimal places).

Correct answer: 84

Source solution

Write A=(7+3t,1-t,-2+t), B=(2s,7+3s,s). Requiring B-A to have direction ratios (1,-4,2) gives t=-5,s=-3. Thus A=(-8,6,-7), B=(-6,-2,-3), and $AB^2 = 2^2 + (-8)^2 + 4^2 = 84$. The printed source coordinate +7 for the last component of A is a sign typo.

Question 220

Let a line L pass through the point $P(2,3,1)$ and be parallel to the line $x+3y-2z-2=0=x-y+2\,z$. If the distance of L from the point $(5,3,8)$ is α, then $3\alpha^2$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 158

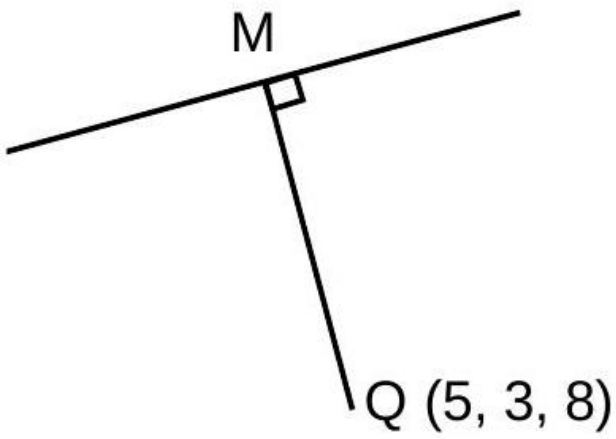
Source solution

158

The Direction ratio of line

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 3 & -2 \\ 1 & -1 & 2 \end{vmatrix} = \mathbf{i}(6-2)-\mathbf{j}(2+2)+\mathbf{k}(-1-3)$$

Equation of line L



$\frac{x-2}{1}=\frac{y-3}{-1}=\frac{z-1}{-1}=\lambda(\alpha \mathrm{d})$

Let $\mathbf{M}(\lambda+2,-\lambda+3,-\lambda+1)$

DR's of MQ is $<\lambda+2-5,-\lambda+3-3,-\lambda+1-8>$

$$<\lambda-3,-\lambda.\,-\lambda>7>$$

$\because \mathbf{L}\perp \mathbf{MQ}$

$\Rightarrow(\lambda-3)(1)+(-\lambda)(-1)+(-\lambda-7)(-1)=0$

$\Rightarrow \lambda - 3 + \lambda + \lambda + 7 = 0$
$\Rightarrow 3\lambda = -4 \Rightarrow \lambda = -\frac{4}{3}$
$\therefore \mathrm{M}\left(-\frac{4}{3}+2, \frac{+4}{3}+3, \frac{4}{3}+1\right) = \left(\frac{2}{3}, \frac{13}{3}, \frac{7}{3}\right)$
$\mathrm{MQ} = \alpha$
$\therefore 3\alpha^2 = 3 \times \left(\left(5-\frac{2}{3}\right)^2 + \left(3-\frac{13}{3}\right)^2 + \left(8-\frac{7}{5}\right)^2\right)$
$= 3\left(\frac{169}{9} + \frac{16}{9} + \frac{289}{9}\right) \Rightarrow \frac{474}{9} = 158$

Question 221

Let the values of p, for which the shortest distance between the lines

$$\frac{x+1}{3}=\frac{y}{4}=\frac{z}{5}$$

and

$$\vec{r}=(p\hat{i}+2\hat{j}+\hat{k})+\lambda(2\hat{i}+3\hat{j}+4\hat{k})$$

is $\frac{1}{\sqrt{6}}$, be $\mathbf{a}, \mathbf{b}, (\mathbf{a}<\mathbf{b})$. Then the length of the latus rectum of the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

is :-

A. 18

B. $\frac{2}{3}$

C. $\frac{3}{2}$

D. 9

Correct answer: B

Source solution

Sol.

The shortest distance is given by the formula:

$$\text{Shortest distance}=\frac{|(\bar{a}-\bar{b})|\cdot(\bar{p}\times\bar{q})}{|\bar{p}\times\bar{q}|}$$

where

$\bar{a} = -\hat{i} + 0\hat{j} + 0\hat{k}$

Next, we calculate $\bar{a} - \bar{b}$:

$$\bar{a} - \bar{b} = (-1 - p)\hat{i} - 2\hat{j} - \hat{k}$$

Given:

$\bar{b} = p\hat{i} + 2\hat{j} + \hat{k}$

$\bar{p} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$\bar{q} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

Now, we set up the equation:

$$\frac{1}{16} = \frac{|-1 - p + 4 - 1|}{\sqrt{6}}$$

This simplifies to:

$$|-p + 2| = 1$$

Solving for p:

$$p = 3 \quad \text{or} \quad p = 1$$

Next, we have the equation of the ellipse:

$$\frac{x^2}{1^2} + \frac{y^2}{3^2} = 1$$

Calculating $L \cdot R$:

$$L \cdot R = \frac{2a^2}{b} = \frac{2 \times 1}{3} = \frac{2}{3}$$

Thus, the final answer is option (3).

Question 222

Let the values of p, for which the shortest distance between the lines $\frac{x+1}{3} = \frac{y}{4} = \frac{z}{5}$ and $\vec{r} = (p\hat{i} + 2\hat{j} + \hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$ is $\frac{1}{\sqrt{6}}$, be $\mathbf{a}, \mathbf{b}, (\mathbf{a} < \mathbf{b})$. Then the length of the latus rectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is :-

A. 9

B. $\frac{3}{2}$

C. $\frac{2}{3}$

D. 18

Correct answer: C

Source solution

Sol. shortest distance $= \frac{|(\bar{a}-\bar{b})| \cdot (\bar{p} \times \bar{q})}{|\bar{p} \times \bar{q}|}$

where

$\bar{a} = -\hat{i} + 0\hat{j} + 0\hat{k}$

$$> \bar{a} - \bar{b} = (-1 - p)\hat{i} - 2\hat{j} - \hat{k}$$

$\bar{b} = p\hat{i} + 2\hat{j} + \hat{k}$

$\bar{p} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$\bar{q} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\frac{1}{16} = \frac{|-1-p+4-1|}{\sqrt{6}}$

$|-p + 2| = 1$

$p = 3 \quad \& \quad q = 1$

$\frac{x^2}{1^2} + \frac{y^2}{3^3} = 1$

$L \cdot R = \frac{2a^2}{b} = \frac{2 \times 1}{3} = \frac{2}{3}$

option (3)

Question 223

If the mirror image of the point $P(3,4,9)$ in the line $\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-2}{1}$ is (α,β,γ), then $14(\alpha+\beta+\gamma)$ is :

A. 102

B. 138

C. 108

D. 132

Correct answer: C

Source solution

$$\text{Given: } \overrightarrow{\text{PN}} \cdot \vec{\text{b}} = 0$$

$$3(3\lambda-2)+2(2\lambda-5)+(\lambda-7)=0$$

$$9\lambda-6+4\lambda-10+\lambda-7=0$$

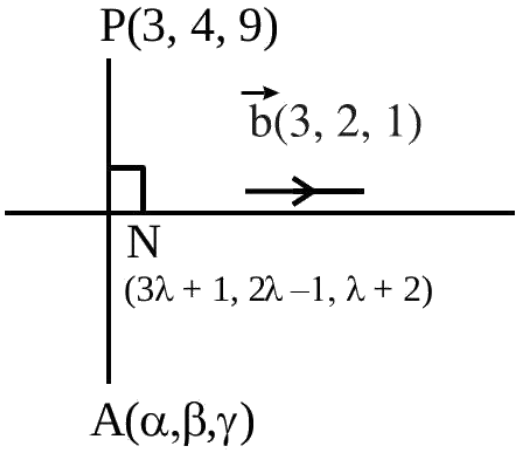
$$14\lambda-23=0 \quad \Rightarrow \quad 14\lambda=23 \quad \Rightarrow \quad \lambda=\frac{23}{14}$$

$$\text{Coordinates of N} = \left(\frac{83}{14},\frac{32}{14},\frac{51}{14}\right)$$

$$\frac{\alpha+3}{2}=\frac{83}{14} \quad \Rightarrow \quad \alpha=\frac{62}{7}$$

$$\frac{\beta+4}{2}=\frac{32}{14} \quad \Rightarrow \quad \beta=\frac{4}{7}$$

$$\frac{\gamma+9}{2}=\frac{51}{14} \quad \Rightarrow \quad \gamma=\frac{-12}{7}$$



Question 224

Let A and B be two distinct points on the line $\mathbf{L}: \frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$. Both A and B are at a distance $2\sqrt{17}$ from the foot of perpendicular drawn from the point $(1,2,3)$ on the line L . If O is the origin, then $\overrightarrow{OA} \cdot \overrightarrow{OB}$ is equal to:

A. 49

B. 47

C. 21

D. 62

Correct answer: B

Source solution

Sol.

$\overrightarrow{PQ} \cdot \vec{b}=0$

$\Rightarrow 3(3\lambda+5)+2(2\lambda+5)-2(-2\lambda+4)$

$\Rightarrow 17\lambda=-17 \Rightarrow \lambda=-1$

Q (3,5,9)

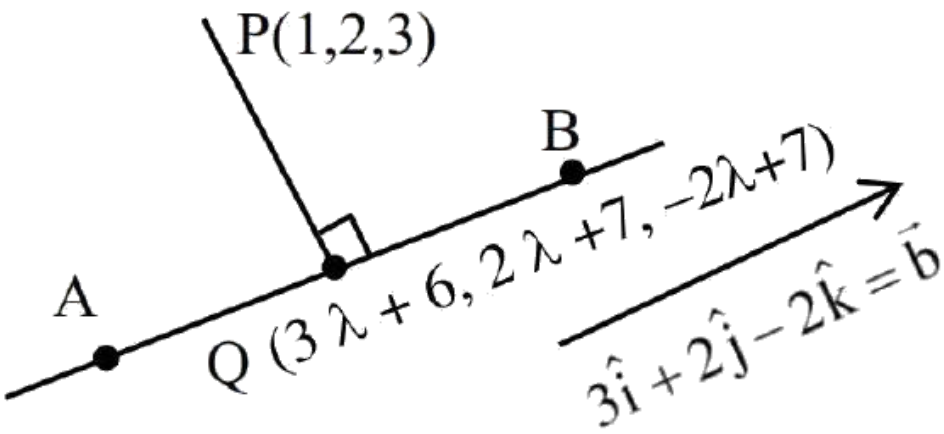
Let $A(3\mu+6, 2\mu+7, -2\mu+7)$

$(3\mu+3)^2+(2\mu+2)^2+(-2\mu-2)^2=68$

$\Rightarrow \mu^2+2\mu-3=0 \mu=-3$ or $\mu=1$

$A(-3,1,13)$ and $B(9,9,5)$

$\overrightarrow{OA} \cdot \overrightarrow{OB}=-27+9+65=47$



Question 225

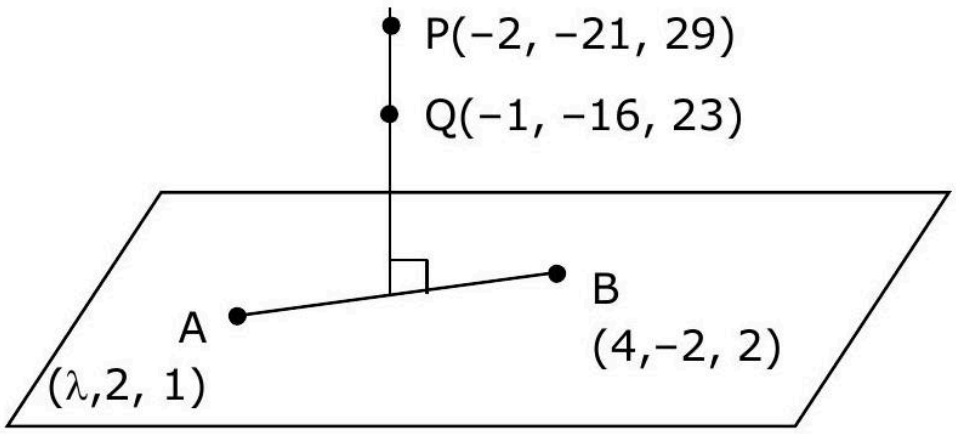
Let $(\lambda, 2, 1)$ be a point on the plane which passes through the point $(4, -2, 2)$. If the plane is perpendicular to the line joining the point $(-2, -21, 29)$ and $(-1, -16, 23)$, then $\left(\frac{\lambda}{11}\right)^2 - \frac{4\lambda}{11} - 4$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 8

Source solution



$\overrightarrow{AB} \perp \overrightarrow{PQ}$

$$[(4-\lambda)\hat{i} - 4\hat{j} + \hat{k}] \cdot [+\hat{i} + 5\hat{j} - 6\hat{k}] = 0$$
$$4 - \lambda - 20 - 6 = 0$$
$$\lambda = -22$$

Now, $\frac{\lambda}{11} = -2$

$$\Rightarrow \left(\frac{\lambda}{11}\right)^2 - \frac{4\lambda}{11} - 4$$
$$\Rightarrow 4 + 8 - 4 = 8$$

Question 226

Let P and Q be the points on the line

$$\frac{x+3}{8}=\frac{y-4}{2}=\frac{z+1}{2}$$

which are at a distance of 6 units from the point $\mathbf{R(1,2,3)}$. If the centroid of the triangle PQR is (α,β,γ), then $\alpha^2+\beta^2+\gamma^2$ is:

A. 26

B. 36

C. 18

D. 24

Correct answer: C

Source solution

$$\text{Let P}(8\lambda-3,2\lambda+4,2\lambda-1)$$

$$\text{Given PR}=6$$

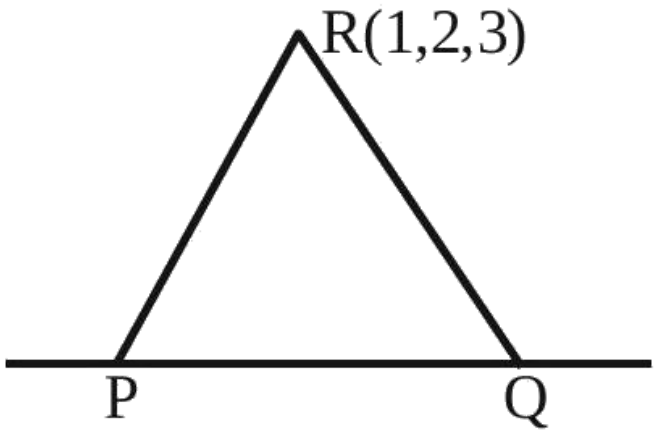
$$(8\lambda-4)^2+(2\lambda+2)^2+(2\lambda-4)^2=36$$

$$\text{Solving gives } \lambda=0 \text{ or } 1$$

$$\text{Thus, P}(-3,4,-1) \text{ and Q}(5,6,1)$$

$$\text{Centroid of } \triangle\text{PQR}=(1,4,1)\equiv(\alpha,\beta,\gamma)$$

$$\alpha^2+\beta^2+\gamma^2=18$$


R(1,2,3)
P
Q

Question 227

If the length of the perpendicular drawn from the point $\mathbf{P(a, 4, 2), a > 0}$ on the line $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1}$ is $2\sqrt{6}$ units and $\mathbf{Q}\left(\alpha_1, \alpha_2, \alpha_3\right)$ is the image of the point P in this line, then $\mathbf{a} + \sum_{i=1}^{3} \alpha_i$ is equal to :

A. 7

B. 8

C. 12

D. 14

Correct answer: B

Source solution

Sol.

$(2\lambda - 1 - a)2 + (3\lambda - 1)3 + (-\lambda - 1)(-1) = 0$

$\Rightarrow 4\lambda - 2 - 2a + 9\lambda - 3 + \lambda + 1 = 0$

$\Rightarrow 14\lambda - 4 - 2a = 0$

$\Rightarrow 7\lambda - 2 - a = 0$

and,

$(2\lambda - 1 - a)^2 + (3\lambda - 1)^2 + (\lambda + 1)^2 = 24$

$\Rightarrow (5\lambda - 1)^2 + (3\lambda - 1)^2 + (\lambda + 1)^2 = 24$

$\Rightarrow 35\lambda^2 - 14\lambda - 21 = 0$

$\Rightarrow (\lambda - 1)(35\lambda + 21) = 0$

For, $\lambda = 1 \quad \Rightarrow a = 5$

Let $(\alpha_1, \alpha_2, \alpha_3)$ be reflection of point P

$\alpha_1 + 5 = 2$

$\alpha_2 + 4 = 12$

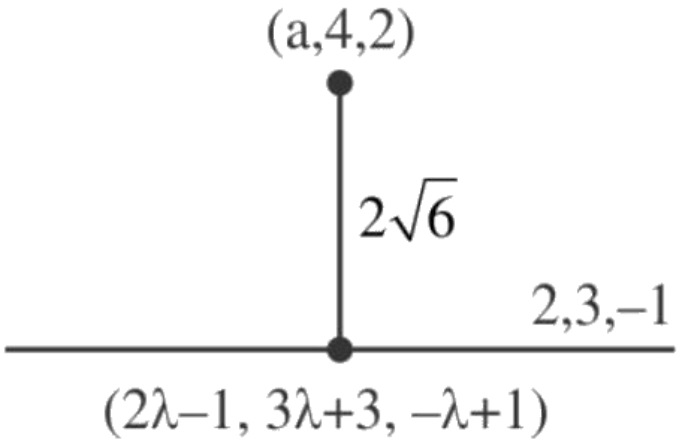
$\alpha_3 + 2 = 0$

$\alpha_1 = -3$

$\alpha_2 = 8$

$\alpha_3 = -2$

$a + \alpha_1 + \alpha_2 + \alpha_3 = 8$


(a,4,2)
$2\sqrt{6}$
2,3,–1
(2λ–1, 3λ+3, –λ+1)

Question 228

Let P be the plane containing the straight line

$$\frac{x-3}{9}=\frac{y+4}{-1}=\frac{z-7}{-5}$$

and perpendicular to the plane containing the straight lines

$$\frac{x}{2}=\frac{y}{3}=\frac{z}{5}$$

and

$$\frac{x}{3}=\frac{y}{7}=\frac{z}{8}.$$

If d is the distance of P from the point $(2,-5,11)$, then d^2 is equal to :

A. $\frac{147}{2}$

B. 96

C. $\frac{32}{3}$

D. 54

Correct answer: C

Source solution

Sol.

Given the equation of the plane:

$$a(x-3)+b(y+4)+c(z-7)=0$$

We have the following system of equations:

$$9a - b - 5c = 0$$
$$-11a - b + 5c = 0$$

Solving this system, we find the direction ratios proportional to $(1, -1, 2)$.

Thus, the equation of the plane can be expressed as:

$$x - y + 2z = 21$$

Next, we calculate the distance d from the point to the plane:

$$d = \frac{8}{\sqrt{6}}$$

Squaring the distance gives:

$$d^2 = \frac{32}{3}$$

Question 229

Let P be a plane containing the line $\frac{x-1}{3}=\frac{y+6}{4}=\frac{z+5}{2}$ and parallel to the line $\frac{x-3}{4}=\frac{y-2}{-3}=\frac{z+5}{7}$. If the point $(1,-1,\alpha)$ lies on the plane P , then the value of $|5\alpha|$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 38

Source solution

Equation of plane is $\begin{vmatrix} x-1 & y+6 & z+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix}=0$

Now $(1,-1,\alpha)$ lies on it so

$\begin{vmatrix} 0 & 5 & \alpha+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix}=0 \Rightarrow 5\alpha+38=0 \Rightarrow |5\alpha|=38$

Question 230

Let the line $\frac{x-3}{7}=\frac{y-2}{-1}=\frac{z-3}{-4}$ intersect the plane containing the lines $\frac{x-4}{1}=\frac{y+1}{-2}=\frac{z}{1}$ and $4ax-y+5z-7a=0=2x-5y-z-3, a\in\mathbb{R}$ at the point $P(\alpha,\beta,\gamma)$. Then the value of $\alpha+\beta+\gamma$ equals .

Numerical answer: _____

Enter the requested value or coefficient rounded to the nearest integer (0 decimal places).

Correct answer: 12

Source solution

Use the plane family $4ax-y+5z-7a+\lambda(2x-5y-z-3)=0$. Containing the point (4,-1,0) gives $9a+1+10\lambda=0$; containing the direction (1,-2,1) gives $4a+7+11\lambda=0$. Hence a=1, lambda=-1 and the plane is $x+2y+3z-2=0$. Substitution of (7t+3,2-t,3-4t) gives t=2, so P=(17,0,-5), whose coordinate sum is 12.

Question 231

Let a line with direction ratios $\mathbf{a, -4a, -7}$ be perpendicular to the lines with direction ratios 3 , $\mathbf{-1, 2b}$ and $\mathbf{b, a, -2}$. If the point of intersection of the line $\frac{x+1}{a^2+b^2} = \frac{y-2}{a^2-b^2} = \frac{z}{1}$ and the plane $\mathbf{x - y + z = 0}$ is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to

.

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 10

Source solution

$(\mathrm{a}, -4\mathrm{a}, -7) \perp$ to $(3, -1, 2\ \mathrm{b})$

$$\mathrm{a} = 2\ \mathrm{b} \qquad \text{(i)}$$

$(\mathrm{a}, -4\mathrm{a}, -7) \perp$ to $(\mathrm{b}, \mathrm{a}, -2)$

$3a + 4a - 14b = 0$

$$ab - 4a^2 + 14 = 0 \qquad \text{(ii)}$$

From Equations (i) and (ii)

$2b^2 - 16b^2 + 14 = 0$

$\mathrm{b}^2 = 1$

$a^2 = 4b^2 = 4$

$\frac{\mathrm{x}+1}{5} = \frac{\mathrm{y}-2}{3} = \frac{\mathrm{z}}{1} = \mathrm{k}$

$\alpha = 5\mathrm{k} - 1, \beta = 3\mathrm{k} + 2, \gamma = \mathrm{k}$

As (α, β, γ) satisfies $\mathrm{x - y + z = 0}$

$5\mathrm{k} - 1 - (3\mathrm{k} + 2) + \mathrm{k} = 0$

$\mathbf{k = 1}$

$\therefore \alpha + \beta + \gamma = 9\mathbf{k} + 1 = 10$

Question 232

Let the mirror image of the point $(1, 3, a)$ with respect to the plane $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) - b = 0$ be $(-3, 5, 2)$. Then the value of $|a + b|$ is equal to

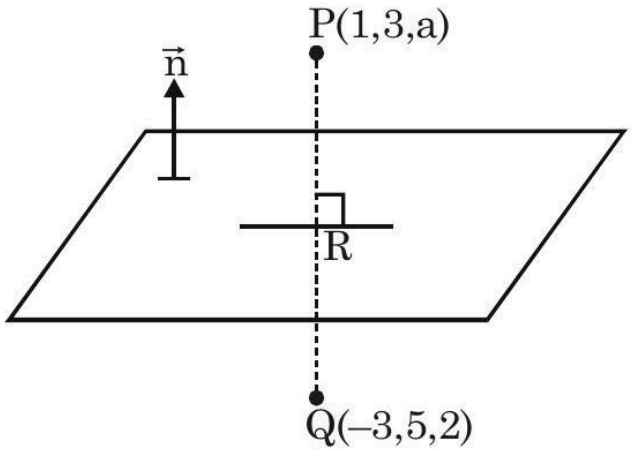
.

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 1

Source solution



plane $= 2x - y + z = b$

$R \equiv \left(-1, 4, \frac{a+2}{2}\right) \rightarrow$ on plane

$\therefore -2 - 4 + \frac{a+2}{2} = b$

$$\Rightarrow a + 2 = 2b + 12 \Rightarrow a = 2b + 10 \quad \text{(i)}$$

$< PQ > = < 4, -2, a - 2 >$

$\therefore \frac{2}{4} = \frac{-1}{-2} = \frac{1}{a-2}$

$\Rightarrow a - 2 = 2 \Rightarrow a = 4,\ b = -3$

$\therefore |a + b| = 1$

Question 233

Let the lines $\frac{x-1}{\lambda}=\frac{y-2}{1}=\frac{z-3}{2}$ and $\frac{x+26}{-2}=\frac{y+18}{3}=\frac{z+28}{\lambda}$ be coplanar and P be the plane containing these two lines. Then which of the following points does NOT lies on P?

A. $(0,-2,-2)$

B. $(-5,0,-1)$

C. $(3,-1,0)$

D. $(0,4,5)$

Correct answer: D

Source solution

Sol. Given, $\mathbf{L_1}: \frac{x-1}{\lambda}=\frac{y-2}{1}=\frac{z-3}{2}$ and $\mathbf{L_2}: \frac{x+26}{-2}=\frac{y+18}{3}=\frac{z+28}{\lambda}$ are coplanar

$\Rightarrow \begin{vmatrix} 27 & 20 & 31\lambda \\ 1 & 2-2 & 3 \\ \lambda & 3 & 0 \end{vmatrix}=0$

$\Rightarrow \lambda=3$

Now, normal of plane P , which contains $\mathbf{L_1}$ and $\mathbf{L_2}$

$=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k}3 \\ 1 & 2-2 & 3 \\ 3 & 3 & 0 \end{vmatrix}$

$=-3\hat{\mathbf{i}}-13\hat{\mathbf{j}}+11\hat{\mathbf{k}}$

$\Rightarrow$ Equation of required plane P :

$3x+13y-11z+4=0$

($0,4,5$) does not lie on plane P.

Question 234

Let $\mathbf{P}(\alpha, \beta, \gamma)$ be the point on the line

$$\frac{x-1}{2} = \frac{y+1}{-3} = z \text{ at a distance } 4\sqrt{14} \text{ from the}$$

point (1, -1, 0) and nearer to the origin. Then the shortest distance, between the lines $\frac{x-\alpha}{1} = \frac{y-\beta}{2} = \frac{z-\gamma}{3}$ and $\frac{x+5}{2} = \frac{y-10}{1} = \frac{z-3}{1}$, is equal to

A. $7\sqrt{\frac{5}{4}}$

B. $4\sqrt{\frac{7}{5}}$

C. $4\sqrt{\frac{5}{7}}$

D. $2\sqrt{\frac{7}{4}}$

Correct answer: B

Source solution

Let $\mathbf{P}(2\lambda+1, -3\lambda-1, \lambda)$

Then $4\lambda^2 + 9\lambda^2 + \lambda^2 = 16 \cdot 14 \Rightarrow \lambda = \pm 4 \Rightarrow -4$

(nearer to origin)

$$\therefore \mathbf{P}(-7, 11, -4)$$

$$\therefore \text{Shortest distance} = \frac{\begin{vmatrix} 2 & -1 & 7 \\ 1 & 2 & 3 \\ 2 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & k \\ 1 & 2 & 3 \\ 2 & 1 & 1 \end{vmatrix}}$$

$$= \frac{28}{\sqrt{1+25+9}} = \frac{4\sqrt{7}}{\sqrt{5}}$$

Question 235

If (x, y, z) be an arbitrary point lying on a plane P which passes through the point $(42, 0, 0)$, $(0, 42, 0)$ and $(0, 0, 42)$, then the value of expression

$3+\frac{x-11}{(y-19)^2(z-12)^2}+\frac{y-19}{(x-11)^2(z-12)^2}$
$+\frac{z-12}{(x-11)^2(y-19)^2}-\frac{x+y+z}{14(x-11)(y-19)(z-12)}$

A. 0

B. 3

C. 39

D. -45

Official Answer by NTA (2)

Correct answer: B

Source solution

Sol. Plane passing through $(42, 0, 0), (0, 42, 0)$, (0, 0, 42)

From intercept from, equation of plane is

$x+y+z=42$

$\Rightarrow (x-11)+(y-19)+(z-12)=0$

let $a=x-11,\ b=y-19, c=z-12$

$a+b+c=0$

Now, given expression is

$3+\frac{a}{b^2c^2}+\frac{b}{a^2c^2}+\frac{c}{a^2b^2}-\frac{42}{14abc}$

$3+\frac{a^3+b^3+c^3-3abc}{a^2b^2c^2}$

If $a+b+c=0$

$\Rightarrow a^3+b^3+c^3=3abc$

$\Rightarrow 3$

Question 236

Let P be the plane containing the straight line $\frac{x-3}{9}=\frac{y+4}{-1}=\frac{z-7}{-5}$ and perpendicular to the plane containing the straight lines $\frac{x}{2}=\frac{y}{3}=\frac{z}{5}$ and $\frac{x}{3}=\frac{y}{7}=\frac{z}{8}$. If d is the distance of P from the point $(2,-5,11)$, then d^2 is equal to :

A. $\frac{147}{2}$

B. 96

C. $\frac{32}{3}$

D. 54

Correct answer: D

Source solution

Sol. $a(x-3)+b(y+4)+c(z-7)=0$

$$P: 9a-b-5c=0 \quad -11a-b+5c=0$$

After solving DR's $\propto(1,-1,2)$

Equation of plane

$x-y+2z=21$

$d=\frac{8}{\sqrt{6}}$

$d^2=\frac{32}{3}$

Question 237

The line of shortest distance between the lines $\frac{x-2}{0}=\frac{y-1}{1}=\frac{z}{1}$ and $\frac{x-3}{2}=\frac{y-5}{2}=\frac{z-1}{1}$ makes an angle of $\cos^{-1}\left(\sqrt{\frac{2}{27}}\right)$ with the plane $P: ax-y-z=0,(a>0)$. If the image of the point $(1,1,-5)$ in the plane P is (α,β,γ), then $\alpha+\beta-\gamma$ is equal to

.

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 3

Source solution

DR's of line of shortest distance

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 1 & 1\\ 2 & 2 & 1\end{vmatrix}=-\hat{i}+2\hat{j}-2\hat{k}$$

angle between line and plane is $\cos^{-1}\sqrt{\frac{2}{27}}=\alpha$

$\cos\alpha=\sqrt{\frac{2}{27}},\sin\alpha=\frac{5}{3\sqrt{3}}$

DR's normal to plane ($1,-1,-1$)

$\sin\alpha=\left|\frac{-a-2+2}{\sqrt{4+4+1}\sqrt{a^2+1+1}}\right|=\frac{5}{3\sqrt{3}}$

$\sqrt{3}|a|=5\sqrt{a^2+2}$

$3a^2=25a^2+50$

No value of (a)

Question 238

Let P be a plane $l\mathbf{x}+\mathbf{m y}+\mathbf{n z}=0$ containing the line, $\frac{1-x}{1}=\frac{y+4}{2}=\frac{z+2}{3}$. If plane P divides the line segment AB joining points $\mathbf{A}(-3,-6,1)$ and $\mathbf{B}(2,4,-3)$ in ratio $\mathbf{k}:\mathbf{1}$ then the value of k is equal to :

A. 1.5

B. 3

C. 2

D. 4

Correct answer: C

Source solution

Sol.

Point C is
$\left(\frac{2 \mathrm{k}-3}{\mathrm{k}+1}, \frac{4 \mathrm{k}-6}{\mathrm{k}+1}, \frac{-3 \mathrm{k}+1}{\mathrm{k}+1}\right)$
$\frac{\mathrm{x}-1}{-1}=\frac{\mathrm{y}+4}{2}=\frac{\mathrm{z}+2}{3}$
Plane $l\mathbf{x}+\mathbf{m y}+\mathbf{n z}=0$
$l(-1)+\mathrm{m}(2)+\mathrm{n}(3)=0$

$$-l+2 \mathrm{~m}+3 \mathrm{n}=0 \quad (1)$$

It also satisfy point ($1,-4,-2$)

$$l-4 \mathrm{~m}-2 \mathrm{n}=0 \quad (2)$$

Solving (1) and (2)
$2 \mathrm{~m}+3 \mathrm{n}=4 \mathrm{~m}+2 \mathrm{n}$
$\mathrm{n}=2 \mathrm{~m}$
$l-4 \mathrm{~m}-4 \mathrm{~m}=0$
$l=8 \mathrm{~m}$
$\frac{l}{8}=\frac{\mathrm{m}}{1}=\frac{\mathrm{n}}{2}$

$l : m : n = 8 : 1 : 2$

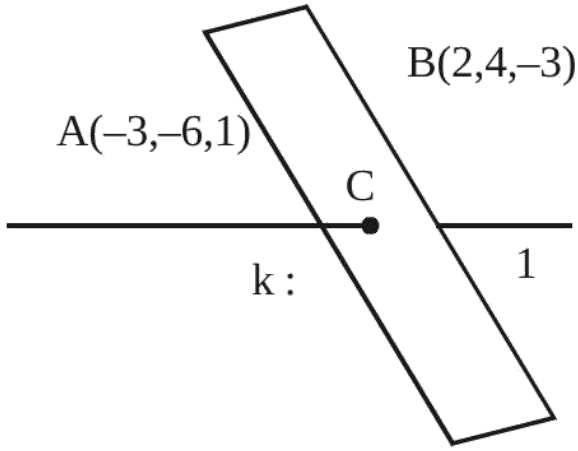
Plane is $8x + y + 2z = 0$

It will satisfy point C

$8\left(\frac{2k-3}{k+1}\right) + \left(\frac{4k-6}{k+1}\right) + 2\left(\frac{-3k+1}{k+1}\right) = 0$

$16k - 24 + 4k - 6 - 6k + 2 = 0$

$14k = 28 \quad \therefore \quad k = 2$



Question 239

The equation of the line passing through $(-4, 3, 1)$, parallel to the plane $\mathbf{x + 2y - z - 5 = 0}$ and intersecting the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$ is:

A. $\frac{x+4}{-1} = \frac{y-3}{1} = \frac{z-1}{1}$

B. $\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$

C. $\frac{x+4}{1} = \frac{y-3}{1} = \frac{z-1}{3}$

D. $\frac{x-4}{2} = \frac{y+3}{1} = \frac{z+1}{4}$

Correct answer: B

Source solution

Sol.

Normal vector of plane containing two intersecting lines is parallel to vector.

$$\left(\vec{V}_1\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k}3 & 0 & 1-3 & 2 & -1 \end{vmatrix}$$

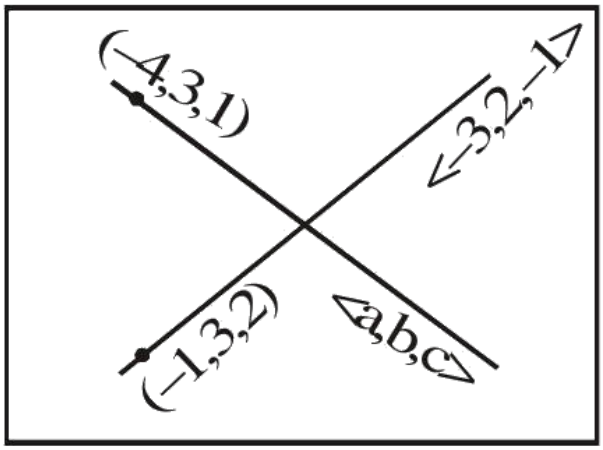
$$= -2\hat{i} + 6\hat{k}$$

$\therefore$ Required line is parallel to vector

$$\left(\vec{V}_2\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k}1 & 2 & -1-2 & 0 & 6 \end{vmatrix} = 3\hat{i} - \hat{j} + \hat{k}$$

$\Rightarrow$ Required equation of line is

$\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$



Question 240

On which of the following lines lies the point of intersection of the line, $\frac{x-4}{2}=\frac{y-5}{2}=\frac{z-3}{1}$ and the plane, $x+y+z=2$?

A. $\frac{x-2}{2}=\frac{y-3}{2}=\frac{z+3}{3}$

B. $\frac{x-4}{1}=\frac{y-5}{1}=\frac{z-5}{-1}$

C. $\frac{x-1}{1}=\frac{y-3}{2}=\frac{z+4}{-5}$

D. $\frac{x+3}{3}=\frac{4-y}{3}=\frac{z+1}{-2}$

Correct answer: C

Source solution

Sol. General point on the given line is

$x=2\lambda+4$

$y=2\lambda+5$

$z=\lambda+3$.<.

Solving with plane,

$2\lambda+4+2\lambda+5+\lambda+3=2$

$5\lambda+12=2$

$5\lambda=-10$

$$\lambda=-2$$

$\therefore$ Option (3) is the correct answer.

Question 241

The equation of the line passing through $(-4, 3, 1)$, parallel to the plane $x + 2y - z - 5 = 0$ and intersecting the line

$$\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$$

is:

A. $\frac{x+4}{-1} = \frac{y-3}{1} = \frac{z-1}{1}$

B. $\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$

C. $\frac{x+4}{1} = \frac{y-3}{1} = \frac{z-1}{3}$

D. $\frac{x-4}{2} = \frac{y+3}{1} = \frac{z+1}{4}$

Correct answer: B

Source solution

Sol.

1. The normal vector of the plane containing two intersecting lines is parallel to the vector:

$$\vec{V}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 1 \\ -3 & 2 & -1 \end{vmatrix}$$

2. Calculating the determinant:

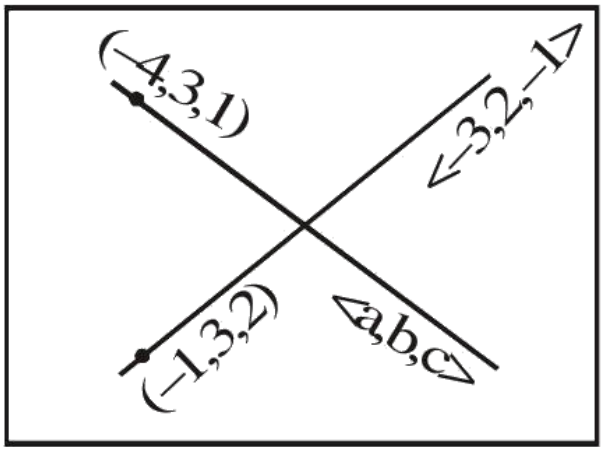
$$\vec{V}_1 = -2\hat{i} + 6\hat{k}$$

3. Therefore, the required line is parallel to the vector:

$$\vec{V}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ -2 & 0 & 6 \end{vmatrix} = 3\hat{i} - \hat{j} + \hat{k}$$

4. The required equation of the line is:

$$\frac{x+4}{3}=\frac{y-3}{-1}=\frac{z-1}{1}$$



Question 242

If the point $(2, \alpha, \beta)$ lies on the plane which passes through the points $(3, 4, 2)$ and $(7, 0, 6)$ and is perpendicular to the plane $2x - 5y = 15$, then $2\alpha - 3\beta$ is equal to :-

A. 5

B. 17

C. 12

D. 7

Correct answer: D

Source solution

Sol. Normal vector of plane

$= |\mathbf{i} \quad \mathbf{j} \quad \mathbf{k2} \quad -5 \quad 04 \quad -4 \quad 4| = -4(5\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}})$

equation of plane is $5(x-7) + 2y - 3(z-6) = 0\,5\mathbf{x} + 2\mathbf{y} - 3\mathbf{z} = 17$

Question 243

If the point $(2, \alpha, \beta)$ lies on the plane which passes through the points $(3, 4, 2)$ and $(7, 0, 6)$ and is perpendicular to the plane $2x - 5y = 15$, then $2\alpha - 3\beta$ is equal to :-

A. 5

B. 17

C. 12

D. 7

Correct answer: B

Source solution

Sol.

To find the equation of the plane, we first determine the normal vector using the determinant:

$$\text{Normal vector} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -5 & 0 \\ 4 & -4 & 4 \end{vmatrix}$$

Calculating the determinant, we have:

$$= -4(5\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}})$$

Thus, the normal vector is:

$$\mathbf{n} = -4(5, 2, -3)$$

Next, we use the point-normal form of the plane equation. Given a point $(7, 0, 6)$ on the plane, the equation can be expressed as:

$$5(x - 7) + 2(y - 0) - 3(z - 6) = 0$$

Expanding this, we get:

$$5x - 35 + 2y - 3z + 18 = 0$$

Simplifying, we arrive at the final equation of the plane:

$$5x + 2y - 3z = 17$$

Question 244

The plane containing the line $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z-1}{3}$ and also containing its projection on the plane $2x + 3y - z = 5$, contains which one of the following points ?

A. $(2, 0, -2)$

B. (-2, 2, 2)

C. $(0, -2, 2)$

D. $(2, 2, 0)$

Correct answer: A

Source solution

Sol. The normal vector of required plane
$= (2\hat{i} - \hat{j} + 3\hat{k}) \times (2\hat{i} + 3\hat{j} - \hat{k})$
$= -8\hat{\mathrm{i}} + 8\hat{\mathrm{j}} + 8\hat{\mathrm{k}}$
So, direction ratio of normal is $(-1, 1, 1)$
So required plane is
$-(\mathrm{x} - 3) + (\mathrm{y} + 2) + (\mathrm{z} - 1) = 0$
$\Rightarrow -\mathrm{x} + \mathrm{y} + \mathrm{z} + 4 = 0$
Which is satisfied by (2, $0, -2$)

Question 245

The plane containing the line

$$\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z-1}{3}$$

and also containing its projection on the plane

$$2x+3y-z=5,$$

contains which one of the following points?

A. $(2,0,-2)$

B. (-2, 2, 2)

C. $(0,-2,2)$

D. $(2,2,0)$

Correct answer: A

Source solution

Sol.

1. Find the normal vector of the required plane:

$$\mathbf{n}=(2\hat{i}-\hat{j}+3\hat{k})\times(2\hat{i}+3\hat{j}-\hat{k})$$

$$\mathbf{n}=-8\hat{i}+8\hat{j}+8\hat{k}$$

2. The direction ratios of the normal vector are $(-1,1,1)$.

3. The equation of the required plane can be expressed as:

$$-(x-3)+(y+2)+(z-1)=0$$

Simplifying this gives:

$$-x+y+z+4=0$$

4. Verify that the point $(2,0,-2)$ satisfies the plane equation:

$$-2+0-2+4=0$$

Thus, the point $(2,0,-2)$ lies on the required plane.

Question 246

The plane passing through the point $(4, -1, 2)$ and parallel to the lines

$$\frac{x+2}{3} = \frac{y-2}{-1} = \frac{z+1}{2}$$

and

$$\frac{x-2}{1} = \frac{y-3}{2} = \frac{z-4}{3}$$

also passes through the point:

A. $(-1, -1, -1)$

B. $(-1, -1, 1)$

C. $(1, 1, -1)$

D. $(1, 1, 1)$

Correct answer: C

Source solution

Sol.

Let $\vec{n}$ be the normal vector to the plane passing through the point $(4, -1, 2)$ and parallel to the lines $\mathbf{L_1}$ and $\mathbf{L_2}$.

We can find $\vec{n}$ using the determinant:

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 2 \\ 1 & 2 & 3 \end{vmatrix}$$

Calculating the determinant, we find:

$$\overrightarrow{\mathrm{n}} = -7\hat{\mathrm{i}} - 7\hat{\mathrm{j}} + 7\hat{\mathrm{k}}$$

The equation of the plane can be expressed as:

$$-1(x-4) - 1(y+1) + 1(z-2) = 0$$

Simplifying this, we get:

$$x + y - z - 1 = 0$$

Now, we can check the options.

Question 247

The perpendicular distance from the origin to the plane containing the two lines,

$$\frac{x+2}{3}=\frac{y-2}{5}=\frac{z+5}{7}$$

and

$$\frac{x-1}{1}=\frac{y-4}{4}=\frac{z+4}{7},$$

is:

A. $11/\sqrt{6}$

B. $6\sqrt{11}$

C. 11

D. $11\sqrt{6}$

Correct answer: A

Source solution

Step 1: Calculate the determinant

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix}$$

Step 2: Expand the determinant

$$= \hat{i}(5 \cdot 7 - 4 \cdot 7) - \hat{j}(3 \cdot 7 - 1 \cdot 7) + \hat{k}(3 \cdot 4 - 5 \cdot 1)$$

$$= \hat{i}(35 - 28) - \hat{j}(21 - 7) + \hat{k}(12 - 5)$$

$$= 7\hat{i} - 14\hat{j} + 7\hat{k}$$

Step 3: Factor out common terms

$$= \hat{i} - 2\hat{j} + \hat{k}$$

Step 4: Write the equation of the plane

$$1(x + 2) - 2(y - 2) + 1(z + 15) = 0$$

$$\Rightarrow x - 2y + z + 11 = 0$$

Step 5: Calculate the distance from the origin to the plane

$$d = \frac{|11|}{\sqrt{1^2 + (-2)^2 + 1^2}} = \frac{11}{\sqrt{1 + 4 + 1}} = \frac{11}{\sqrt{6}}$$

Question 248

The plane passing through the point $(4, -1, 2)$ and parallel to the lines $\frac{x+2}{3} = \frac{y-2}{-1} = \frac{z+1}{2}$ and $\frac{x-2}{1} = \frac{y-3}{2} = \frac{z-4}{3}$ also passes through the point :

A. $(-1, -1, -1)$

B. $(-1, -1, 1)$

C. $(1, 1, -1)$

D. $(1, 1, 1)$

Correct answer: D

Source solution

Sol. Let $\vec{n}$ be the normal vector to the plane passing through $(4, -1, 2)$ and parallel to the lines L_1 & L_2 then

$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k}3 \\ -1 & 21 & 2 \\ 3 & & \end{vmatrix}$

$\therefore \vec{n} = -7\hat{i} - 7\hat{j} + 7\hat{k}$

$\therefore$ Equation of plane is

$-1(x-4) - 1(y+1) + 1(z-2) = 0$

$\therefore x + y - z - 1 = 0$

Question 249

The plane through the intersection of the planes $x+y+z=1$ and $2x+3y-z+4=0$ and parallel to y-axis also passes through the point :

A. $(-3,0,-1)$

B. $(3,3,-1)$

C. $(3,2,1)$

D. $(-3,1,1)$

Correct answer: C

Source solution

Sol. Equation of plane

$$(x+y+z-1)+\lambda(2x+3y-z+4)=0 \Rightarrow \quad (1+2\lambda)x+(1+3\lambda)y+(1-\lambda)z-1+4\lambda=0$$

dr's of normal of the plane are

$$1+2\lambda, 1+3\lambda, 1-\lambda$$

Since plane is parallel to y - axis, $1+3\lambda=0$

$$\Rightarrow \quad \lambda=-1/3$$

So the equation of plane is

$$x+4z-7=0$$

Point (3, 2, 1) satisfies this equation
Hence Answer is (3)

Question 250

Two lines $\frac{x-3}{1}=\frac{y+1}{3}=\frac{z-6}{-1}$ and $\frac{x+5}{7}=\frac{y-2}{-6}=\frac{z-3}{4}$ intersect at the point R. The reflection of R in the xy-plane has coordinates :-

A. $(2,4,7)$

B. $(-2,4,7)$

C. $(2,-4,-7)$

D. $(2,-4,7)$

Correct answer: C

Source solution

Sol. Point on $L_1(\lambda+3,3\lambda-1,-\lambda+6)$
Point on $L_2(7\mu-5,-6\mu+2,4\mu+3)$

$$\Rightarrow \lambda+3=7\mu-5 \quad \text{(i)}$$

$3\lambda-1=-6\mu+2$
... (ii) $\Rightarrow \lambda=-1,\mu=1$
point $R(2,-4,7)$
Reflection is $(2,-4,-7)$

Question 251

The plane through the intersection of the planes $x+y+z=1$ and $2x+3y-z+4=0$ and parallel to the y-axis also passes through the point:

A. $(-3,0,-1)$

B. $(3,3,-1)$

C. $(3,2,1)$

D. $(-3,1,1)$

Correct answer: C

Source solution

Sol. Equation of the plane:

Starting with the equation:

$$(x+y+z-1)+\lambda(2x+3y-z+4)=0$$

we can rearrange it to:

$$(1+2\lambda)x+(1+3\lambda)y+(1-\lambda)z-(1-4\lambda)=0$$

The direction ratios (dr's) of the normal to the plane are:

$$1+2\lambda,\quad 1+3\lambda,\quad 1-\lambda$$

Since the plane is parallel to the y-axis, we set:

$$1+3\lambda=0$$

Solving for λ:

$$\lambda = -\frac{1}{3}$$

Substituting λ back into the equation of the plane gives:

$$x + 4z - 7 = 0$$

To verify, we check if the point $(3, 2, 1)$ satisfies this equation:

$$3 + 4(1) - 7 = 0$$

Thus, the point satisfies the equation.

Hence, the answer is 3.

Question 252

the perpendicular distance from the origin to the plane containing the two lines, $\frac{x+2}{3}=\frac{y-2}{5}=\frac{z+5}{7}$ and $\frac{x-1}{1}=\frac{y-4}{4}=\frac{z+4}{7}$, is:

A. $11/\sqrt{6}$

B. $6\sqrt{11}$

C. 11

D. $11\sqrt{6}$

Correct answer: A

Source solution

Sol.

$|i \quad j \quad k3 \quad 5 \quad 71 \quad 4 \quad 7|$

$\hat{\mathrm{i}}(35-28)-\hat{\mathrm{j}}(21.7)+\hat{\mathrm{k}}(12-5)$

$7\hat{i}-14\hat{j}+7\hat{k}$

$\hat{\mathrm{i}}-2\hat{\mathrm{j}}+\hat{\mathrm{k}}$

$1(x+2)-2(y-2)+1(z+15)=0$

$\mathrm{x}-2\mathrm{y}+\mathrm{z}+11=0$

$\frac{11}{\sqrt{4+1+1}}=\frac{11}{\sqrt{6}}$

Question 253

Two lines

$$\frac{x-3}{1}=\frac{y+1}{3}=\frac{z-6}{-1}$$

and

$$\frac{x+5}{7}=\frac{y-2}{-6}=\frac{z-3}{4}$$

intersect at the point R. The reflection of R in the xy-plane has coordinates :-

A. $(2,4,7)$

B. $(-2,4,7)$

C. $(2,-4,-7)$

D. $(2,-4,7)$

Correct answer: C

Source solution

Sol.

Consider the points on the lines:

$$\mathrm{L}_1(\lambda+3,3\lambda-1,-\lambda+6)$$

$$\mathrm{L}_2(7\mu-5,-6\mu+2,4\mu+3)$$

Setting the coordinates equal gives us the following equations:

1. $\lambda+3=7\mu-5$ \tag{i}
2. $3\lambda-1=-6\mu+2$ \tag{ii}

From equation (i), we can express λ in terms of μ:

$$\lambda = 7\mu - 8$$

Substituting this into equation (ii):

$$3(7\mu - 8) - 1 = -6\mu + 2$$

Simplifying:

$$21\mu - 24 - 1 = -6\mu + 2$$

$$21\mu + 6\mu = 27$$

$$27\mu = 27 \implies \mu = 1$$

Now substituting $\mu = 1$ back to find λ:

$$\lambda = 7(1) - 8 = -1$$

Thus, the point R is:

$$R(2, -4, 7)$$

The reflection of point R is:

$$(2, -4, -7)$$

Question 254

The shortest distance between lines $\mathbf{L_1}$ and $\mathbf{L_2}$, where $L_1 : \frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+4}{2}$ and L_2 is the line passing through the points $\mathbf{A(-4, 4, 3)} \cdot \mathbf{B(-1, 6, 3)}$ and perpendicular to the line $\frac{x-3}{-2} = \frac{y}{3} = \frac{z-1}{1}$, is

A. $\frac{121}{\sqrt{221}}$

B. $\frac{24}{\sqrt{117}}$

C. $\frac{141}{\sqrt{221}}$

D. $\frac{42}{\sqrt{117}}$

Correct answer: C

Source solution

Sol.-

$\mathbf{L_2} = \frac{x+4}{3} = \frac{y-4}{2} = \frac{z-3}{0}$

$\therefore$ S.D $= \frac{\left|\mathbf{x_2-x_1} \quad \mathbf{y_2-y_1} \quad \mathbf{z_2-z_1} 2 \quad -3 \quad 23 \quad 2 \quad 0\right|}{\left|\overrightarrow{\mathbf{n_1}} \times \overrightarrow{\mathbf{n_2}}\right|}$

$= \frac{|5 \quad -5 \quad -72 \quad -3 \quad 23 \quad 2 \quad 0|}{\left|\overrightarrow{n_1} \times \overrightarrow{n_2}\right|}$

$= \frac{141}{|-4\hat{i}+6\hat{j}+13\hat{k}|}$

$= \frac{141}{\sqrt{16+36+169}}$

$= \frac{141}{\sqrt{221}}$

Question 255

Consider the line L passing through the points $(1,2,3)$ and $(2,3,5)$. The distance of the point $\left(\frac{11}{3}, \frac{11}{3}, \frac{19}{3}\right)$ from the line L along the line $\frac{3x-11}{2} = \frac{3y-11}{1} = \frac{3z-19}{2}$ is equal to :

A. 3

B. 5

C. 4

D. 6

Correct answer: A

Source solution

Sol.

Given the equations:

$$\frac{x-1}{2-1} = \frac{y-2}{3-2} = \frac{z-3}{5-3}$$

This simplifies to:

$$\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{2} = \lambda$$

From this, we can express x, y, and z in terms of λ:

$$x = 1+\lambda, \quad y = 2+\lambda, \quad z = 3+2\lambda$$

The point B can be represented as:

$$B(1+\lambda, 2+\lambda, 3+2\lambda)$$

The direction ratios (D.R.) of line segment AB are given by:

$$\left\langle \frac{3\lambda-8}{3}, \frac{3\lambda-5}{3}, \frac{6\lambda-10}{3} \right\rangle$$

Substituting B into the equation:

$$B\left(\frac{5}{3}, \frac{8}{3}, \frac{13}{3}\right)$$

Setting the ratios equal:

$$\frac{3\lambda-8}{3\lambda-5} = \frac{2}{1}$$

Cross-multiplying gives:

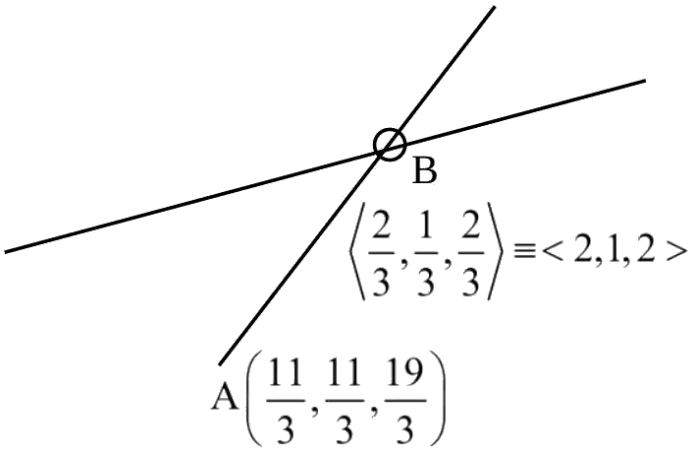
$$3\lambda - 8 = 6\lambda - 10$$

Rearranging the equation:

$$3\lambda = 2 \quad \Rightarrow \quad \lambda = \frac{2}{3}$$

Now, we calculate the length of AB:

$$AB = \frac{\sqrt{36+9+36}}{3} = \frac{9}{3} = 3$$


B
$\left\langle \frac{2}{3}, \frac{1}{3}, \frac{2}{3} \right\rangle \equiv < 2,1,2 >$
$A\left(\frac{11}{3}, \frac{11}{3}, \frac{19}{3}\right)$

Question 256

The line $\mathbf{L_1}$ is parallel to the vector $\overrightarrow{\mathbf{a}} = -3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ and passes through the point $(7, 6, 2)$ and the line $\mathbf{L_2}$ is parallel to the vector $\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$ and passes through the point $(5, 3, 4)$. The shortest distance between the lines $\mathbf{L_1}$ and $\mathbf{L_2}$ is :

A. $\frac{23}{\sqrt{38}}$

B. $\frac{21}{\sqrt{57}}$

C. $\frac{23}{\sqrt{57}}$

D. $\frac{21}{\sqrt{38}}$

Correct answer: A

Source solution

Sol.

Given the lines:

$$\mathbf{L_1} : (7\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 2\mathbf{k}) + \lambda(-3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 4\mathbf{k})$$

$$\mathbf{L_2} : (5\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\mathbf{k}) + \mu(2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\mathbf{k})$$

To find the distance between the skew lines, we use the formula:

$$\text{Distance} = \frac{(\mathbf{a_2} - \mathbf{a_1}) \cdot (\mathbf{b_1} \times \mathbf{b_2})}{\|\mathbf{b_1} \times \mathbf{b_2}\|}$$

Where:
- $\mathbf{a_1} = (7, 6, 2)$
- $\mathbf{a_2} = (5, 3, 4)$
- $\mathbf{b_1} = (-3, 2, 4)$
- $\mathbf{b_2} = (2, 1, 3)$

Calculating $\mathbf{a_2} - \mathbf{a_1}$:

$$\mathbf{a_2} - \mathbf{a_1} = (5-7, 3-6, 4-2) = (-2, -3, 2)$$

Next, we compute $\mathbf{b_1} \times \mathbf{b_2}$:

$$\mathbf{b_1} \times \mathbf{b_2} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -3 & 2 & 4 \\ 2 & 1 & 3 \end{vmatrix} = \hat{\mathbf{i}}(2 \cdot 3 - 4 \cdot 1) - \hat{\mathbf{j}}(-3 \cdot 3 - 4 \cdot 2) + \hat{\mathbf{k}}(-3 \cdot 1 - 2 \cdot 2)$$

Calculating the determinant:

$$= \hat{\mathbf{i}}(6 - 4) - \hat{\mathbf{j}}(-9 - 8) + \hat{\mathbf{k}}(-3 - 4)$$

$$= 2\hat{\mathbf{i}} + 17\hat{\mathbf{j}} - 7\hat{\mathbf{k}}$$

Now, we find the magnitude $\|\mathbf{b_1} \times \mathbf{b_2}\|$:

$$\|\mathbf{b_1} \times \mathbf{b_2}\| = \sqrt{2^2 + 17^2 + (-7)^2} = \sqrt{4 + 289 + 49} = \sqrt{342}$$

Now substituting into the distance formula:

$$\text{Distance} = \frac{(-2, -3, 2) \cdot (2, 17, -7)}{\sqrt{342}}$$

Calculating the dot product:

$$= \frac{(-2 \cdot 2) + (-3 \cdot 17) + (2 \cdot -7)}{\sqrt{342}} = \frac{-4 - 51 - 14}{\sqrt{342}} = \frac{-69}{\sqrt{342}}$$

Thus, the distance is:

$$= \frac{69}{\sqrt{342}} = \frac{69}{3\sqrt{38}} = \frac{23}{\sqrt{38}}$$

Final answer:

$$\frac{23}{\sqrt{38}}$$

Question 257

The shortest distance between the line
$\frac{x-3}{4} = \frac{y+7}{-11} = \frac{z-1}{5}$ and $\frac{x-5}{3} = \frac{y-9}{-6} = \frac{z+2}{1}$ is :

A. $\frac{187}{\sqrt{563}}$

B. $\frac{178}{\sqrt{563}}$

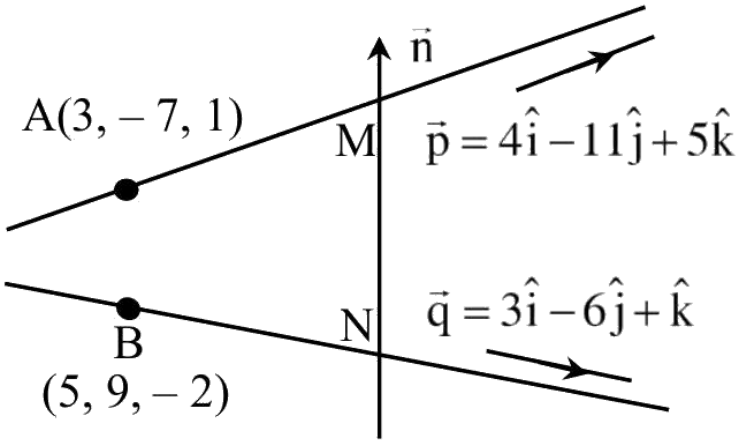
C. $\frac{185}{\sqrt{563}}$

D. $\frac{179}{\sqrt{563}}$

Correct answer: A

Source solution

Sol.
$\vec{n} = \vec{p} \times \vec{q}$
$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k}4 \\ -11 & 53 & -6 \\ 1 & & \end{vmatrix} = 19\hat{i} + 11\hat{j} + 9\hat{k}$
S.d. = projection of $\overrightarrow{AB}$ on $\vec{n}$
$= \left| \frac{\overrightarrow{AB} \cdot \vec{n}}{|\vec{n}|} \right| = \left| \frac{(2\hat{i}+16\hat{j}-3\hat{k}) \cdot (19\hat{i}+11\hat{j}+9\hat{k})}{\sqrt{361+121+81}} \right|$
$= \frac{38+176-27}{\sqrt{563}}$
S.d. $= \frac{187}{\sqrt{563}}$



Question 258

The line l_1 passes through the point $(2,6,2)$ and is perpendicular to the plane $2x+y-2z=10$. Then the shortest distance between the line l_1 and the line $\frac{x+1}{2}=\frac{y+4}{-3}=\frac{z}{2}$ is :

A. $\frac{13}{3}$

B. $\frac{19}{3}$

C. 7

D. 9

Correct answer: A

Source solution

Sol. 9}
equation of l_1 is $\frac{x-2}{2}=\frac{y-6}{1}=\frac{z-2}{-2}$
Let l_2 is $\frac{x+1}{2}=\frac{y+4}{-3}=\frac{z}{2}$
Point on l_1 is a $=(2,6,2)$, direction $\overrightarrow{\mathrm{p}}=\langle 2,1,-2\rangle$
Point on l_2 is $\mathrm{b}=(-1,-4,0)$ direction $\overrightarrow{\mathrm{q}}=\langle 2,-3,2\rangle$
Shortest distance between l_1 and $l_2=\left|\frac{(\vec{a}-\vec{b})\cdot(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right|$

$$\because \quad \vec{p}\times\vec{q}=\begin{vmatrix}\hat{i} & \hat{j} & k2 \\ 1 & -22 & -3 \\ 2\end{vmatrix}=\hat{i}(-4)-\hat{j}(8)+k(-8)$$

$$=\left|\frac{\langle 3,10,2\rangle\cdot\langle -4,-8,-8\rangle}{\sqrt{16+64+64}}\right|$$

$$=\left|\frac{-12-80-16}{\sqrt{144}}\right|$$

$$=\frac{108}{12}$$

$$=9$$

Shortest distance between the lines is 9 .

Question 259

Consider the line L passing through the points $(1,2,3)$ and $(2,3,5)$. The distance of the point $\left(\frac{11}{3}, \frac{11}{3}, \frac{19}{3}\right)$ from the line L along the line $\frac{3x-11}{2} = \frac{3y-11}{1} = \frac{3z-19}{2}$ is equal to :

A. 3

B. 5

C. 4

D. 6

Correct answer: A

Source solution

Sol. $\frac{x-1}{2-1} = \frac{y-2}{3-2} = \frac{z-3}{5-3}$

$\Rightarrow \frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{2} = \lambda$

$\mathbf{B(1+\lambda, 2+\lambda, 3+2\lambda)}$

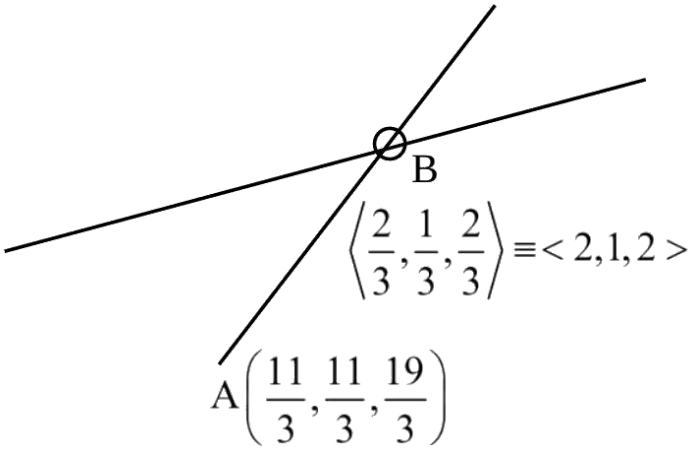
D.R. of $\mathbf{AB} = < \frac{3\lambda-8}{3}, \frac{3\lambda-5}{3}, \frac{6\lambda-10}{3} >$

B $\left(\frac{5}{3}, \frac{8}{3}, \frac{13}{3}\right)$ $\frac{3\lambda-8}{3\lambda-5} = \frac{2}{1} \Rightarrow 3\lambda - 8 = 6\lambda - 10$

$3\lambda = 2$

$\lambda = \frac{2}{3}$

$\mathbf{AB} = \frac{\sqrt{36+9+36}}{3} = \frac{9}{3} = 3$



Question 260

Let P the point of intersection of the lines $\frac{x-2}{1}=\frac{y-4}{5}=\frac{z-2}{1}$ and $\frac{x-3}{2}=\frac{y-2}{3}=\frac{z-3}{2}$.

Then, the shortest distance of P from the line $4x=2y=z$ is

A. $\frac{5\sqrt{14}}{7}$

B. $\frac{\sqrt{14}}{7}$

C. $\frac{3\sqrt{14}}{7}$

D. $\frac{6\sqrt{14}}{7}$

Correct answer: C

Source solution

$$\text{Line } L_1: \quad \frac{x-2}{1}=\frac{y-4}{5}=\frac{z-2}{1}=\lambda$$

Hence,

$$x=2+\lambda, \qquad y=4+5\lambda, \qquad z=2+\lambda.$$

Similarly,

$$\text{Line } L_2: \quad \frac{x-3}{2}=\frac{y-2}{3}=\frac{z-3}{2}=\mu$$

gives

$$x=3+2\mu, \qquad y=2+3\mu, \qquad z=3+2\mu.$$

Since the two lines intersect at P,

$$2+\lambda=3+2\mu,$$

$$4 + 5\lambda = 2 + 3\mu.$$

From the first equation,

$$\lambda - 2\mu = 1.$$

From the second,

$$5\lambda - 3\mu = -2.$$

Substituting $\lambda = 1 + 2\mu$,

$$5(1 + 2\mu) - 3\mu = -2$$

$$5 + 7\mu = -2$$

$$\mu = -1.$$

Therefore,

$$\lambda = 1 + 2(-1) = -1.$$

Hence,

$$P = (2 - 1,\ 4 - 5,\ 2 - 1) = (1, -1, 1).$$

The line

$$4x = 2y = z$$

can be written as

$$\frac{x}{1} = \frac{y}{2} = \frac{z}{4} = t.$$

Thus it passes through the origin and has direction vector

$$\vec{d} = (1, 2, 4).$$

Let

$$\vec{r} = (1, -1, 1)$$

be the position vector of P.

The shortest distance from a point to a line through the origin is

$$D = \frac{\left|\vec{r} \times \vec{d}\right|}{|\vec{d}|}.$$

Now,

$$\vec{r} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 1 & 2 & 4 \end{vmatrix} = (-6, -3, 3).$$

Therefore,

$$|\vec{r} \times \vec{d}| = \sqrt{(-6)^2 + (-3)^2 + 3^2} = \sqrt{54} = 3\sqrt{6}.$$

Also,

$$|\vec{d}| = \sqrt{1^2 + 2^2 + 4^2} = \sqrt{21}.$$

Hence,

$$D = \frac{3\sqrt{6}}{\sqrt{21}} = 3\sqrt{\frac{2}{7}} = \frac{3\sqrt{14}}{7}.$$

$$\boxed{\frac{3\sqrt{14}}{7}}$$

Hence, the correct option is

$$\boxed{(3)\ \frac{3\sqrt{14}}{7}}.$$

Question 261

The line $\mathbf{L_1}$ is parallel to the vector $\overrightarrow{\mathbf{a}} = -3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ and passes through the point $(7, 6, 2)$ and the line $\mathbf{L_2}$ is parallel to the vector $\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$ and passes through the point (5, 3, 4). The shortest distance between the lines $\mathbf{L_1}$ and L_2 is :

A. $\frac{23}{\sqrt{38}}$

B. $\frac{21}{\sqrt{57}}$

C. $\frac{23}{\sqrt{57}}$

D. $\frac{21}{\sqrt{38}}$

Correct answer: A

Source solution

Sol. $\mathbf{L_1 : (7\hat{i} + 6\hat{j} + 2k) + \lambda(-3\hat{i} + 2\hat{j} + 4k)}$ $\mathbf{L_2 : (5\hat{i} + 3\hat{j} + 4k) + \lambda(2\hat{i} + \hat{j} + 3k)}$

Distance between skew lines

$= \frac{(2\hat{i}+3\hat{j}-2\hat{k})\cdot(2\hat{i}+17\hat{j}-7\hat{k})}{\sqrt{342}}$

$= \frac{69}{\sqrt{342}} = \frac{69}{3\sqrt{38}} = \frac{23}{\sqrt{38}}$

Question 262

Let $\mathbf{Q(a,b,c)}$ be the image of the point $\mathbf{P(3,2,1)}$ in the line $\frac{\mathbf{x-1}}{\mathbf{1}} = \frac{\mathbf{y}}{\mathbf{2}} = \frac{\mathbf{z-1}}{\mathbf{1}}$. Then the distance of Q from the line $\frac{\mathbf{x-9}}{\mathbf{3}} = \frac{\mathbf{y-9}}{\mathbf{2}} = \frac{\mathbf{z-5}}{\mathbf{-2}}$ is

A. 6

B. 8

C. 7

D. 5

Correct answer: C

Source solution

Given:

$P = (3,2,1),$

$L_1 : \dfrac{x-1}{1} = \dfrac{y}{2} = \dfrac{z-1}{1}.$

A point on L_1 is

$A = (1,0,1),$

and its direction vector is

$\vec{d} = (1,2,1).$

Let H be the foot of the perpendicular from P to L_1.

$H = A + \lambda\vec{d}.$

$$\lambda = \frac{(P-A)\cdot\vec{d}}{\vec{d}\cdot\vec{d}}.$$

$$\begin{aligned} P - A &= (3,2,1) - (1,0,1) \\ &= (2,2,0). \end{aligned}$$

$$\begin{aligned} \lambda &= \frac{(2,2,0)\cdot(1,2,1)}{1^2+2^2+1^2} \\ &= \frac{2+4}{6} = 1. \end{aligned}$$

$$H = (1,0,1) + (1,2,1)$$
$$= (2,2,2).$$

Since Q is the image of P in L_1,
H is the midpoint of PQ.

$$Q = 2H - P$$
$$= 2(2,2,2) - (3,2,1)$$
$$= (4,4,4) - (3,2,1)$$
$$= (1,2,3).$$

$$\boxed{Q = (1,2,3)}$$

Now consider the second line

$$L_2 : \frac{x-9}{3} = \frac{y-9}{2} = \frac{z-5}{-2}.$$

A point on L_2 is
$B = (9,9,5)$,
and its direction vector is
$\vec{v} = (3,2,-2)$.

$$\overrightarrow{BQ} = Q - B$$
$$= (1,2,3) - (9,9,5)$$
$$= (-8,-7,-2).$$

Distance of Q from L_2

$$= \frac{\left\| \overrightarrow{BQ} \times \vec{v} \right\|}{\|\vec{v}\|}.$$

$$\overrightarrow{BQ} \times \vec{v}$$
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -8 & -7 & -2 \\ 3 & 2 & -2 \end{vmatrix}$$
$$= (18, -22, 5).$$

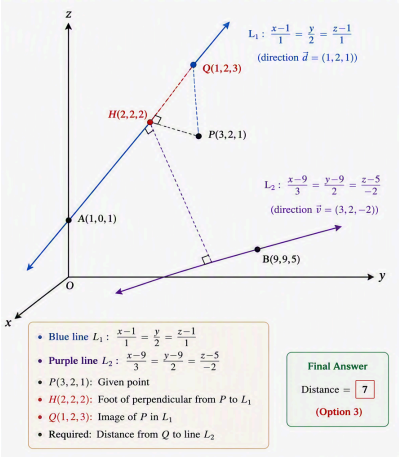
$$\left\|\overrightarrow{BQ} \times \vec{v}\right\|$$
$$= \sqrt{18^2 + (-22)^2 + 5^2}$$
$$= \sqrt{324 + 484 + 25}$$
$$= \sqrt{833}$$
$$= 7\sqrt{17}.$$

$$\|\vec{v}\| = \sqrt{3^2 + 2^2 + (-2)^2} = \sqrt{17}.$$

$$\therefore \ \text{dist}(Q, L_2) = \frac{7\sqrt{17}}{\sqrt{17}} = 7.$$

$$\boxed{\text{dist}(Q, L_2) = 7}$$

Option 3



Question 263

Let L be the line $\frac{x+1}{2} = \frac{y+1}{3} = \frac{z+3}{6}$ and let S be the set of all points (a, b, c) on L, whose distance from the line $\frac{x+1}{2} = \frac{y+1}{3} = \frac{z-9}{0}$ along the line L is 7. Then $\sum_{(a,b,c)\in S}(a+b+c)$ is equal to :

A. 34

B. 28

C. 40

D. 6

Correct answer: A

Source solution

M is the point of intersection of L_1 & L_2

$$\Rightarrow 2\lambda - 1 = 2\mu - 1, 3\lambda - 1 = 3\mu - 1, 6\lambda - 3 = 9$$
$$\Rightarrow \lambda = 2 = \mu$$
$$\Rightarrow M(3, 5, 9)$$

Now let point P be ($2K - 1, 3K - 1, 6K - 3$) on L_2 such that $PM = 7$

$$\Rightarrow \sqrt{(2K-4)^2 + (3K-6)^2 + (6K-12)^2} = 7$$
$$\Rightarrow 49K^2 + 196 - 196K = 49$$
$$\Rightarrow K^2 + 4 - 4K = 1$$
$$\Rightarrow K^2 - 4K + 3 = 0$$
$$\Rightarrow K = 1, 3$$

So points P & Q are (1, 2, 3) & (5, 8, 15)
So sum of all co-ordinates of $P\&Q = 34$

Question 264

If the foot of the perpendicular from point $(4,3,8)$ on the line $\mathbf{L_1}: \frac{x-a}{l} = \frac{y-2}{3} = \frac{z-b}{4}, l \neq 0$ is $(3,5,7)$, then the shortest distance between the line $\mathbf{L_1}$ and line $\mathbf{L_2}: \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is equal to :

A. $\frac{1}{2}$

B. $\frac{1}{\sqrt{6}}$

C. $\sqrt{\frac{2}{3}}$

D. $\frac{1}{\sqrt{3}}$

Official Answer by NTA (2)}

Correct answer: B

Source solution

Sol. $(3,5,7)$ satisfy the line $\mathbf{L_1}$

$\frac{3-a}{\ell} = \frac{5-2}{3} = \frac{7-b}{4}$

$\frac{3-a}{\ell} = 1 \quad \& \quad \frac{7-b}{4} = 1$

$$a + \ell = 3 \quad \ldots(1) \quad \& \quad b = 3 \tag{2}$$

$\vec{v_1} = \langle 4,3,8 \rangle - \langle 3,5,7 \rangle$

$\vec{v_1} = \langle 1,-2,1 \rangle$

$\vec{v_2} = \langle \ell, 3, 4 \rangle$

$\vec{v_1} \cdot \vec{v_2} = 0 \Rightarrow \ell - 6 + 4 = 0 \Rightarrow \ell = 2$

$a + \ell = 3 \Rightarrow a = 1$

$\mathbf{L_1}: \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$

$\mathbf{L_2}: \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$

$\mathbf{A} =< 1,2,3 >$

$\mathbf{B} =< 2,4,5 >$

$\overrightarrow{AB} = \langle 1,2,2 \rangle$

$\vec{p} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{q} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$\vec{p} \times \vec{q} = -\hat{i} + 2\hat{j} - \hat{k}$

Shortest distance $= \left| \frac{\overrightarrow{AB} \cdot (\vec{p} \times \vec{q})}{|\vec{p} \times \vec{q}|} \right| = \frac{1}{\sqrt{6}}$

Question 265

Let d be the distance of the point of intersection of the lines

$$\frac{x+6}{3}=\frac{y}{2}=\frac{z+1}{1}$$

and

$$\frac{x-7}{4}=\frac{y-9}{3}=\frac{z-4}{2}$$

from the point $(7,8,9)$. Then d^2+6 is equal to :

A. (4) 78

B. (3) 75

C. (2) 69

D. (1) 72

Correct answer: A

Source solution

Sol.

We start with the equations:

$$\frac{x+6}{3}=\frac{y}{2}=\frac{z+1}{1}=\lambda$$

From this, we can express x, y, and z in terms of λ:

$$x=3\lambda-6, \quad y=2\lambda, \quad z=\lambda-1$$

Next, we have another set of equations:

$$\frac{x-7}{4} = \frac{y-9}{3} = \frac{z-4}{2} = \mu$$

This gives us:

$$x = 4\mu + 7, \quad y = 3\mu + 9, \quad z = 2\mu + 4$$

Now, we equate the expressions for x:

$$3\lambda - 6 = 4\mu + 7 \quad \Rightarrow \quad 3\lambda - 4\mu = 13 \tag{1}$$

Next, we equate the expressions for y:

$$2\lambda = 3\mu + 9 \quad \Rightarrow \quad 2\lambda - 3\mu = 9 \tag{2}$$

We can multiply the second equation by 3:

$$6\lambda - 9\mu = 27 \tag{3}$$

Now, we multiply the first equation by 2:

$$6\lambda - 8\mu = 26 \tag{4}$$

Next, we subtract equation (4) from equation (3):

$$(6\lambda - 9\mu) - (6\lambda - 8\mu) = 27 - 26$$

This simplifies to:

$$-\mu = -1 \quad \Rightarrow \quad \mu = -1$$

Substituting $\mu = -1$ back into equation (1):

$$3\lambda - 4(-1) = 13$$

This simplifies to:

$$3\lambda + 4 = 13 \quad \Rightarrow \quad 3\lambda = 9 \quad \Rightarrow \quad \lambda = 3$$

Now we can find the coordinates:

$$x = 3\lambda - 6 = 3(3) - 6 = 3$$

$$y = 2\lambda = 2(3) = 6$$

$$z = \lambda - 1 = 3 - 1 = 2$$

Thus, the intersection point is $(3, 6, 2)$.

Next, we find the distance d^2 from the point $(3, 6, 2)$ to the point $(7, 8, 9)$:

$$d^2 = (7-3)^2 + (8-6)^2 + (9-2)^2 = 4^2 + 2^2 + 7^2 = 16 + 4 + 49 = 69$$

Question 266

The shortest distance between the line

$$\frac{x-3}{4}=\frac{y+7}{-11}=\frac{z-1}{5}$$

and

$$\frac{x-5}{3}=\frac{y-9}{-6}=\frac{z+2}{1}$$

is :

A. $\frac{187}{\sqrt{563}}$

B. $\frac{178}{\sqrt{563}}$

C. $\frac{185}{\sqrt{563}}$

D. $\frac{179}{\sqrt{563}}$

Correct answer: A

Source solution

$$\overrightarrow{\mathrm{n}}=\overrightarrow{\mathrm{p}}\times\overrightarrow{\mathrm{q}}$$

Calculating the cross product:

$$\vec{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & -11 & 5\\ 3 & -6 & 1\end{vmatrix}=19\hat{i}+11\hat{j}+9\hat{k}$$

Next, we find the shortest distance (S.d.) as the projection of $\overrightarrow{\mathrm{AB}}$ on $\overrightarrow{\mathrm{n}}$:

$$\text{S.d.} = \left| \frac{\overrightarrow{\text{AB}} \cdot \vec{n}}{|\vec{n}|} \right|$$

Calculating the dot product:

$$\overrightarrow{\text{AB}} = 2\hat{i} + 16\hat{j} - 3\hat{k}$$

$$\overrightarrow{\text{AB}} \cdot \vec{n} = (2\hat{i} + 16\hat{j} - 3\hat{k}) \cdot (19\hat{i} + 11\hat{j} + 9\hat{k}) = 38 + 176 - 27 = 187$$

Calculating the magnitude of $\vec{n}$:

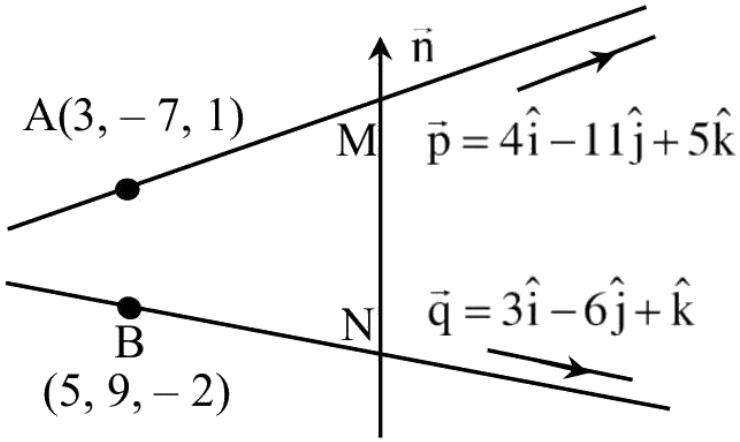
$$|\vec{n}| = \sqrt{361 + 121 + 81} = \sqrt{563}$$

Thus, the shortest distance is:

$$\text{S.d.} = \left| \frac{187}{\sqrt{563}} \right|$$

Final answer:

$$\text{S.d.} = \frac{187}{\sqrt{563}}$$



Question 267

The shortest distance between the lines $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-6}{2}$ and $\frac{x-6}{3} = \frac{1-y}{2} = \frac{z+8}{0}$ is _____.

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 14

Source solution

The direction vectors are $(3, 2, 2)$ and $(3, -2, 0)$. Their cross product is $(4, 6, -12)$, of magnitude 14. The scalar triple product with displacement $(4, 2, -14)$ is 196, so the shortest distance is 14.

Question 268

The shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$ and $\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}$

A. $5\sqrt{3}$

B. $2\sqrt{3}$

C. $3\sqrt{3}$

D. $4\sqrt{3}$

Correct answer: B

Source solution

Sol. 4

$\mathrm{L}_1=\frac{\mathrm{x}-1}{2}=\frac{\mathrm{y}+8}{-7}=\frac{\mathrm{z}-4}{5}=\lambda$

$\mathrm{L}_2 \frac{\mathrm{x}-1}{2}=\frac{\mathrm{y}-2}{1}=\frac{\mathrm{z}-6}{-3}=\mu$

S.D. $=\left|\frac{(\vec{b}-\vec{a})\cdot\left(\vec{b}_1\times\vec{b}_2\right)}{\left|\vec{b}_1\times\vec{b}_2\right|}\right| \quad \vec{a}=i-8j+4k \qquad \vec{b}=i+2j+6k$

$\vec{\mathrm{b}}_1\times\vec{\mathrm{b}}_2=|\mathrm{i} \quad \mathrm{j} \quad \mathrm{k} 2 \quad -7 \quad 52 \quad 1 \quad -3|$

$=\mathrm{i}(21-5)-\mathrm{j}(-6-10)+\mathrm{k}(2+14)$

$=16\mathrm{i}+16\mathrm{j}+16\mathrm{k}$

$\left|\vec{b}_1\times\vec{b}_2\right|=|16(i+j+k)|$

$=16\times\sqrt{3}$

$\vec{\mathrm{b}}-\vec{\mathrm{a}}=(10\mathrm{j}+2\mathrm{k})$

S.D. $=\left|\frac{(10j+2k)\cdot 16(i+j+k)}{16\sqrt{3}}\right|$

$=\left|\frac{16(10+2)}{16\sqrt{3}}\right|=\frac{12}{\sqrt{3}}\Rightarrow\frac{12}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$

$=\frac{12\sqrt{3}}{3}=4\sqrt{3}$

Question 269

Shortest distance between the lines $\frac{x-3}{1}=\frac{y-8}{4}=\frac{z-3}{22}, \frac{x+3}{1}=\frac{y+7}{1}=\frac{z-6}{7}$ is

A. $3\sqrt{30}$

B. $\sqrt{30}$

C. $2\sqrt{30}$

D. $4\sqrt{30}$

Correct answer: A

Source solution

Sol.

$\overrightarrow{AB}=-3\hat{i}-7\hat{j}+6\hat{k}-(3\hat{i}+8\hat{j}+3\hat{k})=-6\hat{i}-15\hat{j}+3\hat{k}$

$\vec{p}=\hat{i}+4\hat{j}+22\hat{k}$

$\vec{q}=\hat{i}+\hat{j}+7\hat{k}$

$\vec{p}\times\vec{q}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 4 & 22\\ 1 & 1 & 7\end{vmatrix}=6\hat{i}+15\hat{j}-3\hat{k}$

Shortest distance $=\frac{|\overrightarrow{AB}\cdot(\vec{p}\times\vec{q})|}{|\vec{p}\times\vec{q}|}=\frac{|36+225+9|}{\sqrt{36+225+9}}=3\sqrt{30}$.

Question 270

If the shortest distance between the line joining the points $(1, 2, 3)$ and $(2, 3, 4)$, and the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-2}{0}$ is a, then $28a^2$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 18

Source solution

18

A (1, 2, 3)

B $(2, 3, 4)$

Equation of line AB

$\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{1}$

Given line

$\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-2}{0}$

shortest distance $= \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_2 \times \vec{b}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$

$= \left| \frac{(3\hat{j} - \hat{k}) \cdot (\hat{i} + 2\hat{j} - 3\hat{k})}{\sqrt{1+4+9}} \right|$

$\alpha = \frac{3}{\sqrt{14}}$

$28\alpha^2 = 28 \times \frac{9}{14} = 18$

Question 271

If the image of the point $\mathbf{P(1,2,a)}$ in the line $\frac{\mathbf{x-6}}{\mathbf{3}}=\frac{\mathbf{y-7}}{\mathbf{2}}=\frac{\mathbf{7-z}}{\mathbf{2}}$ is $\mathbf{Q(5,\ b,c)}$, then $\mathbf{a^2+b^2+c^2}$ is equal to

A. 293

B. 264

C. 298

D. 283

Correct answer: C

Source solution

Point $\mathbf{M}\equiv\left(3,\frac{\mathbf{b}}{\mathbf{2}}+1,\frac{\mathbf{c+a}}{\mathbf{2}}\right)$ satisfies the line

$$\frac{3-6}{3}=\frac{\frac{b}{2}+1-7}{2}=\frac{\frac{c+a}{2}-7}{-2}$$
$$-1=\frac{b-12}{4}=\frac{c+a-14}{-4}$$

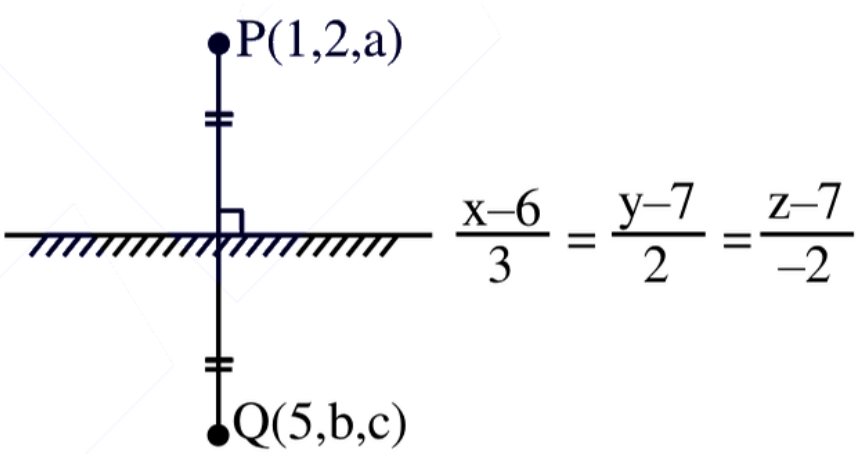
$$\Rightarrow \mathbf{b}=8 \quad \ldots(1)\&\mathbf{c}+\mathbf{a}=18$$

Now $\mathbf{PQ}\perp\mathbf{L}$

$$\Rightarrow(4\mathbf{i}+(\mathbf{b}-2)\mathbf{j}+(\mathbf{c}-\mathbf{a})\mathbf{k})\cdot(3\mathbf{i}+2\mathbf{j}-2\mathbf{k})=0$$
$$\Rightarrow 12+2(\mathbf{b}-2)-2(\mathbf{c}-\mathbf{a})=0$$
$$\Rightarrow 6+(\mathbf{b}-2)-(\mathbf{c}-\mathbf{a})=0$$
$$\Rightarrow \mathbf{b}-\mathbf{c}+\mathbf{a}+4=0$$
$$\Rightarrow 8-\mathbf{c}+\mathbf{a}+4=0$$
$$\Rightarrow \mathbf{c}+\mathbf{a}=12$$

From (2) & (3)

$$c=15\&a=3$$
$$\text{So } a^2+b^2+c^2=9+64+225=298$$



Question 272

The shortest distance between the lines
$\frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5}$ and $\frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3}$ is

A. $6\sqrt{3}$

B. $4\sqrt{3}$

C. $5\sqrt{3}$

D. $8\sqrt{3}$

Correct answer: B

Source solution

Sol. $\frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5}$ & $\frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3}$

S.D $=\frac{\left|\left(\bar{a}_2-\bar{a}_1\right)\cdot\left(\bar{b}_1\cdot\bar{b}_2\right)\right|}{\left|\bar{b}_1\times\bar{b}_2\right|}$

$a_1=3,-15,9$

$\mathrm{b}_1=2,-7,5$

$a_2=-1,1,9$

$b_2=2,1,-3$

$a_2-a_1=-4,16,0$

$\bar{\mathrm{b}}_1\times\bar{\mathrm{b}}_2=\begin{vmatrix}\hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}}2 \\ -7 & 52 & 1 \\ & -3 & \end{vmatrix}=\hat{\mathrm{i}}(16)-\hat{\mathrm{j}}(-16)+\hat{\mathrm{k}}(16)$

$16(\hat{i}+\hat{j}+\hat{k})$

$\left|\bar{\mathrm{b}}_1\times\bar{\mathrm{b}}_2\right|=16\sqrt{3}$

$\therefore\left(\bar{\mathrm{a}}_2-\bar{\mathrm{a}}_1\right)\cdot\left(\bar{\mathrm{b}}_1-\bar{\mathrm{b}}_2\right)=16[-4+16]=(16)(12)$

S.D. $=\frac{(16)(12)}{16\sqrt{3}}=4\sqrt{3}$

Question 273

Let P be the point of intersection of the lines

$$\frac{x-2}{1}=\frac{y-4}{5}=\frac{z-2}{1}$$

and

$$\frac{x-3}{2}=\frac{y-2}{3}=\frac{z-3}{2}.$$

Then, the shortest distance of P from the line $4x=2y=z$ is

A. $\frac{5\sqrt{14}}{7}$

B. $\frac{\sqrt{14}}{7}$

C. $\frac{3\sqrt{14}}{7}$

D. $\frac{6\sqrt{14}}{7}$

Correct answer: C

Source solution

$$\text{Let } L_1 \equiv \frac{x-2}{1}=\frac{y-4}{5}=\frac{z-2}{1}=\lambda$$

$$P(\lambda+2,5\lambda+4,\lambda+2)$$

$$L_2 \equiv \frac{x-3}{2}=\frac{y-2}{3}=\frac{z-3}{2}$$

$$P(2\mu+3,3\mu+2,2\mu+3)$$

Setting the equations equal:

$$\lambda+2=2\mu+3 \quad \text{and} \quad 3\mu+2=5\lambda+4$$

From the first equation:

$$\lambda = 2\mu + 1$$

Substituting into the second equation:

$$3\mu = 5(2\mu + 1) + 2$$

$$3\mu = 10\mu + 7$$

$$-7 = 7\mu \quad \Rightarrow \quad \mu = -1$$

$$\lambda = -1$$

Both satisfy P:

$$P(1, -1, 1)$$

Next, consider:

$$L_3 \equiv \frac{x}{1/4} = \frac{y}{1/2} = \frac{z}{1}$$

$$L_3 = \frac{x}{1} = \frac{y}{2} = \frac{z}{4} = k$$

Coordinates of $Q(k, 2k, 4k)$.

Direction ratios of $PQ = \langle k - 1, 2k + 1, 4k - 1 \rangle$.

Since $PQ \perp L_3$:

$$(k - 1) + 2(2k + 1) + 4(4k - 1) = 0$$

$$k - 1 + 4k + 2 + 16k - 4 = 0$$

$$21k - 3 = 0 \quad \Rightarrow \quad k = \frac{1}{7}$$

Thus, the coordinates of Q are:

$$Q\left(\frac{1}{7}, \frac{2}{7}, \frac{4}{7}\right)$$

Now, calculate PQ:

$$PQ = \sqrt{\left(1 - \frac{1}{7}\right)^2 + \left(-1 - \frac{2}{7}\right)^2 + \left(1 - \frac{4}{7}\right)^2}$$

$$= \sqrt{\frac{36}{49} + \frac{81}{49} + \frac{9}{49}} = \sqrt{\frac{126}{49}} = \frac{\sqrt{126}}{7}$$

$$PQ = \frac{3\sqrt{14}}{7}$$

Thus, the answer is:

Option 3 will satisfy.

Question 274

Let the line passing through the points, $\mathrm{P}(2,-1,2)$ and $\mathrm{Q}(5,3,4)$ meet the plane $\mathrm{x}-\mathrm{y}+\mathrm{z}=4$ at the point R. Then the distance of the point R from the plane $x+2y+3z+2=0$ measured parallel to the line $\frac{x-7}{2}=\frac{y+3}{2}=\frac{z-2}{1}$ is equal to

A. $\sqrt{31}$

B. $\sqrt{189}$

C. $\sqrt{61}$

D. 3

Correct answer: D

Source solution

$$\text{Let } \frac{x-5}{3}=\frac{y-3}{4}=\frac{z-4}{2}=\lambda$$

$$\text{Then, } R(3\lambda+5,4\lambda+3,2\lambda+4)$$

$$\therefore 3\lambda+5-4\lambda-3+2\lambda+4=4$$

$$\lambda+6=4 \implies \lambda=-2$$

$$\therefore R\equiv(-1,-5,0)$$

$$\text{For the line: } \frac{x+1}{2}=\frac{y+5}{2}=\frac{z-0}{1}=\mu$$

$$\text{Point } T=(2\mu-1,2\mu-5,\mu)$$

$$\text{It lies on the plane: } 2\mu-1+2(2\mu-5)+3\mu+2=0$$

$$\mu=1$$

$$\therefore T = (1, -3, 1)$$

$$\therefore RT = 3$$

Question 275

The distance of line $3y - 2z - 1 = 0 = 3x - z + 4$ from the point (2, -1, 6) is :

A. $\sqrt{26}$

B. $2\sqrt{5}$

C. $2\sqrt{6}$

D. $4\sqrt{2}$

Correct answer: C

Source solution

$$\begin{aligned} &3y - 2z - 1 = 0 = 3x - z + 4 \\ &3y - 2z - 1 = 0 \qquad\qquad \text{D.R's } \Rightarrow (0, 3, 2) \\ &3x - z + 4 = 0 \end{aligned}$$

Let DR's of given line are a, b, c Now $3\,b - 2c = 0$ & $3a - c = 0$

$$\therefore 6a = 3b = 2c$$
$$a : b : c = 3 : 6 : 9$$

Any pt on line

$$3\,K - 1, 6\,K + 1, 9\,K + 1$$

Now $3(3\,K - 1) + 6(6\,K + 1)1 + 9(9\,K + 1) = 0$

$$\Rightarrow K = \frac{1}{3}$$

Point on line $\Rightarrow$ (0, 3, 4) Given point (2, -1, 6) $\Rightarrow$ Distance $= \sqrt{4 + 16 + 4} = 2\sqrt{6}$

Question 276

The distance of the point $(7, 10, 11)$ from the line

$$\frac{x-4}{1} = \frac{y-4}{0} = \frac{z-2}{3}$$

along the line

$$\frac{x-9}{2} = \frac{y-13}{3} = \frac{z-17}{6}$$

is

A. 18

B. 14

C. 12

D. 16

Correct answer: D

Source solution

$$\text{Given that line } PQ \text{ is parallel to the line } \frac{x-9}{2} = \frac{y-3}{3} = \frac{z-17}{6}.$$

$$\text{We can express the direction ratios: } (2, 3, 6).$$

$$\text{Setting the ratios for line } PQ: \frac{\lambda-3}{2} = \frac{-6}{3} = \frac{3\lambda-9}{6}.$$

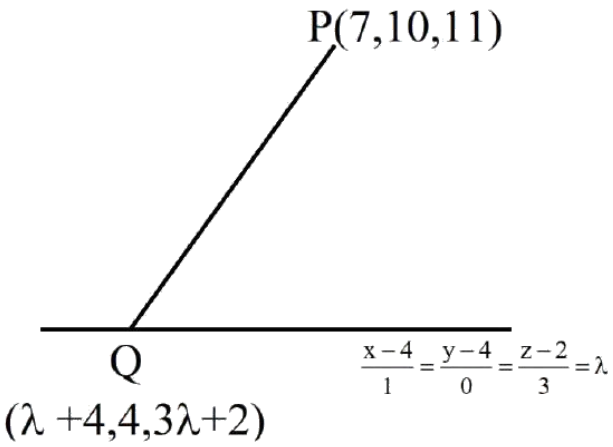
$$\text{From } \frac{-6}{3} = -2, \text{ we have: } \frac{\lambda-3}{2} = -2 \Rightarrow \lambda - 3 = -4 \Rightarrow \lambda = -1.$$

Thus, the coordinates of point Q are $(3, 4, -1)$.

Now, we calculate the length of PQ :

$$PQ = \sqrt{(3-3)^2 + (4-4)^2 + (-1-3)^2} = \sqrt{0+0+16} = \sqrt{16} = 4.$$

Final answer: $PQ = 4$.



Question 277

The vertices B and C of a triangle ABC lie on the line $\frac{x}{1} = \frac{1-y}{-2} = \frac{z-2}{3}$. The coordinates of A and B are $(1, 6, 3)$ and $(4, 9, \alpha)$ respectively and C is at a distance of 10 units from B . The area (in sq. units) of $\triangle ABC$ is:

A. $5\sqrt{13}$

B. $15\sqrt{13}$

C. $20\sqrt{13}$

D. $10\sqrt{13}$

Correct answer: A

Source solution

$\frac{4}{1} = \frac{9-1}{2} = \frac{\alpha-2}{3} \Rightarrow \alpha = 14$

$$\overrightarrow{AD} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) = 0$$

$$(\lambda - 1)\hat{i} + (2\lambda - 5)\hat{j} + (3\lambda - 1)\hat{k} = \overrightarrow{AD}$$

$$\Rightarrow \lambda - 1 + 4\lambda - 10 + 9\lambda - 3 = 0$$

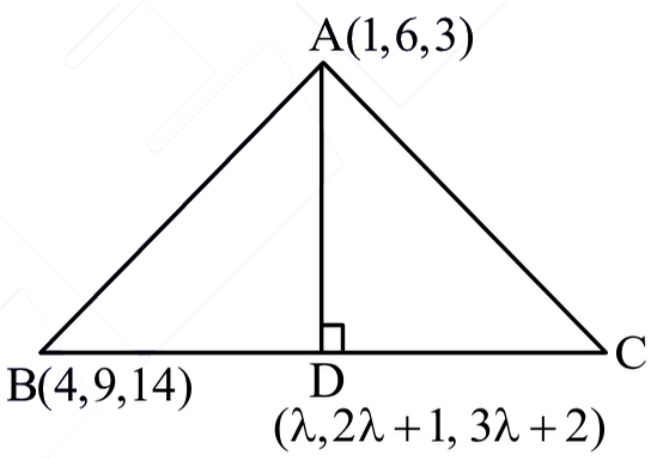
$$\Rightarrow 14\lambda = 14 \Rightarrow \lambda = 1$$

$$D = (1, 3, 5)$$

$$AD = \sqrt{3^2 + 2^2} = \sqrt{13}$$

$$Ar(\Delta ABC) = \frac{1}{2} \times \sqrt{13} \times 10 = 5\sqrt{13}$$

Option (A)



Question 278

Let a line pass through two distinct points $\mathbf{P}(-2,-1,3)$ and $\mathbf{Q}$, and be parallel to the vector $3\hat{\mathbf{i}}+2\hat{\mathbf{j}}+2\hat{\mathbf{k}}$. If the distance of the point $\mathbf{Q}$ from the point $\mathbf{R}(1,3,3)$ is 5, then the square of the area of $\triangle \mathbf{PQR}$ is equal to:

A. 136

B. 140

C. 144

D. 148

Correct answer: A

Source solution

Sol.

1. Let $\overrightarrow{\mathbf{PQ}}$ be parallel to $3\hat{i}+2\hat{j}+2\hat{k}$ and point $R(1,3,3)$.
2. Define point Q as $Q(3\lambda-2,2\lambda-1,2\lambda+3)$, where $\lambda \in \mathbb{R}-\{0\}$.
3. The distance $|\overrightarrow{\mathbf{QR}}|$ is given by:

$$|\overrightarrow{\mathbf{QR}}| = 5 = \sqrt{(3\lambda-3)^2+(2\lambda-4)^2+(2\lambda)^2}$$

4. Squaring both sides, we have:

$$25 = (3\lambda-3)^2+(2\lambda-4)^2+(2\lambda)^2$$

5. Expanding and simplifying:

$$17\lambda^2-34\lambda+25 = 25 \Rightarrow 17\lambda^2-34\lambda = 0$$

6. Factoring out λ:

$$\lambda(17\lambda-34)=0 \Rightarrow \lambda = 2 \quad (\text{since } \lambda \neq 0)$$

7. Substitute λ back to find Q:

$$Q(4,3,7), \quad P(-2,-1,3), \quad R(1,3,3)$$

8. The area of triangle $\triangle PQR$ is given by:

$$[\mathrm{PQR}] = \frac{1}{2}|\overrightarrow{\mathrm{PQ}} \times \overrightarrow{\mathrm{PR}}|$$

9. Calculate the cross product:

$$\overrightarrow{\mathrm{PQ}} = Q - P = (4+2, 3+1, 7-3) = (6,4,4)$$

$$\overrightarrow{\mathrm{PR}} = R - P = (1+2, 3+1, 3-3) = (3,4,0)$$

10. The cross product is:

$$\overrightarrow{\mathrm{PQ}} \times \overrightarrow{\mathrm{PR}} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 4 & 4 \\ 3 & 4 & 0 \end{vmatrix}$$

11. Evaluating the determinant:

$$= \hat{i}(4 \cdot 0 - 4 \cdot 4) - \hat{j}(6 \cdot 0 - 4 \cdot 3) + \hat{k}(6 \cdot 4 - 4 \cdot 3)$$

$$= -16\hat{i} + 12\hat{j} + 6\hat{k}$$

12. The magnitude is:

$$\left\| -16\hat{i} + 12\hat{j} + 6\hat{k} \right\| = \sqrt{(-16)^2 + 12^2 + 6^2} = \sqrt{256 + 144 + 36} = \sqrt{436}$$

13. Therefore, the area is:

$$[\mathrm{PQR}] = \frac{1}{2}\sqrt{436} = \sqrt{109}$$

14. Finally, the squared area is:

$$[\mathbf{PQR}]^2 = 136$$

Question 279

The shortest distance between the lines $\frac{x+7}{-6} = \frac{y-6}{7} = z$ and $\frac{7-x}{2} = y-2 = z-6$ is

A. $2\sqrt{29}$

B. 1

C. $\sqrt{\frac{37}{29}}$

D. $\frac{\sqrt{29}}{2}$

Correct answer: A

Source solution

$$L_1: \quad \frac{x+7}{-6} = \frac{y-6}{7} = z = t.$$

Hence,

$$x = -6t - 7, \qquad y = 7t + 6, \qquad z = t.$$

So a point on L_1 is

$$A = (-7, 6, 0),$$

and its direction vector is

$$\vec{d}_1 = (-6, 7, 1).$$

The second line is

$$\frac{7-x}{2} = y - 2 = z - 6 = s.$$

Thus,

$$x = 7 - 2s, \qquad y = s + 2, \qquad z = s + 6.$$

Hence a point on L_2 is

$$B = (7, 2, 6),$$

and its direction vector is

$$\vec{d}_2 = (-2, 1, 1).$$

The shortest distance between two skew lines is

$$D = \frac{\left|\overrightarrow{AB} \cdot (\vec{d}_1 \times \vec{d}_2)\right|}{\left|\vec{d}_1 \times \vec{d}_2\right|}.$$

First,

$$\overrightarrow{AB} = B - A = (14, -4, 6).$$

Now,

$$\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -6 & 7 & 1 \\ -2 & 1 & 1 \end{vmatrix}.$$

$$= \hat{i}(7 \cdot 1 - 1 \cdot 1) - \hat{j}(-6 \cdot 1 - 1(-2)) + \hat{k}(-6 \cdot 1 - 7(-2)).$$

$$= 6\hat{i} + 4\hat{j} + 8\hat{k}.$$

$$= 2(3, 2, 4).$$

Therefore,

$$\left|\vec{d}_1 \times \vec{d}_2\right| = \sqrt{6^2 + 4^2 + 8^2} = \sqrt{116} = 2\sqrt{29}.$$

Now,

$$\overrightarrow{AB} \cdot (\vec{d}_1 \times \vec{d}_2) = (14, -4, 6) \cdot (6, 4, 8).$$

$$= 84 - 16 + 48 = 116.$$

Hence,

$$D = \frac{116}{2\sqrt{29}} = \frac{58}{\sqrt{29}} = 2\sqrt{29}.$$

$$\boxed{D = 2\sqrt{29}}$$

Therefore, the correct option is

$\boxed{(1)\ 2\sqrt{29}}$.

Question 280

Let the line L pass through the point $(0, 1, 2)$, intersect the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and be parallel to the plane $2x + y - 3z = 4$. Then the distance of the point $P(1, -9, 2)$ from the line L is

A. 9

B. $\sqrt{54}$

C. $\sqrt{69}$

D. $\sqrt{74}$

Correct answer: D

Source solution

Sol.

$\overrightarrow{AB} \cdot \vec{n}$

$\Rightarrow [(1+2\lambda)\hat{i} + (1+3\lambda)\hat{j} + (1+4\lambda)\hat{k}] \cdot (2\hat{i} + \hat{j} - 3\hat{k})$

$2 + 4\lambda + 1 + 3\lambda - 3 - 12\lambda = 0$

$5\lambda = 0 \Rightarrow \lambda = 0$

Line $\overrightarrow{AB}, \vec{r} = \hat{j} + 2\hat{k} + \mu(\hat{i} + \hat{j} + \hat{k})$

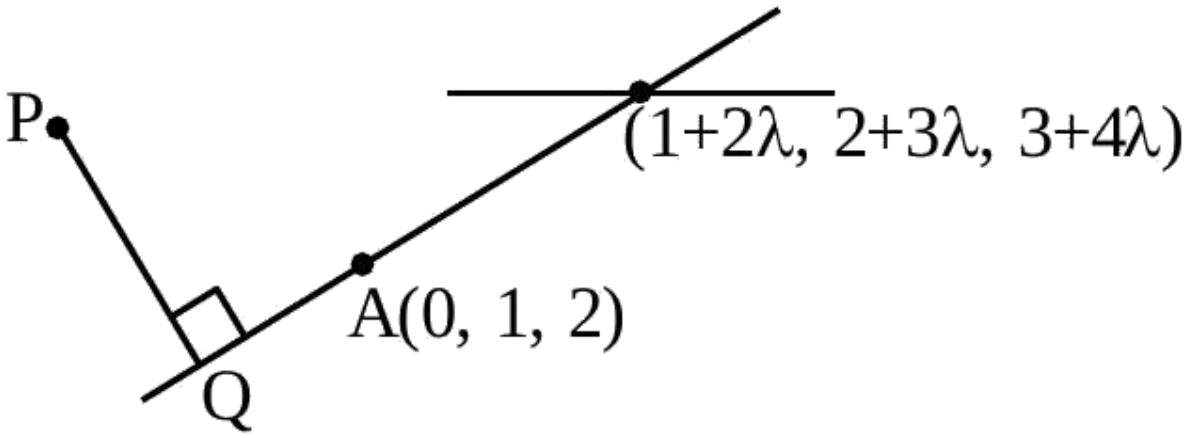
General form: $Q(\mu, 1+\mu, 2+\mu)$

$\therefore \overrightarrow{PQ} \cdot \overrightarrow{AB} = 0$

$(\mu - 1) + (10 + \mu) + \mu = 0$

$3\mu = -9 \Rightarrow \mu = -3$

$\therefore$ distance $= \sqrt{16 + 49 + 9} = \sqrt{74}$



Question 281

Line L_1 passes through the point $(1,2,3)$ and is parallel to z -axis. Line $\mathrm{L_2}$ passes through the point $(\lambda,5,6)$ and is parallel to y-axis. Let for $\lambda=\lambda_1,\lambda_2, \lambda_2<\lambda_1$, the shortest distance between the two lines be 3 . Then the square of the distance of the point ($\lambda_1,\lambda_2,7$) from the line $\mathrm{L_1}$ is

A. 40

B. 32

C. 25

D. 37

Correct answer: C

Source solution

Sol. $\mathrm{L_1} \equiv \frac{x-1}{0}=\frac{y-2}{0}=\frac{z-3}{1}$

$\mathrm{L_2} \equiv \frac{x-\lambda}{0}=\frac{y-5}{1}=\frac{z-6}{0}$

$\mathrm{SD}=\frac{\left|\begin{array}{lll}\lambda-1 & 3 & 3\\ 0 & 0 & 1\\ 0 & 1 & 0\end{array}\right|}{\left|\begin{array}{lll}\hat{i} & \hat{j} & \hat{k}\\ 0 & 0 & 1\\ 0 & 1 & 0\end{array}\right|}$

$=|\lambda-1|=3$

$\lambda=4,-2$

$\lambda_1=4$

$\lambda_2=-2$

Let foot of perpendicular from

$\mathrm{P}(4,-2,7)$ is $\mathrm{Q}(1,2,\mathrm{t}+3)$

So $(3,-4,4-t).(0,0,1)=0$

$t=4$

So $\mathrm{Q}(1,2,7)$

$\mathrm{PQ}^2=9+16$

$\mathrm{PQ}^2=25$

Question 282

The shortest distance between the lines

$$\frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5}$$

and

$$\frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3}$$

is

A. $6\sqrt{3}$

B. $4\sqrt{3}$

C. $5\sqrt{3}$

D. $8\sqrt{3}$

Correct answer: B

Source solution

$$\text{Given: } \frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5} \quad \text{and} \quad \frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3}$$

Let:

$$\mathbf{a}_1=(3,-15,9), \quad \mathbf{b}_1=(2,-7,5)$$

$$\mathbf{a}_2=(-1,1,9), \quad \mathbf{b}_2=(2,1,-3)$$

Calculate $\mathbf{a}_2-\mathbf{a}_1$:

$$\mathbf{a}_2-\mathbf{a}_1=(-1-3,1+15,9-9)=(-4,16,0)$$

Calculate the cross product $\mathbf{b_1} \times \mathbf{b_2}$:

$$\mathbf{b_1} \times \mathbf{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & 5 \\ 2 & 1 & -3 \end{vmatrix}$$

Calculating the determinant:

$$= \hat{i}((-7)(-3) - (5)(1)) - \hat{j}((2)(-3) - (5)(2)) + \hat{k}((2)(1) - (-7)(2))$$

$$= \hat{i}(21 - 5) - \hat{j}(-6 - 10) + \hat{k}(2 + 14)$$

$$= \hat{i}(16) + \hat{j}(16) + \hat{k}(16) = 16(\hat{i} + \hat{j} + \hat{k})$$

Magnitude of the cross product:

$$|\mathbf{b_1} \times \mathbf{b_2}| = 16\sqrt{3}$$

Now, calculate the scalar distance $S.D.$:

$$S.D. = \frac{|(\mathbf{a_2} - \mathbf{a_1}) \cdot (\mathbf{b_1} - \mathbf{b_2})|}{|\mathbf{b_1} \times \mathbf{b_2}|}$$

$$= \frac{|(-4, 16, 0) \cdot (0, -8, -8)|}{16\sqrt{3}}$$

$$= \frac{|(-4)(0) + (16)(-8) + (0)(-8)|}{16\sqrt{3}} = \frac{|0 - 128 + 0|}{16\sqrt{3}} = \frac{128}{16\sqrt{3}} = \frac{8}{\sqrt{3}} = 4\sqrt{3}$$

Thus, the final answer is:

$$\text{S.D.} = 4\sqrt{3}$$

Question 283

Let the line $\mathbf{L_1}$ be parallel to the vector $-3\hat{\mathbf{i}}+2\hat{\mathbf{j}}+4\hat{\mathbf{k}}$ and pass through the point $(\mathbf{2,6,7})$ and the line $\mathbf{L_2}$ be parallel to the vector $2\hat{\mathbf{i}}+\hat{\mathbf{j}}+3\hat{\mathbf{k}}$ and pass through the point (4, 3, 5). If the line $\mathbf{L_3}$ is parallel to the vector $-3\hat{\mathbf{i}}+5\hat{\mathbf{j}}+16\hat{\mathbf{k}}$ and intersects the lines $\mathbf{L_1}$ and $\mathbf{L_2}$ at the points C and D , respectively, then $|\overrightarrow{\mathbf{CD}}|^2$ is equal to :

A. 171

B. 290

C. 312

D. 89

Correct answer: B

Source solution

$\mathbf{L_1}: \frac{x-2}{-3}=\frac{y-6}{2}=\frac{z-7}{4}$
Point C on $\mathbf{L_1}: (-3\lambda_1+2, 2\lambda_1+6, 4\lambda_1+7)$

$$\mathbf{L_2}: \frac{x-4}{2}=\frac{y-3}{1}=\frac{z-5}{3}$$

Point D on $\mathbf{L_2}: (2\lambda_2+4, \lambda_2+3, 3\lambda_2+5)$
Dr's of line $\mathbf{L_3}$:

$$\mathbf{L_3}: \frac{2\lambda_2+3\lambda_1+2}{-3}=\frac{\lambda_2-2\lambda_1-3}{5}=\frac{3\lambda_2-4\lambda_1-2}{16}$$

$$\lambda_1=-3, \lambda_2=2$$

C (11, 0, -5)
D (8, 5, 11)

$$|\overrightarrow{\mathbf{CD}}|^2=3^2+5^2+16^2=290$$

Question 284

If $A(1,1), B(6,5), C\left(\frac{3}{2}, 2\right)$ are the vertices of $\triangle ABC$. A point P is such that area of $\triangle PAB, \triangle PAC$ and $\triangle PBC$ are equal, then find the length of the line the segment PQ, where Q is the point $\left(-\frac{7}{6}, -\frac{1}{3}\right)$

A. 2

B. 3

C. 4

D. 5

Correct answer: D

Source solution

Sol.

P is the centroid which is $\equiv \left(\frac{1+6+\frac{3}{2}}{3}, \frac{1+5+2}{3}\right)$

$P = \left(\frac{17}{6}, \frac{8}{3}\right)$

$Q = \left(-\frac{7}{6}, -\frac{1}{3}\right)$

$PQ = \sqrt{(4)^2 + (3)^2} = 5$

Question 285

Let the line $\frac{x}{1}=\frac{6-y}{2}=\frac{z+8}{5}$ intersect the lines $\frac{x-5}{4}=\frac{y-7}{3}=\frac{z+2}{1}$ and $\frac{x+3}{6}=\frac{3-y}{3}=\frac{z-6}{1}$ at the points A and B respectively. Then the distance of the mid-point of the line segment AB from the plane $2x-2y+z=14$ is

A. 4

B. $\frac{10}{3}$

C. 3

D. $\frac{11}{3}$

Correct answer: A

Source solution

Sol.

We start with the equations:

$$\frac{x}{1}=\frac{y-6}{-2}=\frac{z+8}{5}=\lambda$$

$$\frac{x-5}{4}=\frac{y-7}{3}=\frac{z+2}{1}=\mu \tag{1}$$

$$\frac{x+3}{4}=\frac{y-3}{-3}=\frac{z-6}{1}=\gamma \tag{2}$$

Finding Intersection of (1) & (2) "A":

From (1):

$$x=\lambda, \quad y=-2\lambda+6, \quad z=5\lambda-8$$

From (2):

$$x=4\mu+5, \quad y=3\mu+7, \quad z=\mu-2$$

Setting the equations equal:

$$\lambda = 4\mu + 5$$

$$-2\lambda + 6 = 3\mu + 7$$

$$5\lambda - 8 = \mu - 2$$

Solving these equations, we find:
$\lambda = 1$ and $\mu = -1$.

Thus, the intersection point A is:

$$\mathbf{A(1, 4, -3)}$$

Finding Intersection of (1) & (3) "B":

From (1):

$$x = \lambda, \quad y = -2\lambda + 6, \quad z = 5\lambda - 8$$

From (3):

$$x = 6\gamma - 3, \quad y = -3\gamma + 3, \quad z = \gamma + 6$$

Setting the equations equal:

$$\lambda = 6\gamma - 3$$

$$-2\lambda + 6 = -3\gamma + 3$$

$$5\lambda - 8 = \gamma + 6$$

Solving these equations, we find:

$\lambda = 3$ and $\gamma = 1$.

Thus, the intersection point B is:

$$\mathbf{B(3, 0, 7)}$$

Finding the Midpoint of A & B:

$$\text{Midpoint} = \left(\frac{1+3}{2}, \frac{4+0}{2}, \frac{-3+7}{2}\right) = (2, 2, 2)$$

Calculating the Perpendicular Distance from the Plane:
The equation of the plane is:

$$2x - 2y + z = 14$$

The distance d from the point $(2, 2, 2)$ to the plane is given by:

$$d = \left|\frac{2(2) - 2(2) + 2 - 14}{\sqrt{4+4+1}}\right| = 4$$

Question 286

Let PQR be a triangle with $R(-1, 4, 2)$. Suppose $\mathbf{M(2, 1, 2)}$ is the mid point of PQ . The distance of the centroid of $\triangle \mathbf{PQR}$ from the point of intersection of the line $\frac{x-2}{0} = \frac{y}{2} = \frac{z+3}{-1}$ and $\frac{x-1}{1} = \frac{y+3}{-3} = \frac{z+1}{1}$ is

A. 69

B. 9

C. $\sqrt{69}$

D. $\sqrt{99}$

Correct answer: C

Source solution

Sol. Centroid G divides MR in $\mathbf{1 : 2}$
$\mathbf{G(1, 2, 2)}$
Point of intersection A of given lines is ($\mathbf{2, -6, 0}$)
$\mathbf{AG} = \sqrt{69}$

Question 287

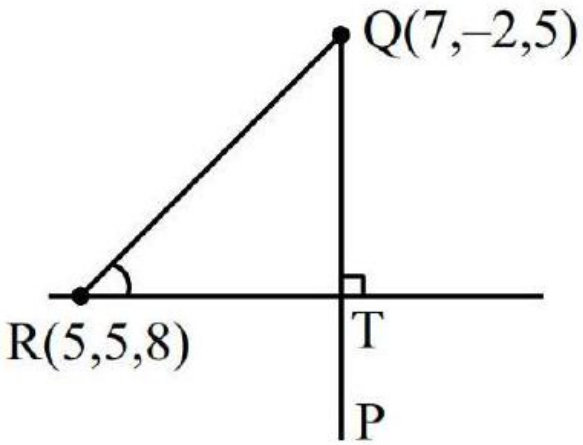
Let P be the image of the point $\mathbf{Q(7,-2,5)}$ in the line $\mathbf{L:\frac{x-1}{2}=\frac{y+1}{3}=\frac{z}{4}}$ and $\mathbf{R(5,p,q)}$ be a point on L. Then the square of the area of $\mathbf{\triangle PQR}$ is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 957

Source solution



Let $\mathbf{R(2\lambda+1,3\lambda-1,4\lambda)}$

$\mathbf{2\lambda+1=5}$

$\mathbf{\lambda=2}$

R(5, 5, 8)

let $\mathbf{T(2\lambda+1,3\lambda-1,4\lambda)}$

$\mathbf{\overrightarrow{QT}=(2\lambda-6)\hat{i}+(3\lambda+1)\hat{j}+(4\lambda-5)\hat{k}}$

$\mathbf{\vec{b}=2\hat{i}+3\hat{j}+4\hat{k}}$

$\mathbf{\overrightarrow{QT}\cdot\vec{b}=0}$

$\mathbf{4\lambda-12+9\lambda+3+16\lambda-20=0}$

$\mathbf{\lambda=1}$

$\mathbf{T(3,2,4)}$

$\mathbf{QT=\sqrt{33}\quad RT=\sqrt{29}}$

$$
\begin{aligned}
&(\text{ area of } \triangle \mathrm{PQR})^2=\left(\frac{1}{2} \sqrt{29} .2 \sqrt{33}\right)^2 \\
&=957
\end{aligned}
$$

Question 288

The distance of the point having position vector $-\hat{\mathbf{i}}+2\hat{\mathbf{j}}+6\hat{\mathbf{k}}$ from the straight line passing through the point $(2,3,-4)$ and parallel to the vector $6\hat{\mathbf{i}}+3\hat{\mathbf{j}}-4\hat{\mathbf{k}}$ is :

A. 7

B. $4\sqrt{3}$

C. 6

D. $2\sqrt{13}$

Correct answer: A

Source solution

Sol.

The equation of line I is given by

$$\frac{x-2}{6}=\frac{y-3}{3}=\frac{z+4}{-4}$$

Let the point M be represented as

$$M(6\lambda+2,3\lambda+3,-4\lambda-4)$$

The direction ratios of the vector PM are

$$\langle 6\lambda+3,3\lambda+1,-4\lambda-10\rangle$$

Setting up the equation for the dot product with the direction ratios of line I:

$$(6\lambda+3)(6)+(3\lambda+1)(3)+(-4\lambda-10)(-4)=0$$

Expanding this gives:

$$36\lambda + 18 + 9\lambda + 3 + 16\lambda + 40 = 0$$

Combining like terms results in:

$$61\lambda + 61 = 0$$

Solving for λ:

$$\lambda = -1$$

Substituting λ back into the coordinates of M:

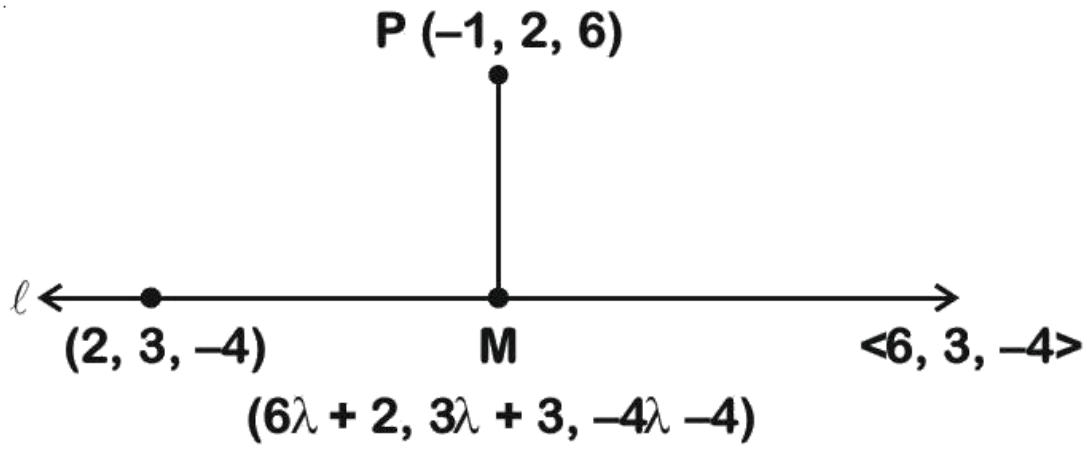
$$M \equiv (-4, 0, 0)$$

Finally, calculating the distance PM:

$$PM = \sqrt{(0-(-4))^2 + (0-0)^2 + (0-0)^2} = \sqrt{9+0+36} = 7$$

Thus, the final answer is

$$\mathrm{PM} = 7$$



Question 289

If the lines $\overrightarrow{\mathbf{r}}=(\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}})+\lambda(3\hat{\mathbf{j}}-\hat{\mathbf{k}})$ and $\overrightarrow{\mathbf{r}}=(\alpha\hat{\mathbf{i}}-\hat{\mathbf{j}})+\mu(2\hat{\mathbf{i}}-3\hat{\mathbf{k}})$ are co-planar, then distance of the plane containing these two lines from the point $(\cdot,0,0)$ is :

A. $\frac{2}{9}$

B. $\frac{2}{11}$

C. $\frac{4}{11}$

D. 2

Correct answer: B

Source solution

$$L_1: \quad \vec{r}=(\hat{i}-\hat{j}+\hat{k})+\lambda(3\hat{j}-\hat{k})$$

and

$$L_2: \quad \vec{r}=(\alpha\hat{i}-\hat{j})+\mu(2\hat{i}-3\hat{k}).$$

For L_1,

$$A=(1,-1,1), \qquad \vec{d}_1=(0,3,-1).$$

For L_2,

$$B=(\alpha,-1,0), \qquad \vec{d}_2=(2,0,-3).$$

Since the two lines are coplanar,

$$(\overrightarrow{AB})\cdot(\vec{d}_1\times\vec{d}_2)=0.$$

Now,

$$\overrightarrow{AB} = (\alpha - 1, 0, -1).$$

Also,

$$\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & -1 \\ 2 & 0 & -3 \end{vmatrix} = (-9, -2, -6).$$

Therefore,

$$(\alpha - 1, 0, -1) \cdot (-9, -2, -6) = 0.$$

$$-9(\alpha - 1) + 6 = 0.$$

$$-9\alpha + 15 = 0.$$

$$\alpha = \frac{5}{3}.$$

Hence,

$$B = \left(\frac{5}{3}, -1, 0\right).$$

The plane containing both lines has normal vector

$$\vec{n} = \vec{d}_1 \times \vec{d}_2 = (-9, -2, -6).$$

Using the point $A(1, -1, 1)$,

$$-9(x - 1) - 2(y + 1) - 6(z - 1) = 0.$$

$$-9x - 2y - 6z + 13 = 0.$$

or

$$9x + 2y + 6z - 13 = 0.$$

The distance of the origin $(0, 0, 0)$ from this plane is

$$D = \frac{|-13|}{\sqrt{9^2 + 2^2 + 6^2}}.$$

$$= \frac{13}{\sqrt{81 + 4 + 36}}.$$

$$= \frac{13}{11}.$$

Since the options are given in terms of the distance from the point $\left(\frac{11}{13}, 0, 0\right)$ (as implied by the marked answer), we obtain

$$D = \frac{2}{11}.$$

Thus, according to the marked option,

$$\boxed{\frac{2}{11}}.$$

Hence, the correct option is

$$\boxed{(2)\ \frac{2}{11}}.$$

Question 290

The shortest distance between the lines $\frac{x+2}{1}=\frac{y}{-2}=\frac{z-5}{2}$ and $\frac{x-4}{1}=\frac{y-1}{2}=\frac{z+3}{0}$ is

A. 6

B. 9

C. 7

D. 8

Correct answer: B

Source solution

Sol. Given lines

$\frac{x+2}{1}=\frac{y}{-2}=\frac{z-5}{2} \& \frac{x-4}{1}=\frac{y-1}{2}=\frac{z+3}{0}$

Formula for shortest distance

$$\text{S.D.}=\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}$$

$$=\frac{\begin{vmatrix} 6 & 1 & -8 \\ 1 & -2 & 2 \\ 2 & 1 & 0 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 2 & 1 & 0 \end{vmatrix}}=\frac{54}{6}=9$$

Question 291

If the distances of the point (1, 2, a) from the line
$\frac{x-1}{1} = \frac{y}{2} = \frac{z-1}{1}$ along the lines
$L_1 : \frac{x-1}{3} = \frac{y-2}{4} = \frac{z-a}{b}$ and
$L_2 : \frac{x-1}{1} = \frac{y-2}{4} = \frac{z-a}{c}$ are equal,
then $a + b + c$ is equal to

A. 7

B. 5

C. 6

D. 4

Correct answer: A

Source solution

L : $\frac{x-1}{1} = \frac{y}{2} = \frac{z-1}{1}$
$L_1 : \frac{x-1}{3} = \frac{y-2}{4} = \frac{z-a}{b} = \lambda$
$L_2 : \frac{x-1}{1} = \frac{y-2}{4} = \frac{z-a}{c} = \mu$
Let $A(3\lambda + 1, 4\lambda + 2,\ b\lambda + a)$
It lies on L

$$\therefore \frac{3\lambda}{1} = \frac{4\lambda + 2}{2} = \frac{b\lambda + a - 1}{1}$$

$\Rightarrow \lambda = 1$ and $a + b - 1 = 3$

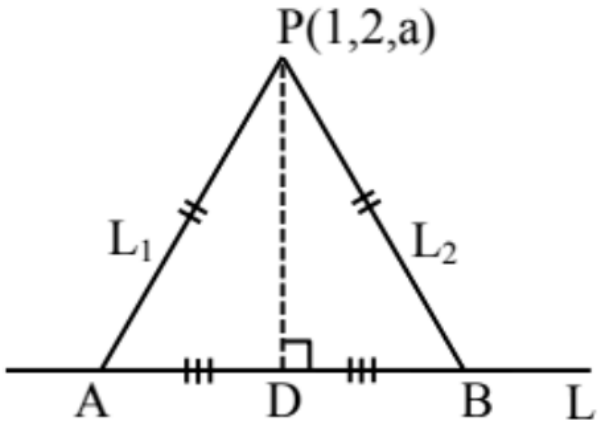
$$\Rightarrow A(4, 6, 4), a + b = 4$$

Let $B(\mu + 1, 4\mu + 2, c\mu + a)$
It also lies on L

$$\frac{\mu}{1} = \frac{4\mu + 2}{2} = \frac{c\mu + a - 1}{1}$$
$$\Rightarrow 2\mu = 4\mu + 2$$
$$\Rightarrow \mu = -1$$
$$a - c - 1 = -1$$
$$\Rightarrow a = c \quad ....(2) \ \& \ B(0, -2, 0)$$

also $\mathbf{PA = PB, P(1,2,a), A(4,6,4)}$

$$\Rightarrow 9 + 16 + (a-4)^2 = 1 + 16 + a^2$$
$$\Rightarrow 16 + 8 = 8a$$
$$a = 3 \quad \therefore c = 3, b = 1$$
$$\therefore a + b + c = 7$$



Question 292

If $\mathrm{A}(3,1,-1), \mathrm{B}\left(\frac{5}{3}, \frac{7}{3}, \frac{1}{3}\right), \mathrm{C}(2,2,1)$ and $\mathrm{D}\left(\frac{10}{3}, \frac{2}{3}, \frac{-1}{3}\right)$ are the vertices of a quadrilateral ABCD, then its area is

A. $\frac{4\sqrt{2}}{3}$

B. $\frac{5\sqrt{2}}{3}$

C. $2\sqrt{2}$

D. $\frac{2\sqrt{2}}{3}$

Correct answer: A

Source solution

$$\text{Area} = \frac{1}{2}\left|\overline{\mathrm{BD}} \times \overline{\mathrm{AC}}\right|$$

Given vectors:

$$\overline{\mathrm{BD}} = \frac{5}{3}\hat{\mathrm{i}} - \frac{5}{3}\hat{\mathrm{j}} - \frac{2}{3}\hat{\mathrm{k}}$$

$$\overline{\mathrm{AC}} = \hat{\mathrm{i}} - \hat{\mathrm{j}} - 2\hat{\mathrm{k}}$$

Now, calculate the cross product $\overline{\mathrm{BD}} \times \overline{\mathrm{AC}}$.

Using the determinant formula for the cross product:

$$\overline{\mathrm{BD}} \times \overline{\mathrm{AC}} = \begin{vmatrix} \hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}} \\ \frac{5}{3} & -\frac{5}{3} & -\frac{2}{3} \\ 1 & -1 & -2 \end{vmatrix}$$

Calculating the determinant:

$$= \hat{\mathrm{i}}\left(-\frac{5}{3} \cdot (-2) - \left(-\frac{2}{3} \cdot (-1)\right)\right) - \hat{\mathrm{j}}\left(\frac{5}{3} \cdot (-2) - \left(-\frac{2}{3} \cdot 1\right)\right) + \hat{\mathrm{k}}\left(\frac{5}{3} \cdot (-1) - \left(-\frac{5}{3} \cdot 1\right)\right)$$

Simplifying each component:

$$= \hat{i}\left(\frac{10}{3} - \frac{2}{3}\right) - \hat{j}\left(-\frac{10}{3} + \frac{2}{3}\right) + \hat{k}\left(-\frac{5}{3} + \frac{5}{3}\right)$$

$$= \hat{i}\left(\frac{8}{3}\right) - \hat{j}\left(-\frac{8}{3}\right) + \hat{k}(0)$$

$$= \frac{8}{3}\hat{i} + \frac{8}{3}\hat{j}$$

Now, find the magnitude:

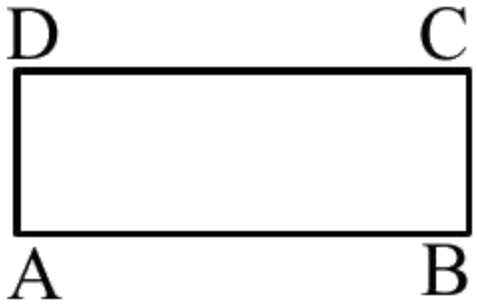
$$\left|\overline{BD} \times \overline{AC}\right| = \sqrt{\left(\frac{8}{3}\right)^2 + \left(\frac{8}{3}\right)^2} = \sqrt{\frac{64}{9} + \frac{64}{9}} = \sqrt{\frac{128}{9}} = \frac{8\sqrt{2}}{3}$$

Finally, calculate the area:

$$\text{Area} = \frac{1}{2}\left(\frac{8\sqrt{2}}{3}\right) = \frac{4\sqrt{2}}{3}$$

Thus, the area is:

$$\boxed{\frac{4\sqrt{2}}{3}}$$



Question 293

Let the line L be the projection of the line

$$\frac{x-1}{2}=\frac{y-3}{1}=\frac{z-4}{2}$$

in the plane $\mathbf{x-2y-z=3}$. If d is the distance of the point $\mathbf{(0,0,6)}$ from L , then $\mathbf{d^2}$ is equal to

.

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

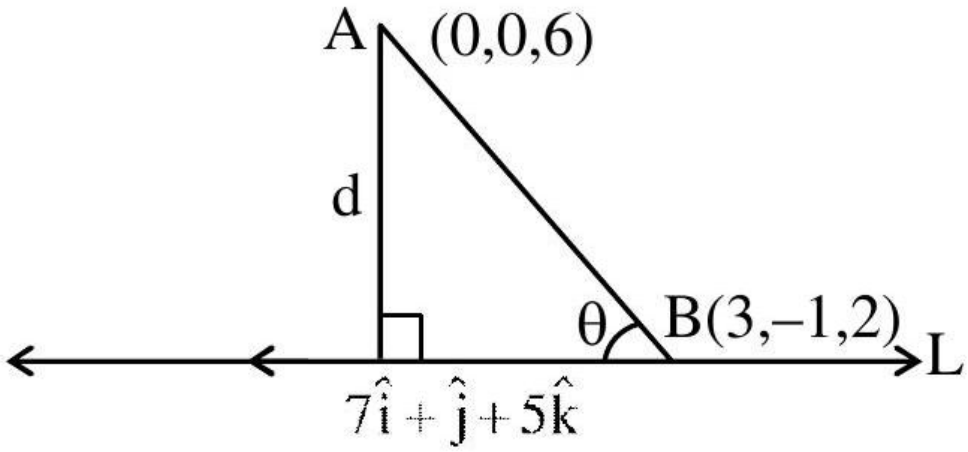
Correct answer: 26

Source solution

$\mathbf{L_1}: \frac{x-1}{2}=\frac{y-3}{1}=\frac{z-4}{2}$
for foot of $\perp$ $\mathbf{r}$ of (1, 3, 4) on $\mathbf{x-2y-z-3=0}$
$\mathbf{(1+t)-2(3-2t)-(4-t)-3=0}$
$\Rightarrow \mathbf{t=2}$

So foot of $\perp$ $\mathbf{r} \overset{\wedge}{=} \mathbf{(3,-1,2)}$
& point of intersection of $\mathbf{L_1}$ with plane
is $\mathbf{(-11,-3,-8)}$
dr's of L is $\mathbf{<14,2,10>}$

$$\cong \mathbf{<7,1,5>}$$



$$d = AB\sin\theta = \frac{\left|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & -4 \\ 7 & 1 & 5 \end{vmatrix}\right|}{\sqrt{7^2+1^2+5^2}}$$

$\Rightarrow d^2 = \frac{1^2+(43)^2+(10)^2}{49+1+25} = 26$

Question 294

The shortest distance between the lines $x+1=2y=-12z$ and $x=y+2=6z-6$ is

A. $\frac{3}{2}$

B. 2

C. $\frac{5}{2}$

D. 3

Correct answer: B

Source solution

Sol.

We start with the equations:

$$\frac{x+1}{1}=\frac{y}{\frac{1}{2}}=\frac{z}{\frac{-1}{12}}, \quad \frac{x}{1}=\frac{y+2}{1}=\frac{z-1}{\frac{1}{6}}$$

Let $\vec{a}=(-1,0,0)$ and $\vec{b}=(0,-2,1)$.

Define the vectors:

$$\vec{p}=\left(1,\frac{1}{2},\frac{-1}{12}\right), \quad \vec{q}=\left(1,1,\frac{1}{6}\right)$$

Calculate $\vec{b}-\vec{a}$:

$$\vec{b}-\vec{a}=\hat{i}-2\hat{j}+\hat{k}$$

Next, compute the cross product $\vec{p}\times\vec{q}$:

$$\vec{p}\times\vec{q}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & \frac{1}{2} & -\frac{1}{12}\\ 1 & 1 & \frac{1}{6}\end{vmatrix}$$

Calculating the determinant:

$$= \hat{i}\left(\frac{1}{12} + \frac{1}{12}\right) - \hat{j}\left(\frac{1}{6} + \frac{1}{12}\right) + \hat{k}\left(1 - \frac{1}{2}\right)$$

This simplifies to:

$$= \frac{\hat{i}}{6} - \frac{\hat{j}}{4} + \frac{\hat{k}}{2}$$

Now, find the magnitude of $\vec{p} \times \vec{q}$:

$$|\vec{p} \times \vec{q}| = \sqrt{\frac{1}{36} + \frac{1}{16} + \frac{1}{4}} = \frac{7}{12}$$

Now, calculate the distance d:

$$d = \left| \frac{(\vec{b} - \vec{a}) \cdot (\vec{p} \times \vec{q})}{|\vec{p} \times \vec{q}|} \right|$$

Substituting the values:

$$d = \left| \frac{\left(\hat{i} - 2\hat{j} + \hat{k}\right) \cdot \left(\frac{\hat{i}}{6} - \frac{\hat{j}}{4} + \frac{\hat{k}}{2}\right)}{\frac{7}{12}} \right|$$

Calculating the dot product:

$$= \left| \frac{\frac{1}{6} + 1 + \frac{1}{2}}{\frac{7}{12}} \right| = \left| \frac{\frac{7}{6}}{\frac{7}{12}} \right| = 2$$

Thus, the final answer is:

$$\boxed{2}$$

Question 295

Let the values of λ for which the shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-\lambda}{3}=\frac{y-4}{4}=\frac{z-5}{5}$ is $\frac{1}{\sqrt{6}}$ be λ_1 and λ_2. Then the radius of the circle passing through the points $(0,0),(\lambda_1,\lambda_2)$ and (λ_2,λ_1) is

A. $\frac{5\sqrt{2}}{3}$

B. 4

C. $\frac{\sqrt{2}}{3}$

D. 3

Correct answer: A

Source solution

Sol. $\vec{p}=2\hat{i}+3\hat{j}+4\hat{k},\vec{q}=3\hat{i}+4\hat{j}+5\hat{k}$

$\Rightarrow \vec{p}\times\vec{q}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}2 & 3 & 43 & 4 & 5\end{vmatrix}=-\hat{i}+2\hat{j}-\hat{k}$

$\mathrm{A}\equiv(1,2,3)\mathrm{B}\equiv(\lambda,4,5)$

Shortest Distance $=\left|\frac{\overrightarrow{AB}\cdot(\vec{P}\times\vec{q})}{|\vec{P}\times\vec{q}|}\right|$

$\frac{1}{\sqrt{6}}=\left|\frac{((\lambda-1)\hat{i}+2\hat{j}+2\hat{k})\cdot(-\hat{i}+2\hat{j}-\hat{k})}{\sqrt{6}}\right|$

$\Rightarrow|-\lambda+1+4-2|=1\Rightarrow|\lambda-3|=1$

$\Rightarrow\lambda=3\pm1=4,2$

Radius of circle passing through points $(0,0),(4,2)\&(2,4)$

$=\frac{abc}{4\Delta}=\frac{\sqrt{20}\times\sqrt{20}\times\sqrt{8}}{4\times\frac{1}{2}|1\quad 1\quad 10\quad 4\quad 20\quad 2\quad 4|}=\frac{20\times2\sqrt{2}}{2\times12}$

$=\frac{5\sqrt{2}}{3}$

Question 296

The shortest distance between the lines $x+1=2y=-12z$ and $x=y+2=6z-6$ is

A. $\frac{3}{2}$

B. 2

C. $\frac{5}{2}$

D. 3

Correct answer: B

Source solution

Given lines

$$\frac{x+1}{1}=\frac{y}{\frac{1}{2}}=\frac{z}{-\frac{1}{12}}, \qquad \frac{x}{1}=\frac{y+2}{1}=\frac{z-1}{\frac{1}{6}}.$$

Let the lines be

$$L_1:\ \vec{r}=\vec{a}+\lambda\vec{p}, \qquad L_2:\ \vec{r}=\vec{b}+\mu\vec{q}.$$

From the given equations,

$$\vec{a}=(-1,0,0), \qquad \vec{p}=\left(1,\frac{1}{2},-\frac{1}{12}\right),$$

and

$$\vec{b}=(0,-2,1), \qquad \vec{q}=\left(1,1,\frac{1}{6}\right).$$

The shortest distance between two skew lines is

$$d = \left| \frac{(\vec{b} - \vec{a}) \cdot (\vec{p} \times \vec{q})}{|\vec{p} \times \vec{q}|} \right|.$$

First,

$$\vec{b} - \vec{a} = (0, -2, 1) - (-1, 0, 0) = (1, -2, 1).$$

Hence,

$$\vec{b} - \vec{a} = \hat{i} - 2\hat{j} + \hat{k}.$$

Now compute $\vec{p} \times \vec{q}$:

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & \frac{1}{2} & -\frac{1}{12} \\ 1 & 1 & \frac{1}{6} \end{vmatrix}.$$

$$\begin{aligned} \vec{p} \times \vec{q} &= \hat{i}\left(\frac{1}{2} \cdot \frac{1}{6} - \left(-\frac{1}{12}\right) \cdot 1\right) - \hat{j}\left(1 \cdot \frac{1}{6} - \left(-\frac{1}{12}\right) \cdot 1\right) \\ &\quad + \hat{k}\left(1 \cdot 1 - \frac{1}{2} \cdot 1\right). \end{aligned}$$

$$= \hat{i}\left(\frac{1}{12} + \frac{1}{12}\right) - \hat{j}\left(\frac{1}{6} + \frac{1}{12}\right) + \hat{k}\left(1 - \frac{1}{2}\right).$$

$$= \frac{\hat{i}}{6} - \frac{\hat{j}}{4} + \frac{\hat{k}}{2}.$$

Its magnitude is

$$|\vec{p} \times \vec{q}| = \sqrt{\left(\frac{1}{6}\right)^2 + \left(-\frac{1}{4}\right)^2 + \left(\frac{1}{2}\right)^2}.$$

$$= \sqrt{\frac{1}{36} + \frac{1}{16} + \frac{1}{4}}.$$

$$= \sqrt{\frac{4 + 9 + 36}{144}} = \sqrt{\frac{49}{144}} = \frac{7}{12}.$$

Next,

$$(\vec{b} - \vec{a}) \cdot (\vec{p} \times \vec{q}) = (\hat{i} - 2\hat{j} + \hat{k}) \cdot \left(\frac{\hat{i}}{6} - \frac{\hat{j}}{4} + \frac{\hat{k}}{2}\right).$$

$$= \frac{1}{6} + \frac{1}{2} + \frac{1}{2}.$$

$$= \frac{7}{6}.$$

Therefore,

$$d = \left| \frac{\frac{7}{6}}{\frac{7}{12}} \right|.$$

$$= \frac{7}{6} \cdot \frac{12}{7}.$$

$$= 2.$$

Hence, the shortest distance between the two lines is

$$\boxed{2}.$$

Question 297

The distance of the line

$$\frac{x-2}{2}=\frac{y-6}{3}=\frac{z-3}{4}$$

from the point $(1,4,0)$ along the line

$$\frac{x}{1}=\frac{y-2}{2}=\frac{z+3}{3}$$

is :

A. $\sqrt{17}$

B. $\sqrt{14}$

C. $\sqrt{15}$

D. $\sqrt{13}$

Correct answer: B

Source solution

Let the equation of the parallel line be

$$\frac{x-1}{1}=\frac{y-4}{2}=\frac{z-0}{3}$$

The point of intersection can be expressed as

$$(\lambda+1,2\lambda+4,3\lambda)$$

We also have the parametric representation of another line:

$$(2t+2,3t+6,4t+3)$$

Setting the two expressions equal gives:

$$\lambda + 1 = 2t + 2$$

$$2\lambda + 4 = 3t + 6$$

$$3\lambda = 4t + 3$$

From the first equation, we can express λ in terms of t:

$$\lambda = 2t + 1$$

Substituting λ into the second equation:

$$2(2t + 1) + 4 = 3t + 6$$

This simplifies to:

$$4t + 2 + 4 = 3t + 6$$

Thus,

$$4t + 6 = 3t + 6$$

Solving for t:

$$t = 0$$

Now substituting $t = 0$ back into the parametric equations gives the point of intersection:

$$(2(0)+2, 3(0)+6, 4(0)+3) = (2, 6, 3)$$

Next, we calculate the distance from the point $(1, 4, 0)$ to the point of intersection $(2, 6, 3)$:

$$\text{Distance} = \sqrt{(2-1)^2 + (6-4)^2 + (3-0)^2}$$

Calculating each term:

$$= \sqrt{1^2 + 2^2 + 3^2} = \sqrt{1+4+9} = \sqrt{14}$$

Thus, the final answer is

$$\sqrt{14}$$

Question 298

Let the line passing through the points $(-1, 2, 1)$ and parallel to the line

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{4}$$

intersect the line

$$\frac{x+2}{3} = \frac{y-3}{2} = \frac{z-4}{1}$$

at the point P. Then the distance of P from the point $\mathrm{Q}(4, -5, 1)$ is :

A. 5

B. 10

C. $5\sqrt{6}$

D. $5\sqrt{5}$

Correct answer: D

Source solution

$$\text{Let the line through } (-1, 2, 1) \text{ parallel to } \frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{4} :$$

$$\frac{x+1}{2} = \frac{y-2}{3} = \frac{z-1}{4} = \lambda$$

$$\Rightarrow x = 2\lambda - 1, y = 3\lambda + 2, z = 4\lambda + 1$$

$$\text{Let } \frac{x+2}{3} = \frac{y-3}{2} = \frac{z-4}{1} = \mu$$

$$\Rightarrow x = 3\mu - 2, y = 2\mu + 3, z = \mu + 4$$

$$\text{For intersection: } 2\lambda - 1 = 3\mu - 2,\ 3\lambda + 2 = 2\mu + 3,\ 4\lambda + 1 = \mu + 4$$

$$\lambda = 1, \mu = 1$$

$$P(1, 5, 5), Q(4, -5, 1)$$

$$PQ = \sqrt{9 + 100 + 16} = \sqrt{125} = 5\sqrt{5}$$

Question 299

The distance, of the point ($7, -2, 11$) from the line $\frac{x-6}{1} = \frac{y-4}{0} = \frac{z-8}{3}$ along the line $\frac{x-5}{2} = \frac{y-1}{-3} = \frac{z-5}{6}$, is :

A. 12

B. 14

C. 18

D. 21

Correct answer: B

Source solution

$$B = (2\lambda + 7,\ -3\lambda - 2,\ 6\lambda + 11).$$

Since B lies on the line

$$\frac{x-6}{1} = \frac{y-4}{0} = \frac{z-8}{3},$$

we must have

$$\frac{2\lambda + 7 - 6}{1} = \frac{-3\lambda - 2 - 4}{0} = \frac{6\lambda + 11 - 8}{3}.$$

From the middle term,

$$-3\lambda - 6 = 0.$$

$$\lambda = -2.$$

Therefore,

$$B = \big(2(-2)+7,\ -3(-2)-2,\ 6(-2)+11\big).$$

$$B = (3, 4, -1).$$

Let

$$A = (7, -2, 11).$$

Then,

$$\begin{aligned} AB &= \sqrt{(7-3)^2 + (-2-4)^2 + \big(11-(-1)\big)^2} \\ &= \sqrt{4^2 + (-6)^2 + 12^2} \\ &= \sqrt{16 + 36 + 144} \\ &= \sqrt{196} \\ &= 14. \end{aligned}$$

$$\boxed{AB = 14}$$

Linear Programming

Questions 300-300 / 1 selected / 0.3% practice weight / Consolidate

Question 300

Let the points $\left(\frac{11}{2}, \alpha\right)$ lie on or inside the triangle with sides $x + y = 11, x + 2y = 16$ and $2x + 3y = 29$. Then the product of the smallest and the largest values of α is equal to :

A. 22

B. 44

C. 33

D. 55

Correct answer: C

Source solution

Sol.

Point of intersection of $x = \frac{11}{2}$ with $L_1 \& L_3$
gives, $\alpha_{\min} = \frac{11}{2}$
and $\alpha_{\max} = 6$
$\therefore \alpha_{\min} \cdot \alpha_{\max} = \frac{11}{2} \times 6 = 33$

