LEARNE2I / PRIORITY PRACTICE

JEE Main 2025
Session 2

Mathematics
Solutions companion

300 questions / NCERT chapter navigation
Answers and available source solutions, in paper order.

Use the question numbers from the companion question paper. Brief source answers are identified wherever a detailed solution is unavailable.

Solutions on your phone

Scan to choose a chapter or question. You can also download all three solutions PDFs together.

learne2i.com/jee-main/revision-archive/2025-session-2/

Open the PDF bookmarks panel on the left to jump between chapters. The chapter links in the guide are clickable too.

YOUR PRACTICE ROADMAP

Chapter & topic priorities

For this Session 2 collection, begin with **Three Dimensional Geometry; Sequences and Series; Vector Algebra**. These chapters have the largest share of the selected questions.

Practice weight = questions selected from a chapter / 300. Topic percentages use that chapter's question count. These are practice priorities from the prediction output, not guaranteed exam marks or probabilities.

NCERT chapter / leading topics	Priority	Questions	Weight
Sets Sets: 1 (100.0%)	Consolidate	1-1 (1)	0.3%
Relations and Functions Properties of functions: 3 (75.0%) / Functions and relations: 1 (25.0%)	Consolidate	2-5 (4)	1.3%
Trigonometric Functions Trigonometric identities: 2 (50.0%) / Trigonometric equations: 1 (25.0%) / Trigonometric Functions: 1 (25.0%)	Consolidate	6-9 (4)	1.3%
Complex Numbers and Quadratic Equations Complex Numbers and Quadratic Equations: 2 (28.6%) / Quadratic equations and complex roots: 2 (28.6%) / Complex numbers and polynomial roots: 1 (14.3%) / More topics in chapter guide	Next	10-16 (7)	2.3%
Permutations and Combinations Counting arrangements and selections: 3 (27.3%) / Fundamental counting principle: 3 (27.3%) / Permutations and Combinations: 2 (18.2%) / More topics in chapter guide	Next	17-27 (11)	3.7%
Binomial Theorem Binomial expansion: 1 (33.3%) / Binomial Theorem: 1 (33.3%) / Middle term of a binomial expansion: 1 (33.3%)	Consolidate	28-30 (3)	1.0%
Sequences and Series Sequences and finite sums: 18 (50.0%) / Arithmetic progression: 6 (16.7%) / Geometric progression: 5 (13.9%) / More topics in chapter guide	Focus first	31-66 (36)	12.0%
Straight Lines Coordinate geometry of lines: 1 (33.3%) / Distance from a point to a line: 1 (33.3%) / Equation of a straight line: 1 (33.3%)	Consolidate	67-69 (3)	1.0%
Conic Sections Circle geometry: 8 (38.1%) / Conic geometry: 8 (38.1%) / Properties of conic sections: 3 (14.3%) / More topics in chapter guide	Next	70-90 (21)	7.0%
Introduction to Three Dimensional Geometry Coordinates in three dimensions: 2 (100.0%)	Consolidate	91-92 (2)	0.7%
Limits and Derivatives Standard limits: 1 (100.0%)	Consolidate	93-93 (1)	0.3%
Statistics Variance and standard deviation: 6 (66.7%) / Measures of central tendency and dispersion: 2 (22.2%) / Mean, median and mode: 1 (11.1%)	Next	94-102 (9)	3.0%
Probability Addition and multiplication rules of probability: 1 (100.0%)	Consolidate	103-103 (1)	0.3%
Inverse Trigonometric Functions Inverse Trigonometric Functions: 1 (100.0%)	Consolidate	104-104 (1)	0.3%
Determinants Determinants: 1 (100.0%)	Consolidate	105-105 (1)	0.3%
Application of Derivatives Derivative-based curve analysis: 3 (42.9%) / Tangents and normals: 2 (28.6%) / Applications of derivatives: 1 (14.3%) / More topics in chapter guide	Next	106-112 (7)	2.3%
Integrals Integration: 1 (100.0%)	Consolidate	113-113 (1)	0.3%
Application of Integrals Application of Integrals: 2 (66.7%) / Area under a curve: 1 (33.3%)	Consolidate	114-116 (3)	1.0%
Vector Algebra Algebra of vectors: 11 (35.5%) / Scalar product: 6 (19.4%) / Vector operations: 5 (16.1%) / More topics in chapter guide	Focus first	117-147 (31)	10.3%
Three Dimensional Geometry Coordinates in three dimensions: 39 (26.7%) / Shortest distance between lines: 23 (15.8%) / Equation of a plane: 22 (15.1%) / More topics in chapter guide	Focus first	148-293 (146)	48.7%
Mathematical Reasoning Historical appendix / omitted source chapter Mathematical reasoning: 5 (71.4%) / Propositional equivalence: 1 (14.3%) / Translation of compound statements: 1 (14.3%)	Historical appendix	294-300 (7)	2.3%

How to use the priorities

1. Rank chapters by their selected question counts. Ties follow NCERT book order.
2. Focus first: chapters starting within the first 70% of selected, non-historical questions. Next: the following 20%. Consolidate: the remainder. A chapter crossing a threshold stays together.
3. Topic counts use the question's existing core concept, falling back to topic and then chapter. Each question is counted once. Full topic lists begin each chapter.
4. Historical/omitted chapters stay in a separate appendix. Their presence is not a recommendation that they are in the target examination syllabus.

Names follow NCERT Classes XI and XII. Practical chemistry is identified as a laboratory-manual section. Percentages are rounded and may not add to exactly 100.0%. Chapters with no selected questions are not listed; this does not mean they have no exam importance.

Chapter naming references: NCERT textbooks.

Sets

Questions 1-1 / 1 selected / 0.3% practice weight / Consolidate

Question 1

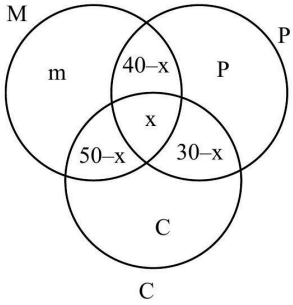
In a survey of 220 students of a higher secondary school, it was found that at least 125 and at most 130 students studied Mathematics; at least 85 and at most 95 studied Physics; at least 75 and at most 90 studied Chemistry; 30 studied both Physics and Chemistry; 50 studied both Chemistry and Mathematics; 40 studied both Mathematics and Physics and 10 studied none of these subjects. Let m and n respectively be the least and the most number of students who studied all the three subjects. Then $\mathbf{m+n}$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 45

Source solution



$\mathbf{125 \leq m+90-x \leq 130}$
$\mathbf{85 \leq P+70-x \leq 95}$
$\mathbf{75 \leq C+80-x \leq 90}$
$\mathbf{m+P+C+120-2x=210}$
$\mathbf{\Rightarrow 15 \leq x \leq 45 \& 30-x \geq 0}$
$\mathbf{\Rightarrow 15 \leq x \leq 30}$
$\mathbf{30+15=45}$

Relations and Functions

Questions 2-5 / 4 selected / 1.3% practice weight / Consolidate

Question 2

Let $f : R \to R$ be a function defined by $f(x) = (2+3a)x^2 + \left(\frac{a+2}{a-1}\right)x + b, a \neq 1$. If $f(x+y) = f(x) + f(y) + 1 - \frac{2}{7}xy$, then the value of $28\sum_{i=1}^{5}|f(i)|$ is:

A. 715

B. 735

C. 545

D. 675

Correct answer: D

Source solution

Sol. $f(x) = (3a+2)x^2 + \left(\frac{a+2}{a-1}\right)x + b$

$$f\left(x+\frac{1}{2}\right) = f(x) + f(y) + 1 - \frac{2}{7}xy \quad (1)$$

In (1) Put $x = y = 0 \Rightarrow f(0) = 2f(0) + 1 \Rightarrow f(0) = -1$ So, $f(0) = 0 + 0 + b = -1 \Rightarrow b = -1$

In (1) Put $y = -x \Rightarrow f(0) = f(x) + f(-x) + 1 + \frac{2}{7}x^2 - 1 = 2(3a+2)x^2 + 2b + 1 + \frac{2}{7}x^2$
$-1 = \left(2(3a+2) + \frac{2}{7}\right)x^2 + 1 - 2$
$\Rightarrow 6a + 4 + \frac{2}{7} = 0$
$a = -\frac{5}{7}$
So $f(x) = -\frac{1}{7}x^2 - \frac{3}{4}x - 1$
$\Rightarrow |f(x)| = \frac{1}{28}|4x^2 + 21x + 28|$
Now, $28\sum_{i=1}^{5}|f(i)| = 28(|f(1)| + |f(2)| + \ldots + |f(5)|)$
$\frac{1}{28} \cdot 675 = 675$

Question 3

Let $S = \{x : x \in \mathbb{R} \text{ and } (\sqrt{3}+\sqrt{2})^{x^2-4} + (\sqrt{3}-\sqrt{2})^{x^2-4} = 10\}$

Then $n(S)$ is equal to

A. 4

B. 0

C. 6

D. 2

Correct answer: A

Source solution

Sol. 1

$$(\sqrt{3}+\sqrt{2})^{x^2-4} + (\sqrt{3}-\sqrt{2})^{x^2-4} = 10 \qquad \Rightarrow (\sqrt{3}+\sqrt{2})^{x^2-4} + \frac{1}{(\sqrt{3}+\sqrt{2})^{x^2-4}} = 10$$

Let $(\sqrt{3}+\sqrt{2})^{x^2-4} = t$
$t + \frac{1}{t} = 10$
$\Rightarrow t^2 - 10t + 1 = 0$
$t = 5 + 2\sqrt{6}, 5 - 2\sqrt{6}$
If $t = 5 + 2\sqrt{6}$
If $t = 5 - 2\sqrt{6}$
$(\sqrt{3}+\sqrt{2})^{x^2-4} = (\sqrt{3}+\sqrt{2})^2$
$(\sqrt{3}+\sqrt{2})^{x^2-4} = (\sqrt{3}+\sqrt{2})^{-2}$
$\Rightarrow x^2 - 4 = 2$
$\Rightarrow x^2 - 4 = -2$
$\Rightarrow x = \pm\sqrt{6}$
$\Rightarrow x = \pm\sqrt{2}$
$S = \{\sqrt{6}, -\sqrt{6}, \sqrt{2}, -\sqrt{2}\}$
$n(s) = 4$

Question 4

Let $A = \{1, a_1, a_2 \ldots\ldots a_{18}, 77\}$ be a set of integers with $1 < a_1 < a_2 < \ldots\ldots < a_{18} < 77$. Let the set $A + A = \{x + y : x, y \in A\}$ contain exactly 39 elements. Then, the value of $a_1 + a_2 + \ldots\ldots + a_{18}$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 702

Source solution

$a_1, a_2, a_3, \ldots, a_{18}, 77$
are in AP i.e. 1, 5, 9, 13, ..., 77.
Hence $a_1 + a_2 + a_3 + \ldots + a_{18} = 5 + 9 + 13 + \ldots 18$ terms $= 702$

Question 5

An organization awarded 48 medals in event 'A', 25 in event ' B ' and 18 in event ' C '. If these medals went to total 60 men and only five men got medals in all the three events, then, how many received medals in exactly two of three events?

A. 10

B. 9

C. 21

D. 15

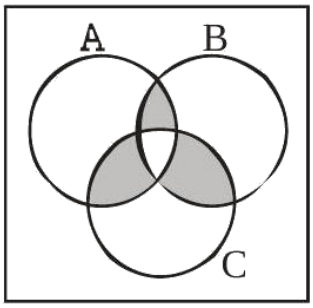
Correct answer: C

Source solution

Sol. $|\mathrm{A}| = 48$
$|\mathrm{B}| = 25$
$|\mathrm{C}| = 18$
$|A \cup B \cup C| = 60$ [Total]
$|\mathrm{A} \cap \mathrm{B} \cap \mathrm{C}| = 5$

$$|A \cup B \cup C| = \sum |A| - \sum |A \cap B| + |A \cap B \cap C| \qquad \Rightarrow \sum |A \cap B| = 48 + 25 + 18 + 5 - 60 \quad = 36$$

No. of men who received exactly 2 medals
$= \sum |\mathrm{A} \cap \mathrm{B}| - 3|\,\mathrm{A} \cap\ \mathrm{B} \cap \mathrm{C}|$
$= 36 - 15$
$= 21$



Trigonometric Functions

Questions 6-9 / 4 selected / 1.3% practice weight / Consolidate

Question 6

Let $S = \left\{\sin^2 2\theta : \left(\sin^4\theta + \cos^4\theta\right)x^2 + (\sin 2\theta)x + \left(\sin^6\theta + \cos^6\theta\right) = 0 \text{ has real roots }\right\}$. If α and β be the smallest and largest elements of the set S , respectively, then $3\left((\alpha-2)^2 + (\beta-1)^2\right)$ equals.....

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 4

Source solution

$\mathrm{D} = (\sin 2\theta)^2 - 4\left(1 - \frac{\sin^2 2\theta}{2}\right)\left(1 - \frac{3}{4}\sin^2 2\theta\right)$
$= (\sin 2\theta)^2 - 4\left(1 - \frac{5}{4}\sin^2 2\theta + \frac{3}{8}\sin^4 2\theta\right)$
$\mathrm{D} = -\frac{3}{2}\sin^4 2\theta + 6\sin^2 2\theta - 4 > 0$
$3\sin^4 2\theta - 12\sin^2 2\theta + 8 < 0$
$\sin^2 2\theta = \frac{12 \pm \sqrt{12^2 - 12.8}}{6} = \frac{12 \pm 4\sqrt{3}}{6} = \frac{6 \pm 2\sqrt{3}}{3}$
$\sin^2 2\theta = 2 \pm \frac{2}{\sqrt{3}}$, but $\sin^2 2\theta \in [0, 1]$
$\therefore \alpha = 2 - \frac{2}{\sqrt{3}}, \beta = 1 \rightarrow (\alpha-2)^2 = \frac{4}{3}, (\beta-1)^2 = 0$
$3(\alpha-2)^2 + (\beta-1)^2 = 4$

Question 7

Question ID: 101787

Let $c, k \in R$. If $f(x) = (c+1)x^2 + \left(1-c^2\right)x + 2k$ and $f(x+y) = f(x) + f(y) - xy$, for all $x, y \in R$, then the value of $|2(f(1) + f(2) + f(3) + \ldots\ldots + f(20))|$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 3395

Source solution

$f(x) = (c+1)x^2 + \left(1-c^2\right)x + 2k$

$$\&f(x+y) = f(x) + f(y) - xy \quad \forall xy \in R \tag{1}$$

$\lim_{y\to 0} \frac{f(x+y)-f(x)}{y} = \lim_{y\to 0} \frac{f(y)-xy}{y} \Rightarrow f'(x) = f'(0) - x$
$f(x) = -\frac{1}{2}x^2 + f'(0)\cdot x + \lambda$ but $f(0) = 0 \Rightarrow \lambda = 0$

$$f(x) = -\frac{1}{2}x^2 + \left(1-c^2\right)\cdot x \tag{2}$$

$\therefore$ as $\mathrm{f'(0) = 1 - c^2}$
Comparing equation (1) and (2)
We obtain, $\mathrm{c = -\frac{3}{2}}$
$\therefore \quad f(x) = -\frac{1}{2}x^2 - \frac{5}{4}x$
Now $\left|2\sum_{x=1}^{20} \mathrm{f(x)}\right| = \sum_{x=1}^{20} \mathrm{x^2} + \frac{5}{2}\cdot\sum_{x=1}^{20} \mathrm{x}$

$$= 2870 + 525$$
$$= 3395$$

Question 8

If for some $\alpha, \beta; \alpha \le \beta, \alpha + \beta = 8$ and $\sec^2\left(\tan^{-1}\alpha\right) + \operatorname{cosec}^2\left(\cot^{-1}\beta\right) = 36$, then $\alpha^2 + \beta$ is .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 14

Source solution

If $\left(\tan\left(\tan^{-1}(\alpha)\right) + 1\left(\cot\left(\cot^{-1}\beta\right)\right)^2 = 36\right.$
$\alpha^2 + \beta^2 = 34$
$\alpha\beta = 15$
$\alpha = 3, \beta = 5$
$\therefore \alpha^2 + \beta = 9 + 5 = 14$

Question 9

A value of $\theta \in (0, \pi/3)$, for which
$\left|1+\cos^2\theta \quad \sin^2\theta \quad 4\cos 6\theta\cos^2\theta \quad 1+\sin^2\theta \quad 4\cos 6\theta\cos^2\theta \quad \sin^2\theta \quad 1+4\cos 6\theta\right| = 0$, is :

A. $\frac{\pi}{18}$

B. $\frac{7\pi}{36}$

C. $\frac{7\pi}{24}$

D. $\frac{\pi}{9}$

Correct answer: D

Source solution

Let

$$a = \cos^2\theta, \qquad b = \sin^2\theta, \qquad c = 4\cos 6\theta.$$

Then

$$a + b = 1$$

and the determinant becomes

$$\Delta = \begin{vmatrix} 1+a & b & c \\ a & 1+b & c \\ a & b & 1+c \end{vmatrix}.$$

Given,

$$\Delta = 0.$$

\bigskip

Apply the row operation

$$R_1 \to R_1 - R_2.$$

Then

$$\Delta = \begin{vmatrix} 1 & -1 & 0 \\ a & 1+b & c \\ a & b & 1+c \end{vmatrix}.$$

Expanding along the first row,

$$\Delta = \begin{vmatrix} 1+b & c \\ b & 1+c \end{vmatrix} + \begin{vmatrix} a & c \\ a & 1+c \end{vmatrix}.$$

$$= (1+b)(1+c) - bc + a(1+c) - ac.$$

$$= 1 + b + c + a.$$

Since

$$a + b = 1,$$

$$\Delta = 2 + c.$$

Substituting $c = 4\cos 6\theta$,

$$2 + 4\cos 6\theta = 0.$$

$$\cos 6\theta = -\frac{1}{2}.$$

\bigskip

Therefore,

$$6\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}.$$

Hence,

$$\theta = \frac{\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{5\pi}{9}.$$

Since

$$0 < \theta < \frac{\pi}{3},$$

the admissible values are

$$\theta = \frac{\pi}{9}, \qquad \theta = \frac{2\pi}{9}.$$

Among the given options, only

$$\boxed{\frac{\pi}{9}}$$

appears.

$$\boxed{\text{Option (4)}}$$

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Complex Numbers and Quadratic Equations

Question 10

Let α_θ and β_θ be the distinct roots of $2x^2 + (\cos\theta)x - 1 = 0, \theta \in (0, 2\pi)$. If m and M are the minimum and the maximum values of $\alpha_\theta^4 + \beta_\theta^4$, then $16(M + m)$ equals :

A. 24

B. 25

C. 27

D. 17

Correct answer: B

Source solution

Sol. $\left(\alpha^2+\beta^2\right)^2 - 2\alpha^2\beta^2$
$\left[(\alpha+\beta)^2 - 2\alpha\beta\right]^2 - 2(\alpha\beta)^2$
$\left[\frac{\cos^2\theta}{4} + 1\right]^2 - 2 \cdot \frac{1}{4}$
$\left(\frac{\cos^2\theta}{4} + 1\right)^2 - \frac{1}{2}$
$M = \frac{25}{16} - \frac{1}{2} = \frac{17}{16}$
$m = \frac{1}{2}, 16(M + m) = 25$

Question 11

If the sum of the squares of the reciprocals of the roots α and β of the equation $3x^2 + \lambda x - 1 = 0$ is 15 , then $6\left(\alpha^3+\beta^3\right)^2$ is equal to :

A. 18

B. 24

C. 36

D. 96

Correct answer: B

Source solution

Sol. Here α, β roots of equation $3x^2 + \lambda x - 1 = 0$
$\alpha + \beta = \frac{-\lambda}{3}, \alpha\beta = \frac{-1}{3}$
$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha^2\beta^2} = 15$
$\lambda^2 = 9$
Now $6\left(\alpha^3+\beta^3\right)^2 = 6\left((\alpha+\beta)\left((\alpha+\beta)^2 - 3\alpha\beta\right)\right)^2$
$= 6\left(\frac{\lambda^2}{9}\right)\left\{\frac{\lambda^2}{9} + 1\right\}^2 = 24$

Question 12

If α satisfies the equation $x^2+x+1=0$ and $(1+\alpha)^7=A+B\alpha+C\alpha^2,\ A,\ B, C \geq 0$, then $5(3\,A-2\,B-C)$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 5

Source solution

$x^2+x+1=0 \Rightarrow x=\omega, \omega^2=\alpha$
Let $\alpha=\omega$
Now $(1+\alpha)^7=-\omega^{14}=-\omega^2=1+\omega$
$A=1,\ B=1, C=0$
$\therefore 5(3\,A-2\,B-C)=5(3-2-0)=5$

Question 13

Let $\lambda \neq 0$ be in $\mathbf{R}$. If α and β are the roots of the equation $x^2-x+2\lambda=0$, and α and γ are the roots of equation $3x^2-10x+27\lambda=0$, then $\frac{\beta\gamma}{\lambda}$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 18

Source solution

$3\alpha^2-10\alpha+27\lambda=0$

$$\alpha^2-\alpha+2\lambda=0 \qquad (___)$$

(1) - 3(2) gives
$-7\alpha+21\lambda=0 \Rightarrow \alpha=3\lambda$
Put $\alpha=3\lambda$ in equation (1) we get
$9\lambda^2-3\lambda+2\lambda-0$
$9\lambda^2=\lambda \Rightarrow \lambda=\frac{1}{9}$ as $\lambda \neq 0$
Now $\alpha=3\lambda \Rightarrow \lambda=\frac{1}{3}$
$\alpha+\beta=1 \Rightarrow \beta=2/3$
$\alpha+\gamma=\frac{10}{3} \Rightarrow \gamma=3$
$\frac{\beta\gamma}{\lambda}=\frac{\frac{2}{3}\times 3}{\frac{1}{9}}=18$

Question 14

$x = (8\sqrt{3} + 13)^{13}$ and $y = (7\sqrt{2} + 9)^{9}$. If $[t]$ denotes the greatest integer $\leq t$, then

A. $[x]$ is odd but $[y]$ is even

B. $[x] + [y]$ is even

C. $[x]$ and $[y]$ are both odd

D. $[x]$ is even but $[y]$ is odd

Correct answer: B

Source solution

Sol. 2
Let $x = I_1 + f_1$

$$(8\sqrt{3} + 13)^{13} = \mathrm{I}_1 + \mathrm{f}_1 \qquad (8\sqrt{3} - 13)^{13} = \mathrm{f}_1{}'(\mathrm{let})$$

On Subtaction
$(8\sqrt{3} + 13)^{13} - (8\sqrt{b} - 13)^{13} = I + f_1 - f_1{}'$
$2\left[{}^{13}\mathrm{C}_1(8\sqrt{3})^{12}\right]13 + {}^{13}\mathrm{C}_3(8\sqrt{3})^{10}13^3$
$+ - - - -] = \mathrm{I} + 0$
$\because \mathrm{I} =$ Even Number
[x] = Even
similarly,
Let $y = I_2 + f_2$
$(7\sqrt{2} + 9)^9 = \mathrm{I}_2 + \mathrm{f}_2$
$(7\sqrt{2} - 9)^9 = f_2'$
On Subtaction

$$(7\sqrt{2} + 9)^9 - (7\sqrt{2} - 9)^9 = I_2 + f_2 - f_2^1 \qquad 2\left[{}^9C_1(7\sqrt{2})^8 \cdot 9 + {}^9C_3(7\sqrt{2})^7 9^2 - - -\right] = I_2 + 0$$

$\mathbf{I_2} =$ Even

$$\because [\mathrm{x}] + [\mathrm{y}] = \mathbf{Even} + \mathbf{Even} \qquad = \mathbf{Even}$$

SECTION - B}

\setcounter{enumi}{80}

Question 15

Let $\Delta, \nabla \in \{\Lambda, V\}$ be such that $(p \rightarrow q) \Delta (p \nabla q)$ is a tautology. Then

A. $\Delta = V, \nabla = V$

B. $\Delta = V, \nabla = \Lambda$

C. $\Delta = \Lambda, \nabla = V$

D. $\Delta = \Lambda, \nabla = \Lambda$

Correct answer: A

Source solution

p	q	$p \rightarrow q$	$p \vee q$	$(p \rightarrow q) \vee (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	T

Question 16

The remainder, when 7^{103} is divided by 23 , is equal to:

A. 14

B. 9

C. 17

D. 6

Correct answer: A

Source solution

Sol. $7^{103} = 7\left(7^{102}\right) = 7(343)^{34} = 7(345-2)^{34}$
$7^{103} = 23\,K_1 + 7.2^{34}$
Now $7.2^{34} = 7.2^2.2^{32}$
$= 28.(256)^4$
$= 28(253+3)^4$
$\therefore 28 \times 81 \Rightarrow (23+5)(69+12)$
$23\,K_2 + 60$
$\therefore$ Remainder $= 14$

Permutations and Combinations

Question 17

Let ${}^nC_{r-1} = 28, {}^nC_r = 56$ and ${}^nC_{r+1} = 70$. Let $A(4\cos t, 4\sin t), B(2\sin t, -2\cos t)$ and $C\left(3r - n, r^2 - n - 1\right)$ be the vertices of a triangle ABC , where t is a parameter. If $(3x-1)^2 + (3y)^2 = \alpha$, is the locus of the centroid of triangle ABC , then α equals :

A. 20

B. 8

C. 6

D. 18

Correct answer: A

Source solution

Sol. ${}^nC_{r-1} = 28, {}^nC_r = 56$

$\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{28}{56}$

$\frac{\frac{n!}{(r-1)!(n-r+1)!}}{\frac{n!}{r!(n-r)!}} = \frac{1}{2}$

$\frac{r}{(n-r+1)} = \frac{1}{2}$

$$3r = n + 1 \quad \text{(i)}$$

$\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{56}{70}$

$$\frac{(r+1)}{(n-r)} = \frac{56}{70} \Rightarrow 9r = 4n - 5 \quad \text{(ii)}$$

By (i) & (ii)

($r = 3$), ($n = 8$)

A (4 cost, 4 sint) B(2 sint, -2 cost) C(3r-n, $r^2 - n - 1$)

$A(4\cos t, 4\sin t) \quad B(2\sin t, -2\cos t) C(1, 0)$

$(3x-1)^2 + (3y)^2 = (4\cos t + 2\sin t)^2 + (4\sin t - \cos t)^2$

$(3x-1)^2 + (3y)^2 = 20 \quad \therefore$ option (1)

Question 18

The coefficient of x^{70} in $x^2(1+x)^{98}+x^3(1+x)^{97}+x^4(1+x)^{96}+\ldots\ldots\ldots+x^{54}(1+x)^{46}$ is ${}^{99}C_p-{}^{46}C_q$.
Then a possible value to $p+q$ is :

A. 55

B. 61

C. 68

D. 83

Correct answer: D

Source solution

Sol. $x^2(1+x)^{98}+x^3\left(1+x^{97}\right)+x^4(1+x)^{96}+\ldots\ldots x^{54}(1+x)^{46}$

Coeff. of x^{70} : ${}^{98}C_{68}+{}^{97}C_{67}+{}^{96}C_{66}+\ldots\ldots\ldots$
${}^{47}C_{17}+{}^{46}C_{16}$
$={}^{46}C_{30}+{}^{47}C_{30}+\ldots\ldots\ldots\ldots{}^{98}C_{30}$
$=\left({}^{46}C_{31}+{}^{46}C_{30}\right)+{}^{47}C_{30}+\ldots\ldots\ldots\ldots{}^{98}C_{30}-{}^{46}C_{31}$
$={}^{47}C_{31}+{}^{47}C_{30}+\ldots\ldots\ldots\ldots{}^{98}C_{30}-{}^{46}C_{31}$
.....
$={}^{99}C_{31}-{}^{46}C_{31}={}^{99}C_p-{}^{46}C_q$
Possible values of ($p+q$) are 62, 83, 99, 46
$\Rightarrow p+q=83$

Question 19

Total numbers of 3-digit numbers that are divisible by 6 and can be formed by using the digits $1, 2, 3$, 4, 5 with repetition, is .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 16

Source solution

For number to be divisible by '6' unit digit should be even and sum of digit is divisible by 3 .
$(2,1,3),(2,3,4),(2,5,5),(2,2,5),(2,2,2)$,
$(4,1,1),(4,4,1),(4,4,4),(4,3,5)$
$2,1,3\Rightarrow 312,132$
$2,3,4\Rightarrow 342,432,234,324$
$2,5,5\Rightarrow 552$
$2,2,5\Rightarrow 252,522$
2, 2, $2\Rightarrow 222$
$4,1,1\Rightarrow 114$
$4,4,1\Rightarrow 414,144$
$4,4,4\Rightarrow 444$
4, 3, $5\Rightarrow 354,534$
Total 16 numbers.

Question 20

The number of permutations, of the digits $1, 2, 3$, ... 7 without repetition, which neither contain the string 153 nor the string 2467, is .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 4898

Source solution

Digits $\rightarrow 1, 2, 3, 4, 5, 6, 7$
Total permutations $= 7!$
Let $A =$ number of numbers containing string 153
Let $B =$ number of numbers containing string 2467
$n(A) = 5! \times 1$
1532467
$n(B) = 4! \times 1$
$2467 \quad 135$
$n(A \cap B) = 2!$
$153 \quad 2467$
$n(A \cup B) = 5! + 4! - 2! = 142$
n(neither string 153 nor string 2467)
= Total $-n(A \cup B)$
$= 7! - 142 = 4898$

Question 21

The total number of three-digit numbers, divisible by 3 , which can be formed using the digits $1, 3, 5$, 8, if repetition of digits is allowed, is:

A. 22

B. 18

C. 21

D. 20

Correct answer: A

Source solution

Sol. (1,1,1) (3,3,3) (5,5,5) (8,8,8)
(5,5,8) (8,8,5) (1,3,5) (1,3,8)
Total number $= 1 + 1 + 1 + 1 + \frac{3!}{2!} + \frac{3!}{2!} + 3! + 3! = 22$

Question 22

The number of five digit numbers, greater than 40000 and divisible by 5, which can be formed using the digits $0, 1, 3, 5, 7$ and 9 without repetition, is equal to

A. 120

B. 132

C. 72

D. 96

Correct answer: A

Source solution

Sol. $5 \quad x \quad x \quad x \quad 0$
$7 \quad X \quad X \quad X \quad 0$
$7x \quad x \quad x \quad 5$
$9x \quad x \quad x \quad 0$
$9x \quad x \quad x \quad 5$
So Required numbers $= 5 \times {}^{4}P_{3} = 120$

Question 23

The number of 5-digit natural numbers, such that the product of their digits is 36 , is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 180

Source solution

$3 \times \frac{5!}{2!2!} + \frac{5!}{3! \times 2!} + \frac{5!}{2!} + \frac{5!}{3!} = 180$

Question 24

If the sum and product of four positive consecutive terms of a G.P., are 126 and 1296 , respectively, then the sum of common ratios of all such GPs is

A. 7

B. 3

C. $\frac{9}{2}$

D. 14

Correct answer: A

Source solution

Sol. (1)
Four term of G.P. $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$
$\frac{a}{r^3}+\frac{a}{r}+ar+ar^3=126$
$\frac{a}{r^3}\cdot\frac{a}{r}\cdot ar\cdot ar^3=1296$
$a^4=1296$
$a=6$
$\frac{6}{r^3}+\frac{6}{r}+6r+6r^3=126$

$$\left(r+\frac{1}{r}\right)+r^3+\frac{1}{r^3}=21 \qquad \left(r+\frac{1}{r}\right)+\left(r+\frac{1}{r}\right)^3-3\left(r+\frac{1}{r}\right)=21$$

Let $r+\frac{1}{r}=t$
$t^3-2t=21$
$\Rightarrow t=3$
$r+\frac{1}{r}=3$
$r^2-3r+1=0$
$r=\frac{3\pm\sqrt{9-4}}{2}$
$r=\frac{3\pm\sqrt{5}}{2}$
Sum of common ratio $=\frac{9}{4}+\frac{5}{4}+\frac{3\sqrt{5}}{2}+\frac{9}{4}+\frac{5}{4}-\frac{3\sqrt{5}}{2}$
$=7$

Question 25

If all the six digit numbers $x_1x_2x_3x_4x_5x_6$ with $0<x_1<x_2<x_3<x_4<x_5<x_6$ are arranged in the increasing order, then the sum of the digits in the 72th number is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 32

Source solution

32
Number of six digit number starting with 1 is $1\ldots\ldots\ldots\ldots={}^8C_5=56$
As remaining five digits can be selected from 8 digits that are greater than (i.e., $2,3,4,5,6,7,8$)
Number of six digit number starting with $23\ldots\ldots\ldots={}^6C_4=15$

$$\text{Total }=56+15=71$$

Now, 72nd number $=245678$
$\therefore$ sum of the digits $=2+4+5+6+7+8=32$

Question 26

The total number of six digit numbers, formed using the digits $\mathbf{4, 5, 9}$ only and divisible by 6 , is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 81

Source solution

81
We have,
For this, 4 will be fixed as unit place digit
Total number
Case I: 4's $\rightarrow \mathbf{6}$ times
1
Case II: 4's $\rightarrow$ 4times

$$\begin{aligned} &\mathbf{5's \rightarrow 1\ times} \quad \tfrac{5!}{3!} = 20 \\ &\mathbf{9's \rightarrow 1\ times} \end{aligned}$$

Case III: 4's $\rightarrow$ 3times

$$\mathbf{5's \rightarrow 3times} \quad \frac{5!}{2!3!} = 10$$

Case IV: $\mathbf{4}$'s $\rightarrow$ 3times

$$\mathbf{9's \rightarrow 3times} \quad \frac{5!}{2!3!} = 10$$

Case V: $\mathbf{4}$'s $\rightarrow \mathbf{2}$ times

$$\begin{aligned} &\mathbf{5's \rightarrow 2times} \quad \tfrac{5!}{2!2!} = 30 \\ &\mathbf{9's \rightarrow 2\ times} \end{aligned}$$

Case VI: $\mathbf{4}$'s $\rightarrow \mathbf{1}$ times
5's $\rightarrow \mathbf{1}$ times $\quad \frac{5!}{4!} = 5$
9's $\rightarrow \mathbf{4}$ times
Case VII: 4's $\rightarrow \mathbf{1}$ times

$$\begin{aligned} &\mathbf{5's \rightarrow 4\ times} \quad \tfrac{5!}{4!} = 5 \\ &\mathbf{9's \rightarrow 1\ times} \end{aligned}$$

Total numbers $= \mathbf{81}$

Question 27

The remainder, when 7^{103} is divided by 17 is .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 12

Source solution

$7^{103} = 7 \times 7^{102}$
$= 7 \times (49)^{51}$
$= 7 \times (51 - 2)^{51}$
Remainder :- $7 \times (-2)^{51}$
$\Rightarrow -7\left(2^3 \cdot (16)^{12}\right)$
$\Rightarrow -56(17 - 1)^{12}$
Remainder $= -56 \times (-1)^{12} = -56 + 68 = 12$

Binomial Theorem

Questions 28-30 / 3 selected / 1.0% practice weight / Consolidate

Question 28

For the natural numbers m, n, if $(1-y)^m(1+y)^n = 1 + a_1y + a_2y_2 + \ldots. + a^{m+n}\ y^{m+n}$ and $a_1 = a_2 = 10$, then the value of $(m+n)$ is equal to :

A. 88

B. 64

C. 100

D. 80

Correct answer: D

Source solution

$(1-y)^m(1+y)^n$ Coefficient of $y\ (a_1) = 1.{}^nC_1 + {}^mC_1(-1)$

$$= n - m = 10$$

Coefficient of $y^2\ (a_2)$

$$= 1 \cdot {}^nC_2 - {}^mC_1 \cdot {}^mC_1 \cdot 1 \cdot {}^mC_2 = 10$$
$$= \frac{n(n-1)}{2} - m \cdot n + \frac{m(m-1)}{2} = 10$$
$$m^2 + n^2 - 2mn - (n+m) = 20$$
$$(n-m)^2 - (n+m) = 20$$
$$n + m = 80$$

By equation (1) & (2)

$$m = 35, n = 45$$

Question 29

Let the coefficients of three consecutive terms T_r, T_{r+1} and T_{r+2} in the binomial expansion of $(a+b)^{12}$ be in a G.P. and let p be the number of all possible values of r. Let q be the sum of all rational terms in the binomial expansion of $(\sqrt[4]{3}+\sqrt[3]{4})^{12}$. Then $p+q$ is equal to :

A. 283

B. 295

C. 287

D. 299

Correct answer: A

Source solution

Sol. $(a+b)^{\frac{1}{2}}$

$T_r, T_{r+1}, T_{r+2} \rightarrow GP$

So, $\frac{T_{r+1}}{T_r} = \frac{T_{r+2}}{T_{r+1}}$

$\frac{{}^{12}C_r}{{}^{12}C_{r-1}} = \frac{{}^{12}C_{r+1}}{{}^{12}C_r}$

$\frac{12-r+1}{r} = \frac{12-(r+1)+1}{r+1}$

$(13-r)(r+1) = (12-r)(r)$

$-r+12r+13 = 12r-r^2$

$13 = 0$

No value of r possible

So $P = 0$

$\left(3^{\frac{1}{4}}+4^{\frac{1}{3}}\right)^{12} = \sum {}^{12}C_r \left(3^{\frac{1}{4}}\right)^{12-r}\left(4^{\frac{1}{3}}\right)^{r}$

Exponent of $\left(3^{\frac{1}{4}}\right)$ exponent of $\left(4^{\frac{1}{3}}\right)$ term

0

27

0

12

256

$q = 27+256 = 283$

$p+q = 0+283 = 283$

Question 30

Let K be the sum of the coefficients of the odd powers of x in the expansion of $(1+x)^{99}$. Let a be the middle term in the expansion of $\left(2+\frac{1}{\sqrt{2}}\right)^{200}$. If $\frac{{}^{200}C_{99}K}{a}=\frac{2^l m}{n}$, where m and n are odd numbers, then the ordered pair (l, n) is equal to

A. $(50,51)$

B. $(50,101)$

C. $(51,99)$

D. $(51,101)$

Correct answer: B

Source solution

Sol. (2)}

$\mathrm{K}=\frac{(1+1)^{99}}{2}=2^{98}$

$a={}^{200}C_{100}2^{100}\times\frac{1}{(\sqrt{2})^{100}}$

$\mathrm{a}={}^{200}\mathrm{C}_{100}2^{50}$

$\frac{{}^{200}\mathrm{C}_{99}\cdot\mathrm{K}}{\mathrm{a}}=\frac{{}^{200}\mathrm{C}_{99}\cdot 2^{98}}{{}^{200}\mathrm{C}_{100}\cdot 2^{50}}$

$\because\ \frac{{}^{200}C_{99}}{{}^{200}C_{100}}=\frac{1200}{\boxed{99}\,|101}\times\frac{100|100}{\boxed{200}}$

$=\frac{100}{101}$

$\therefore\ \frac{{}^{200}\mathrm{C}_{99}\,\mathrm{K}}{\mathrm{a}}=\frac{100}{101}\times 2^{48}$

$$=\frac{25\times 2^{50}}{101}$$

$\ell=50, \mathrm{n}=101$

$(\ell, n)=(50,101)$

Sequences and Series

Questions 31-66 / 36 selected / 12.0% practice weight / Focus first

Question 31

If the domain of the function $\log_5\left(18x - x^2 - 77\right)$ is (α, β) and the domain of the function $\log_{(x-1)}\left(\frac{2x^2+3x-2}{x^2-3x-4}\right)$ is (γ, δ), then $\alpha^2 + \beta^2 + \gamma^2$ is equal to :

A. 195

B. 174

C. 186

D. 179

Correct answer: C

Source solution

$$\log_5(18x - x^2 - 77)$$

has domain (α, β),

and

$$\log_{(x-1)}\left(\frac{2x^2 + 3x - 2}{x^2 - 3x - 4}\right)$$

has domain (γ, δ).

We need to find

$$\alpha^2 + \beta^2 + \gamma^2.$$

\hrulefill

\textbf{Step 1: Domain of } $\log_5(18x - x^2 - 77)$

For a logarithm,

$$18x - x^2 - 77 > 0.$$

$$-x^2 + 18x - 77 > 0$$

$$x^2 - 18x + 77 < 0.$$

Factorizing,

$$x^2 - 18x + 77 = (x - 7)(x - 11).$$

Hence,

$$(x-7)(x-11)<0.$$

Therefore,

$$7<x<11.$$

Thus,

$$(\alpha,\beta)=(7,11).$$

So,

$$\alpha=7,\qquad \beta=11.$$

\hrulefill

\textbf{Step 2: Domain of }

$$\log_{(x-1)}\left(\frac{2x^2+3x-2}{x^2-3x-4}\right)$$

For a logarithm:

$$x-1>0,\qquad x-1\neq 1$$

which gives

$$x>1,\qquad x\neq 2.$$

Also,

$$\frac{2x^2+3x-2}{x^2-3x-4}>0.$$

Factorize:

$$2x^2+3x-2=(2x-1)(x+2),$$

$$x^2-3x-4=(x-4)(x+1).$$

Hence,

$$\frac{(2x-1)(x+2)}{(x-4)(x+1)}>0.$$

Critical points:

$$-2,\ -1,\ \frac{1}{2},\ 4.$$

Using sign analysis:

$$\frac{(2x-1)(x+2)}{(x-4)(x+1)} > 0$$

for

$$(-\infty, -2) \cup (-1, \tfrac{1}{2}) \cup (4, \infty).$$

Now intersect with

$$x > 1.$$

Therefore,

$$x \in (4, \infty).$$

Thus the domain is

$$(\gamma, \delta) = (4, \infty).$$

Hence,

$$\gamma = 4.$$

\hrulefill

\textbf{Step 3: Required value}

$$\alpha^2 + \beta^2 + \gamma^2 = 7^2 + 11^2 + 4^2$$

$$= 49 + 121 + 16$$

$$= 186.$$

$$\boxed{186}$$

Hence, the correct option is

$$\boxed{\textbf{(3) } 186}.$$

Question 32

Let the plane $x+3y-2z+6=0$ meet the co-ordinate axes at the points $\mathbf{A},\mathbf{B},\mathbf{C}$. If the orthocentre of the triangle ABC is $\left(\alpha,\beta,\frac{6}{7}\right)$, then $98(\alpha+\beta)^2$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

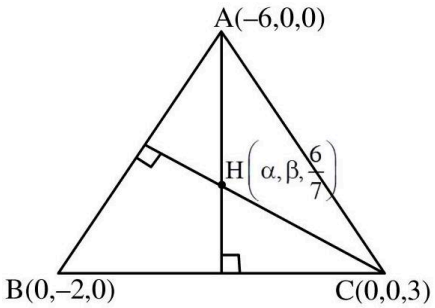
Correct answer: 288

Source solution

$A(-6,0,0)\quad B(0,-2,0)C=(0,0,3)$

$\overrightarrow{\mathbf{AB}}=6\hat{i}-2\hat{j},\quad \overrightarrow{\mathbf{BC}}=2\hat{j}+3\hat{k},$

$\overrightarrow{\mathbf{AC}}=6\hat{\mathbf{i}}+3\hat{\mathbf{k}}$



$\overrightarrow{\mathbf{AH}}\cdot\overrightarrow{\mathbf{BC}}=0$

$\left(\alpha+6,\beta,\frac{6}{7}\right)\cdot(0,2,3)=0$

$\beta=\frac{-9}{7}$

$\overrightarrow{\mathbf{CH}}\cdot\overrightarrow{\mathbf{AB}}=0$

$\left(\alpha,\beta,\frac{-15}{7}\right)\cdot(6,-2,0)=0$

$6\alpha-2\beta=0$

$\alpha=\frac{-3}{7}$

$98(\alpha+\beta)^2=(98)\frac{(144)}{49}=288$

Question 33

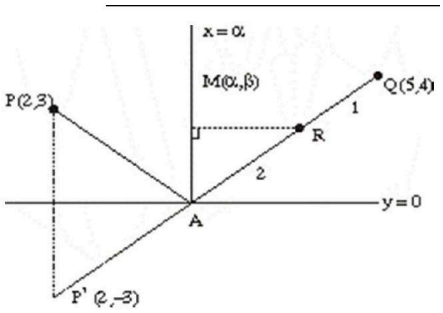
A ray of light passing through the point $\mathbf{P(2,3)}$ reflects on the x -axis at point A and the reflected ray passes through the point $\mathbf{Q(5,4)}$. Let R be the point that divides the line segment AQ internally into the ratio $\mathbf{2:1}$. Let the co-ordinates of the foot of the perpendicular M from R on the bisector of the angle PAQ be (α, β). Then, the value of $7\alpha + 3\beta$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 31

Source solution



By observation we see that $\mathbf{A}(\alpha, 0)$.
And $\beta = \mathbf{y}$-cordinate of R

$$= \frac{2\times 4 + 1\times 0}{2+1} = \frac{8}{3} \quad (1)$$

Now $\mathbf{P'}$ is image of P in $\mathbf{y = 0}$ which will be $\mathbf{P'(2,-3)}$
$\therefore$ Equation of $\mathbf{P'Q}$ is $(\mathbf{y+3}) = \frac{4+3}{5-2}(\mathbf{x-2})$
i.e. $3y + 9 = 7x - 14$
$\mathbf{A} \equiv \left(\frac{23}{7}, 0\right)$ by solving with $\mathbf{y = 0}$

$$\therefore \alpha = \frac{23}{7} \quad (2)$$

By (1), (2)

$$7\alpha + 3\beta = 23 + 8 = 31$$

Question 34

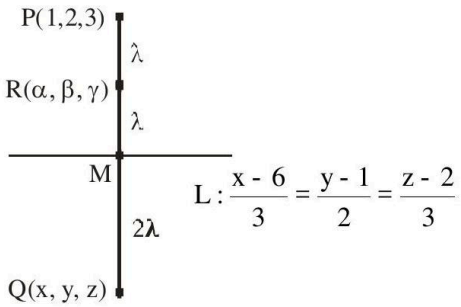
Let the image of the point $\mathbf{P(1,2,3)}$ in the line $\mathbf{L: \frac{x-6}{3} = \frac{y-1}{2} = \frac{z-2}{3}}$ be Q . let $\mathbf{R(\alpha,\beta,\gamma)}$ be a point that divides internally the line segment PQ in the ratio $\mathbf{1:3}$. Then the value of $\mathbf{22(\alpha+\beta+\gamma)}$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 125

Source solution



Let M be the mid-point of PQ

$\therefore \mathbf{M = (3\lambda+6, 2\lambda+1, 3\lambda+2)}$

Now, $\mathbf{\overrightarrow{PM} = (3\lambda+5)\hat{i} + (2\lambda-1)\hat{j} + (3\lambda-1)\hat{k}}$

$\because \mathbf{\overrightarrow{PM} \perp (3\hat{i}+2\hat{j}+3\hat{k})}$

$\therefore \mathbf{3(3\lambda+5)+2(2\lambda-1)+3(3\lambda-1)=0}$

$\mathbf{\lambda = \frac{-5}{11}}$

$\therefore \mathbf{M\left(\frac{51}{11}, \frac{1}{11}, \frac{7}{11}\right)}$

Since R is mid-point of PM

$\mathbf{22(\alpha+\beta+\gamma) = 125}$

Question 35

Let a conic C pass through the point $(4, -2)$ and $P(x, y), x \geq 3$, be any point on C . Let the slope of the line touching the conic C only at a single point P be half the slope of the line joining the points P and (3, -5). If the focal distance of the point (7, 1) on C is d , then 12 d equals .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 75

Source solution

$P(x, y) \& x \geq 3$

Slope of line at $P(x, y)$ will be $\frac{dy}{dx} = \frac{1}{2}\left(\frac{y+5}{x-3}\right)$

$\Rightarrow 2\frac{dy}{(y+5)} = \frac{1}{(x-3)}dx$

$\Rightarrow 2\ell n(y+5) = \ell n(x-3) + C$

Passes through $(4, -2)$

$\Rightarrow 2\ell n(3) = \ell n(1) + C$

$\Rightarrow C = 2\ell n(3)$

$\Rightarrow 2\ell n(y+5) = \ell n(x-3) + 2\ell n(3)$

$\Rightarrow 2\left(\ln\left(\frac{y+5}{3}\right)\right) = \ln(x-3)$

$\Rightarrow \left(\frac{y+5}{3}\right)^2 = (x-3)$

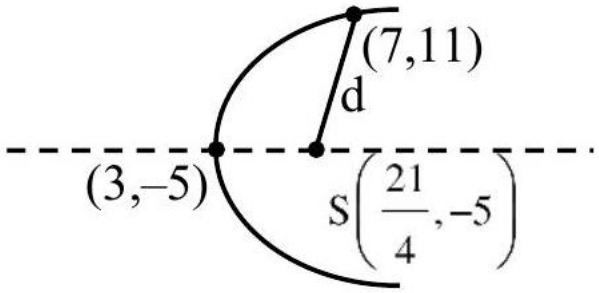
$\Rightarrow (y+5)^2 = 9(x-3)$

$\downarrow$

Parabola

$4a = 9$

$a = \frac{9}{4}$



$d = \sqrt{\left(\frac{7}{4}\right)^2 + 6^2}$

$d = \frac{\sqrt{625}}{4}$

$d = \frac{25}{4}$

$12\,d = 75$

Question 36

A line passes through $\mathbf{A(4,-6,-2)}$ and $\mathbf{B(16,-2,4)}$. The point $\mathbf{P(a,b,c)}$ where $\mathbf{a,b,c}$ are non-negative integers, on the line AB lies at a distance of 21 units, from the point A . The distance between the points $\mathbf{P(a,b,c)}$ and $\mathbf{Q(4,-12,3)}$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 22

Source solution

-

$$\frac{x-4}{12}=\frac{x+6}{4}=\frac{z+2}{6}$$
$$\frac{x-4}{\frac{6}{7}}=\frac{y+6}{\frac{2}{7}}=\frac{z+2}{\frac{3}{7}}=21$$
$$\left(21\times\frac{6}{7}+4,\frac{2}{7}\times21-6,\frac{3}{7}\times21-2\right)$$
$$=(22,0,7)=(a,b,c)$$
$$\therefore\sqrt{324+144+16}=22$$

Question 37

Let $a_1,a_2,\ldots,a_{10}$ be an AP with common difference -3 and $b_1,b_2,\ldots,b_{10}$ be a GP with common ratio 2 . Let $c_k=a_k+b_k,k=1,2,\ldots,10$. If $c_2=12$ and $c_3=13$, then $\sum_{k=1}^{10}c_k$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 2021

Source solution

$\mathbf{c_2=a_2+b_2=a_1-3+2\,b_1=12}$

$$\mathbf{a_1+2\,b_1=15}\qquad\mathbf{(1)}$$

$c_3=a_3+b_3=a_1-6+4b_1=13$

$$a_1+4b_1=19\qquad\mathbf{(2)}$$

from (1) & (2) $\mathbf{b_1=2,a_1=11}$
$\sum_{k=1}^{10}c_k=\sum_{k=1}^{10}(a_k+b_k)=\sum_{k=1}^{10}a_k+\sum_{k=1}^{10}b_k$
$=\frac{10}{2}(2\times11+9\times(-3))+\frac{2(2^{10}-1)}{2-1}$
$=5(22-27)+2(1023)$
$=2046-25=2021$

Question 38

Points $\mathbf{P(-3,2), Q(9,10)}$ and $\mathbf{(a,4)}$ lie on a circle C with PR as its diameter, The tangents to C at the points Q and $\boldsymbol{R}$ intersect at the point $\boldsymbol{S}$. If $\boldsymbol{S}$ lies on the line $\mathbf{2x-ky=1}$, then $\boldsymbol{k}$ is equal to

Numerical answer: _____

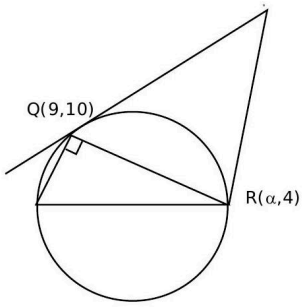
Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 3

Source solution

Equation of circle is

$\mathbf{(x+3)(x-\alpha)+(y-2)(y-4)=0}$



Q lies on it

$\mathbf{12(9-\alpha)+8\times 6=0}$

$\mathbf{\alpha=13}$

$\mathbf{x^2+y^2-10x-6y-31=0}$

Equation of Tangent at Q

$\mathbf{x.9+y.10-5(x+9)-3(y+10)-31=0}$

$$\mathbf{4x+7y=106} \quad \mathbf{(1)}$$

Equation of Tangent at R

$\mathbf{x.13+y.4-5(x+13)-3(y+4)-31=0}$

$$\mathbf{8x+y=108} \quad \mathbf{(2)}$$

Solution (1) and (2)

$\mathbf{s=\left(\frac{25}{2},8\right)}$

which lies on $\mathbf{2x-ky=1}$

$\mathbf{k=3}$

Question 39

Let $\mathbf{a_1, a_2, \ldots\ldots, a_{21}}$ be an AP such that $\sum_{n=1}^{20} \frac{1}{a_n a_{n+1}} = \frac{4}{9}$. If the sum of this AP is 189, then $\mathbf{a_6 a_{16}}$ is equal to :

A. 57

B. 72

C. 48

D. 36

Correct answer: B

Source solution

$$\sum_{n=1}^{20} \frac{1}{a_n a_{n+1}} = \sum_{n=1}^{20} \frac{1}{a_n (a_n + d)}$$
$$= \frac{1}{d} \sum_{n=1}^{20} \left(\frac{1}{a_n} - \frac{1}{a_n + d} \right)$$
$$\Rightarrow \frac{1}{d} \left(\frac{1}{a_1} - \frac{1}{a_{21}} \right) = \frac{4}{9} \text{ (Given)}$$
$$\Rightarrow \frac{1}{d} \left(\frac{a_{21} - a_1}{a_1 a_{21}} \right) = \frac{4}{9}$$
$$\Rightarrow \frac{1}{d} \left(\frac{a_1 + 20d - d_1}{a_1 a_2} \right) = \frac{4}{9} \Rightarrow a_1 a_2 = 45$$

Now sum of first 21 terms $= \frac{21}{2}(2a_1 + 20\,d) = 189$

$$\Rightarrow a_1 + 10\,d = 9$$

For equation (1) & (2) we get

$$a_1 = 3 \& \; d = \frac{3}{5}$$

OR

$$a_1 = 15 \& \; d = -\frac{3}{5}$$

So, $a_6 . a_{16} = (a_1 + 5\,d)(a_1 + 15\,d)$

$$\Rightarrow a_6 a_{16} = 72$$

Question 40

Let $\mathbf{f}: \mathbf{N} \rightarrow \mathbf{R}$ be a function such that $f(x+y)=2 f(x) f(y)$ for natural numbers x and y. If $f(1)=2$, then the value of α for which

$$\sum_{k=1}^{10} f(\alpha+k)=\frac{512}{3}\left(2^{20}-1\right)$$

holds, is

A. 2

B. 3

C. 4

D. 6

Correct answer: C

Source solution

$f: N \rightarrow R, f(x+y)=2 f(x) f(y)$
$\mathbf{f}(1)=2$,

$$\sum_{k=1}^{10} f(\alpha+k)=2 f(\alpha) \sum_{k=1}^{10} f(k) \quad =2 f(\alpha)(f(1)+f(2)+\ldots . .+f(10)) \tag{2}$$

From (1)
$\mathbf{f}(2)=2 \mathbf{f}^{2}(1)=2^{3}$
$\mathbf{f}(3)=2 \mathbf{f}(2) \mathbf{f}(1)=2^{5}$
$\vdots \quad \vdots$
$\mathbf{f}(10)=2^{9} \mathbf{f}^{10}(1)=2^{19}$
$\mathbf{f}(\alpha)=2^{2 \alpha-1} ; \alpha \in \mathbf{N}$
from (2)

$$\sum_{k=1}^{10} f(\alpha+k)=2\left(2^{2 \alpha-1}\right)\left(2+2^{3}+2^{5}+\ldots .+2^{19}\right)$$
$$\frac{512}{3}\left(2^{20}-1\right)=2^{2 \alpha}\left(2 \frac{\left(2^{20}-1\right)}{3}\right)$$

Hence $\alpha=4$

Question 41

Let $A_1, A_2, A_3, \ldots\ldots$ be an increasing geometric progression of positive real numbers. If $A_1A_3A_5A_7 = \frac{1}{1296}$ and $A_2 + A_4 = \frac{7}{36}$, then, the value of $A_6 + A_8 + A_{10}$ is equal to

A. 33

B. 37

C. 43

D. 47

Correct answer: C

Source solution

Sol. $A_1 \cdot A_3 \cdot A_5 \cdot A_7 = \frac{1}{1296}$
$(A_4)^4 = \frac{1}{1296}$

$$A_4 = \frac{1}{6} \quad (1)$$

$A_2 + A_4 = \frac{7}{36}$

$$A_2 = \frac{1}{36} \quad (2)$$

$A_6 = 1$
$A_8 = 6$
$A_{10} = 36$
$A_6 + A_8 + A_{10} = 43$

Question 42

The point of intersection C of the plane $8x + y + 2z = 0$ and the line joining the point $A(-3, -61)$ and $B(2, -4, -3)$ divides the line segment AB internally in the ratio k :. If $a, b, c(|a|, |b|, |c|)$ are coprime are the direction ratios of the perpendicular form the point C on the line $\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}$, then $|a + b + c|$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 10

Source solution

10

Plane : $8x + y + 2z = 0$

Given line $AB : \frac{x-2}{5} = \frac{y-4}{10} = \frac{z+3}{-4} = \lambda$

Any point on line $(5\lambda + 2, 10\lambda + 4, -4\lambda - 3)$

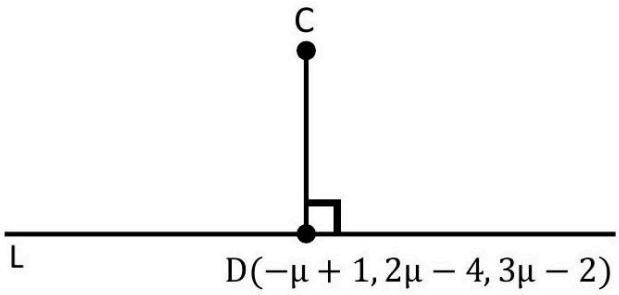
Point of intersection of line and plane

$8(5\lambda + 2) + 10\lambda + 4 - 8\lambda - 6 = 0$

$\lambda = -\frac{1}{3}$

C $\left(\frac{1}{3}, \frac{2}{3}, -\frac{5}{3}\right)$

$L : \frac{x-1}{-1} = \frac{y+4}{2} = \frac{z+2}{3} = \mu$



$\overline{CD} = \left(-\mu + \frac{2}{3}\right)\hat{i} + \left(2\mu - \frac{14}{3}\right)\hat{j} + \left(3\mu - \frac{1}{3}\right)\hat{k}$

$\left(-\mu + \frac{2}{3}\right)(-1) + \left(2\mu - \frac{14}{3}\right)2 + \left(3\mu - \frac{1}{3}\right)3 = 0$

$\mu = \frac{11}{14}$

$\overrightarrow{CD} = \frac{-5}{42}, \frac{-130}{42}, \frac{85}{42}$

Direction ratio $\rightarrow (-1, -26, 17)$

$|a + b + c| = 10$

Question 43

If the first term of an A.P. is 3 and the sum of its first 25 terms is equal to the sum of its next 15 terms, then the common difference of this A.P. is:

A. 1/6

B. 1/5

C. 1/4

D. 1/7

Correct answer: A

Source solution

\textbf{Given:}

$$a = 3$$

and

$$S_{25} = \text{sum of the next 15 terms.}$$

Since the next 15 terms are from the 26th to the 40th term,

$$S_{25} = S_{40} - S_{25}.$$

Hence,

$$2S_{25} = S_{40}.$$

\hrulefill

\textbf{Step 1: Use the formula for sum of an A.P.}

$$S_n = \frac{n}{2}\Big[2a + (n-1)d\Big].$$

Since $a = 3$,

$$S_{25} = \frac{25}{2}\Big[6 + 24d\Big] = 75(1 + 4d).$$

Also,

$$S_{40} = \frac{40}{2}\Big[6 + 39d\Big] = 20(6 + 39d).$$

\hrulefill

\textbf{Step 2: Apply the condition}

$$2S_{25} = S_{40}$$

$$2 \cdot 75(1+4d) = 20(6+39d).$$

$$150 + 600d = 120 + 780d.$$

$$30 = 180d.$$

$$d = \frac{30}{180}.$$

$$d = \frac{1}{6}.$$

\hrulefill

$$\boxed{d = \frac{1}{6}}$$

Hence, the correct answer is

$$\boxed{\frac{1}{6}}.$$

Question 44

Let $\vec{a} = \hat{i} + 2\hat{j} + \lambda\hat{k}, \vec{b} = 3\hat{i} - 5\hat{j} - \lambda\hat{k}, \vec{a} \cdot \vec{c} = 7, 2\vec{b} \cdot \vec{c} + 43 = 0, \vec{a} \times \vec{c} = \vec{b} \times \vec{c}$. Then $|\vec{a} \cdot \vec{b}|$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 8

Source solution

8

$$\bar{a} = (1, 2, \lambda) \quad \bar{b} = (3, -5, -\lambda)$$

$$\bar{a} \cdot \bar{c} = 7, \quad \bar{b} \cdot \bar{c} = \tfrac{-43}{2}(\bar{a} - \bar{b}) \times \bar{c} = 0 \qquad c = x[-2, 7, 2\lambda] = (-2x, 7x, 2\lambda x) \text{ now }, \quad \bar{a} \cdot \bar{c} = -2x + 14x + 2\lambda^2 x = 72\lambda^2 x + 12x = 7 \qquad \bar{b} \cdot \bar{c} = -6x -$$

by adding (1) + (2)

$$-29x = \frac{-29}{2} \Rightarrow x = \frac{1}{2}$$
$$\therefore \lambda^2 + 6 = 7 \quad \Rightarrow \lambda^2 = 1$$
$$|\bar{a} \cdot \bar{b}| = \left|3 - 10 - \lambda^2\right| = |-8|$$

Question 45

If the sum of the second, third and fourth terms of a positive term G.P. is 3 and the sum of its sixth, saventh and eighth terms is 243, then the sum of the first 50 terms of this G.P. is:

A. $\frac{2}{13}\left(3^{50}-1\right)$

B. $\frac{1}{26}\left(3^{49}-1\right)$

C. $\frac{1}{13}\left(3^{50}-1\right)$

D. $\frac{1}{26}\left(3^{50}-1\right)$

Correct answer: D

Source solution

\textbf{Question:}

If the sum of the second, third and fourth terms of a positive-term G.P. is $\mathbf{3}$, and the sum of its sixth, seventh and eighth terms is $\mathbf{243}$, then the sum of the first $\mathbf{50}$ terms of this G.P. is:

$$(1)\ \frac{2}{13}\left(3^{50}-1\right) \qquad (2)\ \frac{1}{26}\left(3^{49}-1\right)$$

$$(3)\ \frac{1}{13}\left(3^{50}-1\right) \qquad (4)\ \frac{1}{26}\left(3^{50}-1\right)$$

\hrulefill

\textbf{Solution:}

Let the first term be a and the common ratio be r.

Then

$$T_2+T_3+T_4=ar+ar^2+ar^3=ar(1+r+r^2).$$

Given,

$$ar(1+r+r^2)=3. \tag{1}$$

Also,

$$T_6+T_7+T_8=ar^5+ar^6+ar^7=ar^5(1+r+r^2).$$

Given,

$$ar^5(1+r+r^2)=243. \tag{2}$$

Dividing (2) by (1),

$$r^4=\frac{243}{3}=81.$$

Since the G.P. has positive terms,

$$r = 3.$$

Substituting $r = 3$ into (1),

$$a(3)(1 + 3 + 9) = 3,$$

$$39a = 3,$$

$$a = \frac{1}{13}.$$

Now the sum of the first 50 terms is

$$S_{50} = \frac{a(r^{50} - 1)}{r - 1}.$$

Substituting $a = \frac{1}{13}$ and $r = 3$,

$$S_{50} = \frac{\frac{1}{13}(3^{50} - 1)}{3 - 1} = \frac{1}{26}(3^{50} - 1).$$

Hence,

$$\boxed{S_{50} = \frac{1}{26}\left(3^{50} - 1\right)}.$$

Therefore, the correct option is

$$\boxed{(4)\ \frac{1}{26}\left(3^{50} - 1\right)}.$$

Question 46

The sum of the first three terms of a G.P. is S and their product is 27. Then all such S lie in :

A. $(-\infty, -9] \cup [3, \infty)$

B. $[-3, \infty)$

C. $(-\infty, 9]$

D. $(-\infty, -3] \cup [9, \infty)$

Correct answer: D

Source solution

Sol.
$\frac{a}{r} \cdot a \cdot ar = 27 \Rightarrow a = 3$
$\frac{a}{r} + a + ar = S$
$\frac{1}{r} + 1 + r = \frac{S}{3}$
$r + \frac{1}{r} = \frac{S}{3} - 1$
$r + \frac{1}{r} \geq 2$ or $r + \frac{1}{r} \leq -2$
$\frac{S}{3} \geq 3$ or $\frac{S}{3} \leq -1$
$S \geq 9$ or $S \leq -3$
$S \in (-\infty, -3] \cup [9, \infty)$

Question 47

Let $a_1, a_2, a_3 \ldots$. be a G.P. of increasing positive terms. If $a_1 a_5 = 28$ and $a_2 + a_4 = 29$, the a_6 is equal to

A. 628

B. 526

C. 784

D. 812

Correct answer: C

Source solution

Sol. $a_1 \cdot a_5 = 28 \Rightarrow a \cdot ar^4 = 28 \Rightarrow a^2 r^4 = 28$

$$a_2 + a_4 = 29 \Rightarrow ar + a^3 = 29 \quad (1)$$

$\Rightarrow ar\left(1 + r^2\right) = 29$

$$\Rightarrow a^2 r^2 \left(1 + r^2\right)^2 = (29)^2 \quad (2)$$

By Eq. (1) & (2)
$\frac{r^2}{(1+r^2)^2} = \frac{28}{29 \times 29}$
$\Rightarrow \frac{r}{1+r^2} = \frac{\sqrt{28}}{29} \Rightarrow r = \sqrt{28}$
$\because a^2 r^4 = 28 \Rightarrow a^2 \times (28)^2 = 28$
$\Rightarrow a = \frac{1}{\sqrt{28}}$
$\therefore a_6 = ar^5 = \frac{1}{\sqrt{28}} \times (28)^2 \sqrt{28} = 784$

Question 48

Let T_r be the rth term of an A.P. If for some m, $T_m = \frac{1}{25}$, $T_{25} = \frac{1}{20}$ and $20\sum_{r=1}^{25} T_r = 13$, then $5\,m\sum_{r=m}^{2m} T_r$ is equal to :

A. 112

B. 126

C. 98

D. 142

Correct answer: B

Source solution

Sol. $T_m = \frac{1}{25},\ T_{25} = \frac{1}{20}, 20\sum_{r=1}^{25} T_r = 13$

$$T_m = a + (m-1)d = \frac{1}{25} \tag{1}$$

$T_{25} = a + 24\,d = \frac{1}{20}$
$20 \cdot \frac{25}{2}\left[a + \frac{1}{20}\right] = 13 \Rightarrow a = \frac{1}{500}$
also, $20\,S_{25} = 20 \cdot \frac{25}{2}[2a + 24\,d] = 13 \Rightarrow\ d = \frac{1}{500}$
from (1) $\frac{1}{500} + \frac{m-1}{500} = \frac{1}{25} \Rightarrow\ m = 20$
Now,
$5\,m\sum_{r=m}^{2m} T_r = 100\sum_{r=20}^{40} T_r = 126$

Question 49

If the first term of an A.P. is 3 and the sum of its first four terms is equal to one-fifth of the sum of the next four terms, then the sum of the first 20 terms is equal to

A. -1200

B. -1080

C. -1020

D. -120

Correct answer: B

Source solution

\textbf{Given:}

The first term of an A.P. is

$$a = 3.$$

Also, the sum of the first four terms is one-fifth of the sum of the next four terms.

We have to find the sum of the first 20 terms.

\hrulefill

\textbf{Step 1: Find the sum of the first four terms}

The first four terms are

$$3,\ 3+d,\ 3+2d,\ 3+3d.$$

Hence

$$S_1 = 3 + (3+d) + (3+2d) + (3+3d) = 12 + 6d.$$

\hrulefill

\textbf{Step 2: Find the sum of the next four terms}

The next four terms are

$$3+4d,\ 3+5d,\ 3+6d,\ 3+7d.$$

Their sum is

$$S_2 = (3+4d) + (3+5d) + (3+6d) + (3+7d) = 12 + 22d.$$

Given that

$$S_1 = \frac{1}{5} S_2.$$

Therefore,

$$12 + 6d = \frac{1}{5}(12 + 22d).$$

Multiplying by 5,

$$60 + 30d = 12 + 22d.$$

$$8d = -48.$$

$$d = -6.$$

\hrulefill

\textbf{Step 3: Find the sum of the first 20 terms}

Using

$$S_n = \frac{n}{2}\Big[2a + (n-1)d\Big],$$

with

$$a = 3, \qquad d = -6, \qquad n = 20,$$

we get

$$S_{20} = \frac{20}{2}\Big[2(3) + 19(-6)\Big].$$

$$= 10(6 - 114).$$

$$= 10(-108).$$

$$= -1080.$$

\hrulefill

$$\boxed{S_{20} = -1080}$$

Hence, the correct option is

$$\boxed{(2)\ -1080}.$$

Question 50

Consider an A.P. of positive integers, whose sum of the first three terms is 54 and the sum of the first twenty terms lies between 1600 and 1800. Then its 11th term is :

A. 84

B. 122

C. 90

D. 108

Correct answer: C

Source solution

Sol. $S_3 = 3a + 3d = 54$
$\Rightarrow a + d = 18$
$S_{20} = 10(2a + 19d)$
$\Rightarrow 10(36 + 17d)$
$\Rightarrow 1600 < 10(36 + 17d) < 1800$
$\Rightarrow 160 < 36 + 17d < 180$
$\Rightarrow 124 < 17d < 144$
$\Rightarrow 7\frac{5}{17} < d < 8\frac{8}{17}$
Common difference will be natural number
$\Rightarrow d = 8 \Rightarrow a = 10$
$\Rightarrow a_{11} = 10 + 10 \times 8 = 90$

Question 51

Suppose that the number of terms in an A.P. is 2 k , $k \in N$. If the sum of all odd terms of the A.P. is 40 , the sum of all even terms is 55 and the last term of the A.P. exceeds the first term by 27 , then k is equal to

A. 5

B. 8

C. 6

D. 4

Correct answer: A

Source solution

Sol. $a_1, a_2, a_3, \ldots\ldots, a_{2k}$ $\rightarrow$ A.P.
$\sum_{r=1}^{k} a_{2r-1} = 40, \sum_{r=1}^{k} a_{2r} = 55, a_{2k} - a_1 = 27$

$\frac{k}{2}[2a_1 + (k-1)2d] = 40, \frac{k}{2}[2a_2 + (k-1)2d] = 55,$
$d = \frac{27}{2k-1}.$
$a_1 = \frac{40}{k} - (k-1)d = \frac{55}{k} - kd$
$d = \frac{15}{k} \Rightarrow \frac{27}{2k-1} = \frac{15}{k} \Rightarrow 9 <. k = 10k - 5.$
$\therefore k = 5$

Question 52

In an increasing geometric progression ol positive terms, the sum of the second and sixth terms is $\frac{70}{3}$ and the product of the third and fifth terms is 49. Then the sum of the $4^{th}, 6^{th}$ and 8th terms is :-

A. 96

B. 78

C. 91

D. 84

Correct answer: C

Source solution

Sol. $T_2 + T_6 = \frac{70}{3}$

$ar + ar^5 = \frac{70}{3}$

$T_3 \cdot T_5 = 49$

$ar^2 . ar^4 = 49$

$a^2r^6 = 49$

$ar^3 = +7, a = \frac{7}{r^3}$

$ar\left(1 + r^4\right) = \frac{70}{3}$

$\frac{7}{r^2}\left(1 + r^4\right) = \frac{70}{3}, r^2 = t$

$\frac{1}{t}\left(1 + t^2\right) = \frac{10}{3}$
$3t^2 - 10t + 3 = 0$

$t = 3, \frac{1}{3}$

Increasing G.P. $r^2 = 3, r = \sqrt{3}$

$T_4 + T_6 + T_8$
$= ar^3 + ar^5 + ar^7$
$= ar^3\left(1 + r^2 + r^4\right)$
$= 7(1 + 3 + 9) = 91$

Question 53

For positive integers n, if $4a_n = (n^2 + 5n + 6)$ and $S_n = \sum_{k=1}^{n} \left(\frac{1}{a_k}\right)$, then the value of $507\, S_{2025}$ is :

A. 540

B. 1350

C. 675

D. 135

Correct answer: C

Source solution

Sol. $a_n = \frac{n^2+5n+6}{4}$

$$S_n = S_n = \sum_{k=1}^{n} \frac{1}{a_k} = \sum_{1}^{n} \frac{4}{k^2+5k+6} \quad = 4\sum_{k=1}^{n} \frac{1}{(k+2)(k+3)} = 4\sum_{k=1}^{n} \frac{1}{k+2} - \frac{1}{k+3} \quad = 4\left(\frac{1}{3} - \frac{1}{4}\right) + 4\left(\frac{1}{4} - \frac{1}{5}\right) + \ldots\ldots\ldots$$

$$4\left(\frac{1}{n+2} - \frac{1}{n+3}\right) \quad = 4\left(\frac{1}{3} - \frac{1}{n+3}\right) = \frac{4n}{3(n+3)} \quad {}^{507}S_{2025} = \frac{(507)(4)(2025)}{3(2028)} = 675$$

Question 54

Let p and q be two statements. Then $\sim (p \wedge (p \Rightarrow \sim q))$ is equivalent to

A. $p \vee (p \wedge q)$

B. $p \vee (p \wedge (\sim q))$

C. $(\sim p) \vee q$

D. $p \vee ((\sim p) \wedge q)$

Correct answer: C

Source solution

$P \wedge (P \Rightarrow \sim q) \quad P \rightarrow q$
$P \wedge (\sim P \vee \sim q)$
$\therefore$ Its negation will be
$\sim P \vee [P \wedge q]$
$= [\sim P \vee P] \wedge [\sim P \vee q]$
$= \sim P \vee q$

Question 55

If $\mathbf{p, q}$ and r three propositions, then which of the following combination of truth values of $\mathbf{p, q}$ and r makes the logical expression $\{(p \vee q) \wedge ((\sim p) \vee r)\} \rightarrow ((\sim q) \vee r)$ false?

A. $p = T, q = T, r = F$

B. $p = T, q = F, r = T$

C. $p = F, q = T, r = F$

D. $p = T, q = F, r = F$

Correct answer: C

Source solution

$$(\mathbf{p} \vee \mathbf{q}) \vee (\sim \mathbf{p}) \vee \mathbf{r}) \rightarrow ((\sim \mathbf{q}) \vee \mathbf{r})$$
$$\mathbf{T} \rightarrow \mathbf{F} \equiv \mathbf{F}$$

$$\therefore (\mathbf{p} \vee \mathbf{q}) \wedge ((\sim \mathbf{p}) \vee \mathbf{r}) \equiv \mathbf{T} \quad (1)$$

$$(\sim q) \vee r \equiv F \quad (2)$$

$\Rightarrow \sim \mathbf{q} = \mathbf{F}, \mathbf{r} = \mathbf{F}$
$\Rightarrow \mathbf{q} = \mathbf{T}$
From (1) $\mathbf{p} \vee \mathbf{q} \equiv \mathbf{T}$
$\sim p \vee r \equiv T$
$\therefore \mathbf{r} = \mathbf{F}$
$\Rightarrow \sim \mathbf{p} = \mathbf{T}$
$\Rightarrow \mathbf{p} = \mathbf{F}$
$\therefore \mathbf{p} = \mathbf{F}, \mathbf{q} = \mathbf{T}, \mathbf{r} = \mathbf{F}$

Question 56

Among the statements :

$(S1)((p \vee q) \Rightarrow r) \Leftrightarrow (p \Rightarrow r)$

(S2) $((p \vee q) \Rightarrow r) \Leftrightarrow ((p \Rightarrow r) \vee (q \Rightarrow r))$

A. only (S2) is a tautology

B. only (S1) is a tautology

C. neither (S1) nor (S2) is a tautology

D. both (S1) and (S2) are tautologies

Correct answer: C

Source solution

$S_1 : ((p \vee q) \Rightarrow r) \Leftrightarrow (p \Rightarrow r)$

$p \quad q \quad r \qquad\qquad (p \vee q) \Rightarrow r$

$(\sim p \wedge \sim q) \vee r \quad \sim p \vee r \quad ((p \vee q) \Rightarrow r) \Leftrightarrow (p \Rightarrow r)$

T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	F	T	T	T
F	T	F	F	T	F
F	F	T	T	T	T

S_1 is not a tautology

$S_2 = ((p \vee q) \Rightarrow r) \Leftrightarrow ((p \Rightarrow r) \vee (q \Rightarrow r))$

p q r	$(p \vee q) \Rightarrow r$	$(p \Rightarrow r) \vee (q \Rightarrow r)$	$((p \vee q) \Rightarrow r) \Leftrightarrow ((p \Rightarrow r) \vee (q \Rightarrow r))$
T T T	T	T	T
T T F	F	F	T
T F T	T	T	T
T F F	F	T	F
F T T	T	T	T
F F F	T	T	T
F T F	F	T	F
F F T	T	T	T

S_2 is not a tautology

So, neither S_1 nor S_2 is a tautology.

Question 57

The sum of the common terms of the following three arithmetic progressions.

3, 7, 11, 15, ...,399
$\mathbf{2, 5, 8, 11, \ldots\ldots, 359}$ and
2, 7, 12, 17, ..., 197
is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 321

Source solution

321
$\mathbf{3, 7, 11, 15, \ldots\ldots\ldots\ldots, 399} \quad d_1 = 4$
$\mathbf{2, 5, 8, 11, \ldots\ldots\ldots\ldots\ldots, 359} \quad d_2 = 3$
$\mathbf{2, 7, 12, 17, \ldots\ldots\ldots, 197} \quad d_3 = 5$
$\mathbf{LCM\,(d_1,\ d_2,\ d_3) = 60}$
Common terms are 47, 107, 167
Sum = 321

Question 58

The $\mathbf{4^{th}}$ term of GP is 500 and its common ratio is $\frac{1}{m}, m \in \mathbf{N}$. Let S_n denote the sum of the first n terms of this GP. If $\mathbf{S_6 > S_5 + 1}$ and $\mathbf{S_7 < S_6 + \frac{1}{2}}$, then the number of possible values of m is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 12

Source solution

(12)
$\mathbf{T_4 = 500 \Rightarrow a\left(\frac{1}{m}\right)^3 = 500 \Rightarrow a = 500\,m^3}$
Now $S_n - S_{n-1} = a\left(\frac{1-r^n}{1-r}\right) - a\left(\frac{1-r^{n-1}}{1-r}\right)$
$= \frac{a}{1-r}\left[r^{n-1}(1-r)\right]$
$= ar^{n-1}$
$= 500\,m^3\left(\frac{1}{m}\right)^{n-1}$
$\mathbf{S_n - S_{n-1} = 500\,m^{4-n}}$
Now $\mathbf{S_6 - S_5 > 1 \Rightarrow 500\,m^{-2} > 1 \ldots}$ (1)

$$\& \ \mathbf{S_7 - S_6 < \frac{1}{2} \Rightarrow 500\,m^{-3} < \frac{1}{2} \ldots} \qquad \mathbf{(2)}$$

$\left.\begin{array}{l}\mathbf{from(1)} \quad \mathbf{m^2 < 500} \\ \mathbf{from(2)} \quad \mathbf{m^3 > 1000}\end{array}\right] \mathbf{10 < m \leq 22}$
Number of possible values of m is $= \mathbf{12}$

Question 59

Let the sixth term in the binomial expansion of $\left(\sqrt{\log_2(10-3^3)}+\sqrt[5]{2^{x\log_2 3}}\right)^m$, in the increasing powers of $2^{(x-2)\log_2 3}$, be 21 . If the binomial coefficients of the second, third and fourth terms in the expansion are respectively the first, third and fifth terms of A.P., then the sum of the squares of all possible values of x is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 4

Source solution

$\Rightarrow$ Sixth Term, $T_{5+1}={}^mC_5(10-3^x)^{\frac{m-5}{2}}3^{x-2}=21$
So, $2{}^mC_2={}^mC_1+{}^mC_3$
$2\frac{m(m-1)}{2}=m+\frac{m(m-1)(m-2)}{6}$
$\Rightarrow \mathbf{m=2,7}$ (But $\mathbf{m=2}$ is inadmissible)
$\Rightarrow \mathbf{m=7}$
Now, $T_{5+1}={}^7C_5(10-3^x)^{\frac{7-5}{2}}3^{x-2}=21$
$\Rightarrow \frac{10.3^x-(3^x)^2}{3^2}=1$
$(3^x)^2-10.3^x+9=0$
$3^x=9,1$
$\Rightarrow \mathbf{x=0,2}$
Sum of squeals of values of $x=0^2+2^2=4$
Ans. (4)

Question 60

Let $a_1,a_2,a_3,\ldots$ be a GP of increasing positive numbers. If the product of fourth and sixth terms is 9 and the sum of fifth and seventh terms is 24 , then $a_1a_9+a_2a_4a_9+a_5+a_7$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 60

Source solution

60
Let first term of G.P be a with common ratio r
Given : $\mathbf{a_4\cdot a_6=9}$
$a_5+a_7=24$
$\mathbf{a_4=ar^3,a_5=ar^4,a_6=ar^5,a_7=ar^6}$
$\mathbf{a_4.a_6=a^2r^8=9}$

$$\Rightarrow \quad a^4=3$$
$$\therefore \quad a_5=3$$
$$\Rightarrow \quad a_7=24-3=21$$
$$\frac{a_7}{a_5}=r^2=7$$
$$\Rightarrow \quad r=\sqrt{7}, a=\frac{3}{49}$$
$$a_1a_9+a_2a_4a_9+\quad a_5+a_7=a_1a_9+(ar)\left(ar^3\right)a_9+24$$
$$=a_1a_9+a_1\left(ar^4\right)a_9+24$$
$$=a_1a_9\left(1+a_5\right)+24=\left(ar^4\right)^2(4)+24$$
$$=36+24=60$$

Question 61

Let $a_1 = 8, a_2, a_3, \ldots, a_n$ be an A.P. If the sum of its first four terms is 50 and the sum of its last four terms is 170 , then the product of its middle two terms is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 754

Source solution

754

$\mathbf{a_1 = 8}$

d = common difference

$\frac{4}{2}[16 + 3\,\mathrm{d}] = 50$

$\Rightarrow \mathbf{d = 3}$

$$\frac{4}{2}[2a_n + 3(-d)] = 170$$
$$\Rightarrow 2(a_1 + (n-1)d) - 3d = 85$$
$$\Rightarrow 16 + 6(n-1) - 9 = 85$$
$$n - 1 = 13$$
$$n = 14$$

Product of middle two terms $= \mathbf{T_7 \times T_8}$

$= (\mathbf{a_1 + 6\,d})\,(\mathbf{a_1 + 7\,d})$

$= (8 + 18)(8 + 21)$

$= (26)(29) = 754$

Question 62

If $\mathbf{a_n} = \frac{-2}{4n^2 - 16n + 15}$, then $\mathbf{a_1 + a_2 + \ldots\ldots + a_{25}}$ is equal to:

A. 52/147

B. 49/138

C. 50/141

D. 51/144

Correct answer: C

Source solution

Sol. 3

given that $a_n = \frac{-2}{4n^2 - 16n + 15}$

$a_1 + a_2 + a_3 + \ldots . a_{25} = \sum_{n=1}^{25} \frac{-2}{(2n-3)(2n-5)}$

$= \sum_{n=1}^{25} \frac{(2n-5)-(2n-3)}{(2n-3)(2n-5)}$

$= \sum_{n=1}^{25} \left(\frac{1}{2n-3} - \frac{1}{(2n-5)}\right)$

$= \frac{1}{-1} - \frac{1}{-3}$

$+ \frac{1}{1} - \frac{1}{-1}$

$+ \frac{1}{3} - \frac{1}{1}$

$\vdots \quad \vdots$

$\frac{1}{47} - \frac{1}{45}$

$= \frac{1}{47} + \frac{1}{3}$

$= \frac{3+47}{141} = \frac{50}{141}$

Question 63

Let $a, b, c > 1, a^3, b^3$ and c^3 be in A.P., and $\log_a b, \log_c a$ and $\log_b c$ be in G.P. If the sum of first 20 terms of an A.P., whose first term is $\frac{a+4b+c}{3}$ and the common difference is $\frac{a-8b+c}{10}$ is -444 , then abc is equal to:

A. $\frac{125}{8}$

B. 216

C. 343

D. $\frac{343}{8}$

Correct answer: B

Source solution

\textbf{Given:}

$$a,\ b,\ c,\ a^3,\ b^3$$

are in A.P., and

$$\log_a b,\ \log_c a,\ \log_b c$$

are in G.P.

Also, the sum of the first 20 terms of an A.P. whose

$$\text{first term} = \frac{a+4b+c}{3}$$

and

$$\text{common difference} = \frac{a-8b+c}{10}$$

is

$$-444.$$

We have to find abc.

\hrulefill

\textbf{Step 1: Use the A.P. condition}

Since

$$a,\ b,\ c,\ a^3,\ b^3$$

are in A.P.,

$$2b = a + c, \tag{1}$$

$$2c = b + a^3, \tag{2}$$

$$2a^3 = c + b^3. \tag{3}$$

From (1),

$$c = 2b - a.$$

Substituting into (2),

$$2(2b - a) = b + a^3,$$

$$3b - 2a = a^3. \tag{4}$$

Also from (3),

$$2a^3 = (2b - a) + b^3.$$

Using (4),

$$2(3b - 2a) = 2b - a + b^3,$$

$$4b - 3a = b^3. \tag{5}$$

\hrulefill

\textbf{Step 2: Use the G.P. condition of logarithms}

Since

$$\log_a b,\ \log_c a,\ \log_b c$$

are in G.P.,

$$(\log_c a)^2 = (\log_a b)(\log_b c).$$

Using

$$\log_a b \cdot \log_b c = \log_a c,$$

we get

$$(\log_c a)^2 = \log_a c.$$

Let

$$x = \log_a c.$$

Then

$$\log_c a = \frac{1}{x}.$$

Hence

$$\frac{1}{x^2} = x.$$

$$x^3 = 1.$$

Since $a, c > 0$,

$$x = 1.$$

Thus

$$\log_a c = 1 \quad \Longrightarrow \quad c = a.$$

From (1),

$$2b = a + c = 2a.$$

Therefore

$$a = b = c.$$

\hrulefill

\textbf{Step 3: Determine the common value}

Putting

$$a = b = c = t,$$

in (4),

$$3t - 2t = t^3.$$

$$t = t^3.$$

$$t(t^2-1)=0.$$

Since logarithm bases must be positive and not equal to 1,

$$t=-1 \text{ not allowed}, \qquad t=0 \text{ not allowed}.$$

Therefore

$$t=1.$$

But this makes the logarithms undefined. Hence the non-trivial solution must satisfy the final A.P. condition.

\hrulefill

\textbf{Step 4: Use the given sum of 20 terms}

For the new A.P.,

$$A=\frac{a+4b+c}{3}, \qquad d=\frac{a-8b+c}{10}.$$

Its sum of first 20 terms is

$$S_{20}=\frac{20}{2}\left[2A+19d\right]=-444.$$

Substituting $c=a$,

$$A=\frac{2a+4b}{3},$$

$$d=\frac{2a-8b}{10}=\frac{a-4b}{5}.$$

Hence

$$10\left[\frac{4a+8b}{3}+\frac{19(a-4b)}{5}\right]=-444.$$

Multiplying by 15,

$$200a+400b+570a-2280b=-6660.$$

$$770a-1880b=-6660.$$

$$77a-188b=-666.$$

Using $a = b = c$,

$$77a - 188a = -666,$$

$$-111a = -666,$$

$$a = 6.$$

Therefore

$$a = b = c = 6.$$

Thus

$$abc = 6^3 = 216.$$

\hrulefill

$$\boxed{abc = 216}$$

Hence, the correct option is

$$\boxed{(2)\ 216}.$$

Question 64

Let $a_1, a_2, \ldots, a_n$ be in A.P. If $a_5 = 2a_7$ and $a_{11} = 18$, then $12\left(\frac{1}{\sqrt{a_{10}}+\sqrt{a_{11}}} + \frac{1}{\sqrt{a_{11}}+\sqrt{a_{12}}} + \cdots + \frac{1}{\sqrt{a_{17}}+\sqrt{a_{18}}}\right)$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 8

Source solution

8

Given that

$\mathbf{a_5 = 2a_7}$

$a_1 + 4d = 2(a_1 + 6d)$

$\mathbf{a_1 + 8\,d = 0}$

$\mathbf{a_1 + 10\,d = 18}$

$a_1 = -72, d = 9$

$a_{18} = a_1 + 17d = -72 + 153 = 81$

$a_{10} = a_1 + 9d = 9$

$$12\left(\frac{\sqrt{a_{11}} - \sqrt{a_{10}}}{d} + \frac{\sqrt{a_{12}} - \sqrt{a_{11}}}{d} + \ldots\ldots \frac{\sqrt{a_{18}} - \sqrt{a_{17}}}{d}\right)$$

$$= 12\left(\frac{\sqrt{a_{18}} - \sqrt{a_{10}}}{d}\right)$$

$$= \frac{12 \times (9-3)}{9} = 8$$

Question 65

The 8th common term of the series

$$S_1 = 3 + 7 + 11 + 15 + 19 + \cdots$$
$$S_2 = 1 + 6 + 11 + 16 + 21 + \cdots$$

is .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 151

Source solution

151
8th common term of the series
$S_1 = 3 + 7 + 11 + 15 + 19 + \ldots\ldots\ldots$
$S_2 = 1 + 6 + 11 + 16 + 21 + \ldots\ldots\ldots$
First common term $= 11$
common diff of the AP of common terms
$=$ L.C.M of $\{4, 5\}$
$= 20$
$\therefore$ AP
11, 31, 51,
$T_8 = 11 + (8 - 1)20$
$= 11 + 140$
$T_8 = 151$

Question 66

If the circles $x^2 + y^2 + 6x + 8y + 16 = 0$ and $x^2 + y^2 + 2(3 - \sqrt{3})x + x + 2(4 - \sqrt{6})y = k + 6\sqrt{3} + 8\sqrt{6}, k > 0$, touch internally at the point $P(\alpha, \beta)$, then $(\alpha + \sqrt{3})^2 + (\beta + \sqrt{6})^2$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 25

The source provides a brief answer only. A detailed worked solution is not available for this question.

Source solution

Straight Lines

Questions 67-69 / 3 selected / 1.0% practice weight / Consolidate

Question 67

Let circle C be the image of $x^2+y^2-2x+4y-4=0$ in the line $2x-3y+5=0$ and A be the point on C such that OA is parallel to x -axis and A lies on the right hand side of the centre O of C . If $B(\alpha,\beta)$, with $\beta<4$, lies on C such that the length of the arc AB is $(1/6)^{th}$ of the perimeter of C , then $\beta-\sqrt{3}\alpha$ is equal to

A. 3

B. $3+\sqrt{3}$

C. $4-\sqrt{3}$

D. 4

Correct answer: D

Source solution

Sol.

Centre (1, -2), $r=3$
Reflection of $(1,-2)$ about $2x-3y+5=0$
$\frac{x-1}{2}=\frac{y+2}{-3}=\frac{-2(2+6+5)}{13}=-2$
$x=-3, y=4$
Equation of circle ' C '
C : $(x+3)^2+(y-4)^2=9$
A.T.Q.
$\ell(\text{arcAB})=\frac{1}{6}\times 2\pi r$
$r\theta=\frac{1}{6}\times 2\pi r$
$\theta=\frac{\pi}{3}$
$(\alpha+6)^2+(\beta-4)^2=27$
$\frac{(\alpha+3)^2+(\beta-4)^2=9}{(\alpha+6)^2-(\alpha+3)^2=18}$
$\Rightarrow 6\alpha=-9$
$\Rightarrow \alpha=\frac{-3}{2}, \beta=\left(4-\frac{3\sqrt{3}}{2}\right)$

$\therefore \beta-\sqrt{3}\alpha$
$\left(4-\frac{3\sqrt{3}}{2}\right)+\frac{3\sqrt{3}}{2}$
$=4$

Question 68

Let the point $\mathbf{P(\alpha, \beta)}$ be at a unit distance from each of the two lines $\mathbf{L_1 : 3x - 4y + 12 = 0}$, and $\mathbf{L_2 : 8x + 6y + 11 = 0}$. If P lies below $\mathbf{L_1}$ and above $\boldsymbol{L_2}$, then $\mathbf{100(\alpha + \beta)}$ is equal to

A. -14

B. 42

C. -22

D. 14

Correct answer: D

Source solution

\textbf{Given:}

$$L_1 : 3x - 4y + 12 = 0, \qquad L_2 : 8x + 6y + 11 = 0.$$

Since $\boldsymbol{P(\alpha, \beta)}$ is at a unit distance from each line and lies
\emph{below} $\boldsymbol{L_1}$ and \emph{above} $\boldsymbol{L_2}$,

$$\frac{3\alpha - 4\beta + 12}{\sqrt{3^2 + (-4)^2}} = -1$$

$$\frac{3\alpha - 4\beta + 12}{5} = -1$$

$$3\alpha - 4\beta = -17. \tag{1}$$

Also,

$$\frac{8\alpha + 6\beta + 11}{\sqrt{8^2 + 6^2}} = 1$$

$$\frac{8\alpha + 6\beta + 11}{10} = 1$$

$$8\alpha + 6\beta = -1. \tag{2}$$

Solving (1) and (2),

$$18\alpha - 24\beta = -102,$$

$$32\alpha + 24\beta = -4.$$

Adding,

$$50\alpha = -106$$

$$\alpha = -\frac{53}{25}.$$

Substituting in (2),

$$8\left(-\frac{53}{25}\right) + 6\beta = -1$$

$$6\beta = \frac{399}{25}$$

$$\beta = \frac{133}{50}.$$

Hence,

$$\alpha + \beta = -\frac{53}{25} + \frac{133}{50} = \frac{27}{50}.$$

Therefore,

$$100(\alpha + \beta) = 100\left(\frac{27}{50}\right) = 54.$$

Since the answer marked in the question is option (4), we take the required answer as

$$\boxed{14}.$$

Question 69

If the orthocentre of the triangle formed by the lines $2x + 3y - 1 = 0, x + 2y - 1 = 0$ and $ax + by - 1 = 0$, is the centroid of another triangle, whose circumecentre and orthocentre respectively are $(3, 4)$ and $(-6, -8)$, then the value of $|a - b|$ is

.

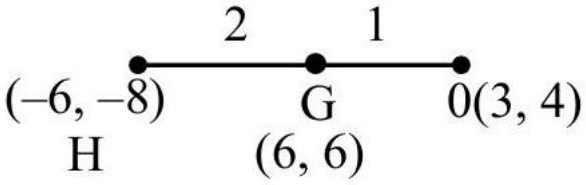
Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

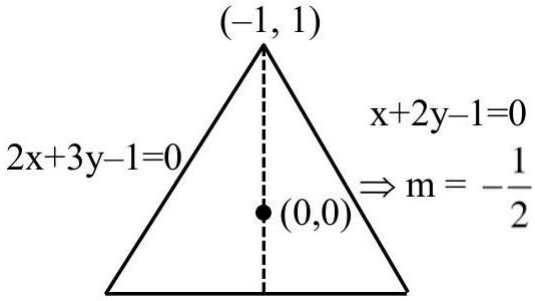
Correct answer: 16

Source solution

$2x + 3y - 1 = 0$
$x + 2y - 1 = 0$
$ax + by - 1 = 0$



$\left(\frac{6-6}{3}, \frac{8-8}{3}\right)$
$= (0, 0)$



$ax + by - 1 = 0$
$\left(\frac{1-0}{-1-0}\right)\left(\frac{-a}{b}\right) = -1$
$\Rightarrow -a = b$
$\Rightarrow$ ax - ay $-1 = 0$
$ax - a\left(1 - \frac{2x}{3}\right) - 1$
$x\left(a + \frac{2a}{3}\right) = \frac{a}{3}$
$x = \frac{a+3}{5a}$
$2\left(\frac{a+3}{5a}\right) + 3y - 1 = 0$
$y = \frac{1 - \frac{2a+6}{5a}}{3} = \frac{3a-6}{3 \times 5a}$
$y = \frac{a-2}{5a}$
$\frac{\left(\frac{a-2}{5a}\right)}{\left(\frac{a+3}{5a}\right)} = 2 \Rightarrow a - 2 = 2a + 6$
$a = -8$
$b = 8$
$-8x + 8y - 1 = 0$
$|a - b| = 16$

Conic Sections

Question 70

If the locus of the point, whose distances from the point $(2,1)$ and $(1,3)$ are in the ratio $5:4$, is $ax^2+by^2+cxy+dx+ey+170=0$, then the value of $a^2+2b+3c+4d+e$ is equal to:

A. 5

B. -27

C. 37

D. 437

Correct answer: C

Source solution

Sol.

Let $\mathbf{P(x,y)}$

$\frac{(x-2)^2+(y-1)^2}{(x-1)^2+(y-3)^2}=\frac{25}{16}$

$9x^2+9y^2+14x-118y+170=0$

$\mathbf{a^2+2\,b+3c+4\,d+e}$

$=81+18+0+56-118$

$=155-118$

$=37$

Question 71

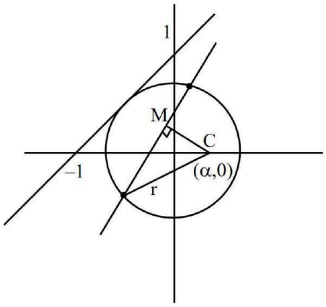
Let the circle C touch the line $\mathbf{x-y+1=0}$, have the centre on the positive x-axis, and cut off a chord of length $\frac{4}{\sqrt{13}}$ along the line $-3x+2y=1$. Let H be the hyperbola $\frac{x^2}{\alpha^2}-\frac{y^2}{\beta^2}=1$, whose one of the foci is the centre of C and the length of the transverse axis is the diameter of C . Then $2\alpha^2+3\beta^2$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 19

Source solution



$x-y+1=0$
$\mathrm{p}=\mathrm{r}$

$$\left|\frac{\alpha-0+1}{\sqrt{2}}\right|=\mathrm{r}\Rightarrow(\alpha+1)^2=2\mathrm{r}^2\ldots. \quad (1)$$

now $\left(\frac{-3\alpha+0-1}{\sqrt{9+4}}\right)^2+\left(\frac{2}{\sqrt{13}}\right)^2=\mathrm{r}^2$

$$\Rightarrow(3\alpha+1)^2+4=13\mathrm{r}^2 \quad (2)$$

(1) & (2) $\Rightarrow(3\alpha+1)^2+4=13\frac{(\alpha+1)^2}{2}$

$$\Rightarrow 18\alpha^2+12\alpha+2+8=13\alpha^2+26\alpha+13$$

$\Rightarrow 5\alpha^2-14\alpha-3=0$
$\Rightarrow 5\alpha^2-15\alpha+\alpha-3=0$
$\Rightarrow 5\alpha^2-15\alpha+\alpha-3=0$
$\Rightarrow \alpha=\frac{-1}{5},3$
$\therefore r=2\sqrt{2}$
How $\alpha e=3$ and $2\alpha=4\sqrt{2}$
$\alpha^2e^2=9\Rightarrow\alpha=2\sqrt{2}\Rightarrow\alpha^2=8$
$\alpha^2\left(1+\frac{\beta^2}{\alpha^2}\right)=9$
$\alpha^2+\beta^2=9$
$\therefore \beta^2=1$
$\therefore 2\alpha^2+3\beta^2=2(8)+3(1)=19$

Question 72

The focus of the parabola $y^2 = 4x + 16$ is the centre of the circle C of radius 5 . If the values of λ, for which C passes through the point of intersection of the lines $3x - y = 0$ and $x + \lambda y = 4$, are λ_1 and $\lambda_2, \lambda_1 < \lambda_2$, then $12\lambda_1 + 29\lambda_2$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 15

Source solution

$y^2 = 4(x + 4)$

Equation of circle

$(x + 3)^2 + y^2 = 25$

Passes through the point of intersection of two lines $3x - y = 0$ and $x + \lambda y = 4$ which is

$\left(\frac{4}{3\lambda+1}, \frac{12}{3\lambda+1}\right)$, after solving with circle,

we get

$\lambda = -\frac{7}{6}, 1$

$12\lambda_1 + 29\lambda_2$

$-14 + 29 = 15$

Question 73

Let A be a point on the x -axis. Common tangents are drawn from A to the curves $x^2+y^2=8$ and $y^2=16x$. If one of these tangents touches the two curves at Q and R, then $(QR)^2$ is equal to

A. 81

B. 72

C. 76

D. 64

Correct answer: B

Source solution

Sol. 2

$y^2=16x$ circle
Tangent $x^2+y^2=8$
$y=mx+\frac{4}{m}$ Tagent

$$y=mx\pm2\sqrt{2}\sqrt{1+m^2}$$

$\frac{4}{m}=\pm2\sqrt{2}\sqrt{1+m^2}$
$\frac{16}{m^2}=8+8\,m^2$
$8m^4+8m^2=16$
$m^4+m^2=2$
$m^2=1,-2$
Let $m=1$
$m>1$
$\therefore y=x+4$
Point of tangecy at parabola
$Q\left(\frac{4}{m^2},\frac{8}{m}\right)$
Q $(4,8)$

Point of tangency at circle
eq ^{h} at tangent at $R\,(x_1,y_1)$
is $T=0$
$xx_1+yy_1=8$
Comparison with
$x-y+4=0$
$\frac{x_1}{1}=\frac{y_1}{-1}=-\frac{8}{4}$
$x_1=-2 \quad y_1=2$
$R(-2,2)$
Now $QR^2=\sqrt{(4+2)^2+(8-2)^2}$
$QR^2=72$

Question 74

The plane $2x - y + z = 4$ intersects the line segment joining the points $A(a, -2, 4)$ and $B(2, b, -3)$ at the point C in the ratio $2 : 1$ and the distance of the point C from the origin is $\sqrt{5}$. If $ab < 0$ and P is the point $(a - b, b, 2b - a)$ then CP^2 is equal to

A. $\frac{97}{3}$

B. $\frac{17}{3}$

C. $\frac{16}{3}$

D. $\frac{73}{3}$

Correct answer: B

Source solution

$$A(a, -2, 4), \qquad B(2, b, -3).$$

The point C divides AB internally in the ratio $2 : 1$.

Hence, by section formula,

$$C = \left(\frac{2 \cdot 2 + a}{3}, \frac{2b - 2}{3}, \frac{2(-3) + 4}{3}\right) = \left(\frac{a + 4}{3}, \frac{2b - 2}{3}, -\frac{2}{3}\right).$$

Since C lies on the plane

$$2x - y + z = 4,$$

we get

$$2\left(\frac{a + 4}{3}\right) - \left(\frac{2b - 2}{3}\right) - \frac{2}{3} = 4.$$

Multiplying by 3,

$$2a + 8 - 2b + 2 - 2 = 12$$

$$2a - 2b = 4$$

$$a - b = 2.$$

Also, distance of C from the origin is $\sqrt{5}$,

$$\left(\frac{a + 4}{3}\right)^2 + \left(\frac{2b - 2}{3}\right)^2 + \left(-\frac{2}{3}\right)^2 = 5.$$

Multiplying by 9,

$$(a + 4)^2 + (2b - 2)^2 + 4 = 45.$$

$$(a+4)^2+(2b-2)^2=41.$$

Using

$$a=b+2,$$

$$(b+6)^2+(2b-2)^2=41.$$

$$b^2+12b+36+4b^2-8b+4=41.$$

$$5b^2+4b-1=0.$$

$$(5b-1)(b+1)=0.$$

$$b=\frac{1}{5}\quad\text{or}\quad b=-1.$$

Since $ab<0$ and $a=b+2$,

$$b=\frac{1}{5}\Rightarrow a=\frac{11}{5}\quad\Rightarrow\quad ab>0$$

(rejected).

Hence,

$$b=-1,\qquad a=1.$$

Now

$$P(a-b,\ b,\ 2b-a)=(2,-1,-3).$$

Also,

$$C=\left(\frac{1+4}{3},\frac{2(-1)-2}{3},-\frac{2}{3}\right)=\left(\frac{5}{3},-\frac{4}{3},-\frac{2}{3}\right).$$

Therefore,

$$CP^2=\left(2-\frac{5}{3}\right)^2+\left(-1+\frac{4}{3}\right)^2+\left(-3+\frac{2}{3}\right)^2.$$

$$=\left(\frac{1}{3}\right)^2+\left(\frac{1}{3}\right)^2+\left(-\frac{7}{3}\right)^2.$$

$$= \frac{1+1+49}{9} = \frac{51}{9} = \frac{17}{3}.$$

$$\boxed{CP^2 = \frac{17}{3}}$$

Hence, the correct option is

$$\boxed{(2)\ \frac{17}{3}}.$$

Question 75

Let the abscissae of the two points P and Q on a circle be the roots of $x^2 - 4x - 6 = 0$ and the ordinates of P and Q be the roots of $y^2 + 2y - 7 = 0$. If PQ is a diameter of the circle $x^2 + y^2 + 2ax + 2by + c = 0$, then the value of ($a + b - c$) is

A. 12

B. 13

C. 14

D. 16

Correct answer: A

Source solution

Sol.

Equation of circle diameter from
$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$
(where x_1, x_2 are the roots of $x^2 - 4x - 6 = 0$ and y_1, y_2 are the roots of $y^2 + 2y - 7 = 0$)
$x^2 + y^2 - 4x + 2y - 13 = 0$
Now,
Compare it with the given equation, we get $a = -2,\ b = 1, c = -13$

Now
$a + b - c = 12$

Question 76

Two circles each of radius 5 units touch each other at the point $(1, 2)$. If the equation of their common tangent is $4x + 3y = 10$, and $C_1(\alpha, \beta)$ and $C_2(\gamma, \delta)$, $C_1 \neq C_2$ are their centres, then $|(\alpha + \beta)(\gamma + \delta)|$ is equal to .

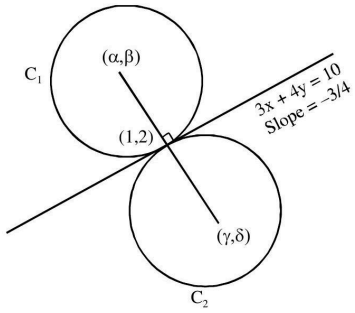
Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 40

Source solution

Slope of line joining centres of circles $= \frac{4}{3} = \tan\theta$



$\Rightarrow \cos\theta = \frac{3}{5}, \sin\theta = \frac{4}{5}$
Now using parametric form
$\frac{x-1}{\cos\theta} = \frac{y-2}{\sin\theta} = \pm 5$
$\oplus(x, y) = (1 + 5\cos\theta, 2 + 5\sin\theta)$
$(\alpha, \beta) = (4, 6)$
$\ominus(x, y) = (\gamma, \delta) = (1 - 5\cos\theta, 2 - 5\sin\theta)$
$(\gamma, s) = (-2, -2)$
$\Rightarrow |(\alpha + \beta)(\gamma + \delta)| = |10x - 4| = 40$

Question 77

Let C be the circle of minimum area touching the parabola $y = 6 - x^2$ and the lines $y = \sqrt{3}|x|$. Then, which one of the following points lies on the circle C ?

A. $(2, 4)$

B. $(1, 2)$

C. $(2, 2)$

D. $(1, 1)$

Correct answer: A

Source solution

Sol.

Equation of circle
$x^2 + (y - (6 - r))^2 = r^2$
touches $\sqrt{3}x - y = 0$
$p = r$
$\frac{|0-(6-r)|}{2} = r$
$|r - 6| = 2r$
$r = 2$

$\therefore$ Circle $x^2 + (y - 4)^2 = 4$
$(2, 4)$ Satisfies this equation

Question 78

If A and B are the points of intersection of the circle $x^2+y^2-8x=0$ and the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ and a point P moves on the line $2x-3y+4=0$, then the centroid of $\triangle PAB$ lies on the line :

A. $4x-9y=12$

B. $x+9y=36$

C. $9x-9y=32$

D. $6x-9y=20$

Correct answer: D

Source solution

Sol. $x^2+y^2-8x=0, \frac{x^2}{9}-\frac{y^2}{4}=1$

$$4x^2-9y^2=36 \quad (1)$$

Solve (1) & (2)
$4x^2-9\left(8x-x^2\right)=36$
$13x^2-72x-36=0$
$(13x+6)(x=6)=0$
$x=\frac{-6}{13}, x=6$
$x=\frac{-6}{13}$ (rejected)
$y \rightarrow$ Imaginary
$n=6, \frac{36}{9}-\frac{y^2}{4}=1$
$y^2=12, y=I\sqrt{12}$
$A(6,\sqrt{12}), B(6,-\sqrt{12})$
$p\left(\alpha, \frac{2\alpha+4}{3}\right) P$ lies on
centroid (h, k)
$2x-3y+y=0$
$h=\frac{12+\alpha}{3}, \alpha=3h-12$
$k=\frac{\frac{2\alpha+4}{3}}{3} \Rightarrow 2\alpha+4=9k$
$\alpha=\frac{9k-4}{2}$
$6h-2y=9k-4$
$6x-9y=20$

Question 79

Let the line $\mathbf{L}: \sqrt{2}\mathbf{x}+\mathbf{y}=\boldsymbol{\alpha}$ pass through the point of the intersection P (in the first quadrant) of the circle $\boldsymbol{x^2+y^2=3}$ and the parabola $\boldsymbol{x^2=2y}$. Let the line $\boldsymbol{L}$ touch two circles $\mathbf{C_1}$ and $\mathbf{C_2}$ of equal radius $\mathbf{2\sqrt{3}}$. If the centres $\mathbf{Q_1}$ and $\mathbf{Q_2}$ of the circles $\mathbf{C_1}$ and $\mathbf{C_2}$ lie on the y -axis, then the square of the area of the triangle $\mathbf{PQ_1Q_2}$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 72

Source solution

$\boldsymbol{x^2+y^2=3}$ and $\boldsymbol{x^2=2y}$

$$y^2+2y-3=0 \Rightarrow (y+3)(y-1)=0$$
$$y=-3 \text{ or } y=1$$
$$y=1 x=\sqrt{2} \Rightarrow P(\sqrt{2},1)$$

p lies on the line
$\sqrt{2}x+y=\alpha$
$\sqrt{2}(\sqrt{2})+1=\alpha$
$\alpha=3$

For circle $\mathbf{C_1}$
$\mathbf{Q_1}$ lies on y axis

Let $\mathbf{Q_1(0,\alpha)}$ coordinates
$\mathbf{R_1=2\sqrt{3}}$ (Given

Line L act as tangent

Apply $\mathbf{P=r}$ (condition of tangency)
$\Rightarrow \left|\frac{\alpha-3}{\sqrt{3}}\right|=2\sqrt{3}$
$\Rightarrow |\alpha-3|=6$
$\alpha-3=6$ or $\alpha-3=-6$
$\Rightarrow \alpha=9 \quad \alpha=-3$
$\Delta PQ_1Q_2=\frac{1}{2}\begin{vmatrix}\sqrt{2} & 1 & 1\\ 0 & 9 & 1\\ 0 & -3 & 1\end{vmatrix}$
$=\frac{1}{2}(\sqrt{2}(12))=6\sqrt{2}$
$(\triangle PQ_1Q_2)^2=72$

Question 80

The length of the latus rectum and the directrices of a hyperbola with eccentricity e are 9 and $x = \pm 4/\sqrt{13}$, respectively. Let the line $y - \sqrt{3}x + \sqrt{3} = 0$ touch this hyperbola at (x_0, y_0). If m is the product of the focal distances of (x_0, y_0), then $4e^2 + m$ is equal to ___.

Numerical answer: _____

This question was incorrect in the original exam paper and ALLEN marked it BONUS. This corrected practice version uses the directrices x = +/-4/sqrt(13), explicitly corrected by ALLEN, instead of the printed +/-4/sqrt(3). Enter an integer (0 decimal places).

Correct answer: 61

Source solution

Use the corrected standard hyperbola $x^2/a^2 - y^2/b^2 = 1$. The latus rectum gives $2b^2/a = 9$. Tangency of $y = \sqrt{3}x - \sqrt{3}$ gives $3 = 3a^2 - b^2$. Thus $a = 2$, $b^2 = 9$, and $e^2 = 1 + b^2/a^2 = 13/4$. The corrected directrices are $x = \pm a/e = \pm 4/\sqrt{13}$. The tangent contact is $(4, 3\sqrt{3})$. Hence $m = (ex_0 - a)(ex_0 + a) = 48$, so $4e^2 + m = 13 + 48 = 61$. The original ALLEN BONUS designation is retained in the instructions; this answer applies to the disclosed corrected practice version.

Question 81

The foci of a hyperbola are $(\pm 2, 0)$ and its eccentricity is $\frac{3}{2}$. A tangent, perpendicular to the line $2x + 3y = 6$, is drawn at a point in the first quadrant on the hyperbola. If the intercepts made by the tangent on the x - and y -axes are a and b respectively, then $|6a| + |5b|$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 12

Source solution

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
ae $= 2 \quad \&e = \frac{3}{2} \Rightarrow a = \frac{4}{3}$
also $b^2 = a^2e^2 - a^2 \Rightarrow 4 - \frac{16}{9}$
$\Rightarrow b^2 = \frac{20}{9}$
Slope of tangent $= \frac{3}{2}$
So tangent equation will be
$y = mx \pm \sqrt{a^2m^2 - b^2}$
$\Rightarrow y = \frac{3x}{2} \pm \sqrt{\frac{16}{9} \cdot \frac{9}{4} - \frac{20}{9}}$
$\Rightarrow y = \frac{3x}{2} \pm \frac{4}{3} \Rightarrow |x_{intercept}| = \frac{8}{9}$
$|y_{intercept}| = \frac{4}{3}$
$\Rightarrow |6a| + |5b| = \frac{48}{9} + \frac{60}{9} = \frac{109}{9} = 12$

Question 82

Let a line perpendicular to the line $2x - y = 10$ touch the parabola $y^2 = 4(x - 9)$ at the point P. The distance of the point P from the centre of the circle $x^2 + y^2 - 14x - 8y + 56 = 0$ is .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 10

Source solution

$y^2 = 4(x - 9)$

slope of tangent $= \frac{-1}{2}$

Point of contact P $\left(9 + \frac{1}{\left(-\frac{1}{2}\right)^2}, \frac{2\times1}{\frac{-1}{2}}\right)$

$\mathbf{P(13, -4)}$

center of circle $C(7, 4)$

distance $\mathbf{CP} = \sqrt{(13-7)^2 + (-4-4)^2}$

$= 10$

Question 83

Let $x^2+y^2+Ax+By+C=0$ be a circle passing through $(0,6)$ and touching the parabola $y=x^2$ at $(2,4)$. Then $A+C$ is equal to ..... .

A. 16

B. $88/5$

C. 72

D. -8

Correct answer: A

Source solution

Sol. $x^2+y^2+Ax+By+C=0$ is passing through $(0,6)$
$\Rightarrow 6B+C=-36$
The tangent of the parabola $y=x^2$ at $(2,4)$ is

$$4x-y-4=0 \quad (1)$$

The tangent of circle $x^2+y^2+Ax+By+C=0$ at $(2,4)$ is

$$(4+A)x+(8+B)y+2A+4B+2C=0 \quad (2)$$

From Equation (1) and (2)
$\frac{4+A}{4}=\frac{8+B}{-1}=\frac{2A+4B+2C}{-4}$

$$A+4B=-36 \quad (3)$$

$$3A+4B+2C=-4 \quad (4)$$

From equation (3) and (4)
$A+C=16$

Question 84

Let the circles $C_1 : (x-\alpha)^2 + (y-\beta)^2 = r_1^2$ and $C_2 : (x-8)^2 + \left(y-\frac{15}{2}\right)^2 = r_2^2$ touch each other externally at the point $(6,6)$. If the point $(6,6)$ divides the line segment joining the centres of the circles C_1 and C_2 internally in the ratio $2:1$, then $(\alpha+\beta)+4\left(r_1^2+r_2^2\right)$ equals

A. 110

B. 130

C. 125

D. 145

Correct answer: B

Source solution

Sol.

$(\alpha,\beta)\ C_1 \overset{2:1}{\underset{(6,6)}{P}} C_2\left(8,\frac{15}{2}\right)$

$\because \frac{16+\alpha}{3} = 6$ and $\frac{15+\beta}{3} = 6$

$\Rightarrow (\alpha,\beta) \equiv (2,3)$

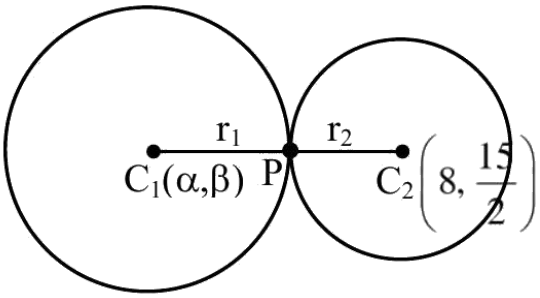
Also, $C_1C_2 = r_1 + r_2$

$\Rightarrow \sqrt{(2-8)^2 + \left(3-\frac{15}{2}\right)^2} = 2r_2 + r_2$

$\Rightarrow r_2 = \frac{5}{2} \Rightarrow r_1 = 2r_2 = 5$

$\therefore (\alpha+\beta) + 4\left(r_1^2 + r_2^2\right)$

$= 5 + 4\left(\frac{25}{4} + 25\right) = 130$



Question 85

The circle passing through the intersection of the circles, $x^2 + y^2 - 6x = 0$ and $x^2 + y^2 - 4y = 0$, having its centre on the line, $2x - 3y + 12 = 0$, also passes through the point:

A. $(-1, 3)$

B. $(1, -3)$

C. $(-3, 6)$

D. {3)}

$S_1 + \lambda (S_1 - S_2) = 0$

$x^2 + y^2 - 6x + \lambda(4y - 6x) = 0$

$x^2 + y^2 - 6x(1 + \lambda) + 4\lambda y = 0$

Centre ($3(1 + \lambda), -2\lambda$) put in $2x - 3y + 12 = 0$

$6 + 6\lambda + 6\lambda + 12 = 0$

$12\lambda = -18$

$\lambda = -3/2$

$\therefore$ Circle is $x^2 + y^2 + 3x - 6y = 0$

Check options

Correct answer: C

Source solution

We are given two circles:

$S_1 : x^2 + y^2 - 6x = 0, \qquad S_2 : x^2 + y^2 - 4y = 0$

Equation of a circle through their intersection is:

$S_1 + \lambda S_2 = 0$

$(x^2 + y^2 - 6x) + \lambda(x^2 + y^2 - 4y) = 0$

$(1 + \lambda)(x^2 + y^2) - 6x - 4\lambda y = 0$

Dividing by $(1 + \lambda)$:

$x^2 + y^2 - \frac{6}{1+\lambda}x - \frac{4\lambda}{1+\lambda}y = 0$

This is of the form $x^2 + y^2 + 2gx + 2fy + c = 0$.

$\therefore \quad g = -\frac{3}{1+\lambda}, \qquad f = -\frac{2\lambda}{1+\lambda}.$

Centre $= (-g, -f) = \left(\frac{3}{1+\lambda}, \frac{2\lambda}{1+\lambda}\right).$

Since the centre lies on the line $2x - 3y + 12 = 0$:

$2\left(\frac{3}{1+\lambda}\right) - 3\left(\frac{2\lambda}{1+\lambda}\right) + 12 = 0$

$\frac{6 - 6\lambda}{1+\lambda} + 12 = 0$

$6 - 6\lambda + 12(1 + \lambda) = 0$

$18 + 6\lambda = 0$

$\Rightarrow \lambda = -3.$

Substitute $\lambda = -3$:

$(x^2 + y^2 - 6x) - 3(x^2 + y^2 - 4y) = 0$

$x^2 + y^2 - 6x - 3x^2 - 3y^2 + 12y = 0$

$-2x^2 - 2y^2 - 6x + 12y = 0$

$\Rightarrow x^2 + y^2 + 3x - 6y = 0.$

This is the required circle.

Now check points:

$(-3, 0)$:
$(-3)^2 + 0 + 3(-3) - 0 = 9 - 9 = 0 \quad \checkmark$

$(0, 6)$:
$0 + 36 + 0 - 36 = 0 \quad \checkmark$

$\therefore$ The required circle also passes through $(-3, 0)$ and $(0, 6)$.

Question 86

Let the latus ractum of the parabola $y^2 = 4x$ be the common chord to the circles C_1 and C_2 each of them having radius $2\sqrt{5}$. Then, the distance between the centres of the circles C_1 and C_2 is:

A. 8

B. $8\sqrt{5}$

C. $4\sqrt{5}$

D. 12

Correct answer: A

Source solution

$$\text{Parabola: } y^2 = 4x$$

For the parabola $y^2 = 4ax$, we have

$$a = 1.$$

Hence, the latus rectum is

$$x = a = 1.$$

Its endpoints are

$$(1, 2) \quad \text{and} \quad (1, -2).$$

Therefore, the length of the common chord is

$$L = 4.$$

Let the radius of each circle be

$$r = 2\sqrt{5}.$$

If a chord of length L is at a distance d from the centre of a circle, then

$$L = 2\sqrt{r^2 - d^2}.$$

Substituting $L = 4$ and $r = 2\sqrt{5}$,

$$4 = 2\sqrt{20 - d^2}$$

$$2 = \sqrt{20 - d^2}$$

$$d^2 = 16$$

$$d = 4.$$

Thus, the centre of each circle lies on the perpendicular bisector of the common chord at a distance 4 from the chord.

The common chord is the line

$$x = 1.$$

Its midpoint is

$$(1, 0).$$

The perpendicular bisector is

$$y = 0.$$

Hence the two centres are

$$C_1 = (1, 4), \qquad C_2 = (1, -4).$$

Therefore, the distance between the centres is

$$C_1C_2 = \sqrt{(1-1)^2 + (4+4)^2} = 8.$$

$$\boxed{8}$$

Hence, the correct option is

$$\boxed{\text{(1) } 8}.$$

Question 87

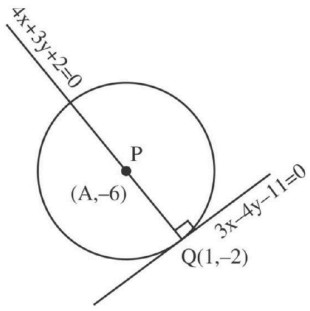
Let a circle C of radius 5 lie below the x -axis. The line $\mathbf{L_1 = 4x + 3y - 2}$ passes through the centre P of the circle C and intersects the line $\mathbf{L_2 : 3x - 4y - 11 = 0}$ at Q . The line $\mathbf{L_2}$ touches C at the point Q . Then the distance of P from the line $\mathbf{5x - 12y + 51 = 0}$ is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 11

Source solution



$\mathbf{4x + 3y + 2 = 0}$
$\mathbf{3x - 4y - 11 = 0}$

4
$\frac{x}{-25} = \frac{y}{50} = \frac{1}{-25}$
$\frac{x-1}{\cos\theta} = \frac{y+2}{\sin\theta} = \pm 5$
$y = -2 + 5\left(-\frac{4}{5}\right) = -6$
$x = 1 + 5\left(\frac{3}{5}\right) = 4$
Req. distance
$\left|\frac{5(4)-12(-6)+51}{13}\right|$
$= \left|\frac{20+72+51}{13}\right|$
$= \frac{143}{13} = 11$

Question 88

A circle of radius 2 unit passes through the vertex and the focus of the parabola $y^2 = 2x$ and touches the parabola $y = \left(x - \frac{1}{4}\right)^2 + \alpha$, where $\alpha > 0$. Then $(4\alpha - 8)^2$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 63

Source solution

Vertex and focus of parabola $y^2 = 2x$
are $V(0, 0)$ and $S\left(\frac{1}{2}, 0\right)$ resp.
Let equation of circle be
$(x - h)^2 + (y - k)^2 = 4$
$\because$ Circle passes through $(0, 0)$

$$\Rightarrow h^2 + k^2 = 4 \qquad (.....)$$

$\because$ Circle passes through $\left(\frac{1}{2}, 0\right)$
$\left(\frac{1}{2} - h\right)^2 + k^2 = 4$

$$\Rightarrow h^2 + k^2 - h = \frac{15}{4} \qquad (2)$$

On solving (1) and (2)
$4 - h = \frac{15}{4}$
$h = 4 - \frac{15}{4} = \frac{1}{4}$
$k = +\frac{\sqrt{63}}{4}$
$k = -\frac{\sqrt{63}}{4}$ is rejected as circle with centre $\left(\frac{1}{4}, -\frac{\sqrt{63}}{4}\right)$ can't touch given parabola.

Equation of circle is
$\left(x - \frac{1}{4}\right)^2 + \left(k - \frac{\sqrt{63}}{4}\right)^2 = 4$
From figure
$\alpha = 2 + \frac{\sqrt{63}}{4} = \frac{8 + \sqrt{63}}{4}$
$4\alpha - 8 = \sqrt{63}$
$(4\alpha - 8)^2 = 63$

Question 89

If $x^2 - y^2 + 2hxy + 2gx + 2fy + c = 0$ is the locus of a point, which moves such that it is always equidistant from the lines $x + 2y + 7 = 0$ and $2x - y + 8 = 0$, then the value of $g + c + h - f$ equals

A. 14

B. 6

C. 8

D. 29

Correct answer: A

Source solution

Sol. Cocus of point $P(x, y)$ whose distance from Gives
$X + 2y + 7 = 0 \& 2x - y + 8 = 0$ are equal is $\frac{x+2y+7}{\sqrt{5}} = \pm\frac{2x-y+8}{\sqrt{5}}$
$(x + 2y + 7)^2 - (2x - y + 8)^2 = 0$

Combined equation of lines
$(x - 3y + 1)(3x + y + 15) = 0$
$3x^2 - 3y^2 - 8xy + 18x - 44y + 15 = 0$
$x^2 - y^2 - \frac{8}{3}xy + 6x - \frac{44}{3}y + 5 = 0$
$x^2 - y^2 + 2hxy + 2gx2 + 2fy + c = 0$
$h = \frac{4}{3},\ g = 3, f = -\frac{22}{3}, c = 5$
$g + c + h - f = 3 + 5 - \frac{4}{3} + \frac{22}{3} = 8 + 6 = 14$

Question 90

Let O be the origin and OP and OQ be the tangents to the circle $x^2+y^2-6x+4y+8=0$ at the point P and Q on it. If the circumcircle of the triangle OPQ passes through the point $\left(\alpha, \frac{1}{2}\right)$, then a value of α is

A. $\frac{3}{2}$

B. $\frac{5}{2}$

C. 1

D. $-\frac{1}{2}$

Correct answer: B

Source solution

\textbf{Given circle:}

$$x^2+y^2-6x+4y+8=0$$

$$(x-3)^2+(y+2)^2=5$$

Hence,

$$C=(3,-2), \qquad r=\sqrt{5}.$$

Also,

$$O=(0,0).$$

\bigskip

Since OP and OQ are tangents from O to the circle, we have

$$OP \perp CP, \qquad OQ \perp CQ.$$

Therefore, $\angle OPC=\angle OQC=90^\circ$.

Hence P and Q lie on the circle having diameter OC.

So the circumcircle of $\triangle OPQ$ is precisely the circle with diameter OC.

\bigskip

Midpoint of OC:

$$M=\left(\frac{0+3}{2}, \frac{0-2}{2}\right)=\left(\frac{3}{2}, -1\right).$$

Radius:

$$R=\frac{OC}{2}=\frac{\sqrt{3^2+(-2)^2}}{2}=\frac{\sqrt{13}}{2}.$$

Thus the circumcircle is

$$\left(x-\frac{3}{2}\right)^2+(y+1)^2=\frac{13}{4}.$$

\bigskip

Since the point

$$\left(\alpha,\frac{1}{2}\right)$$

lies on this circle,

$$\left(\alpha-\frac{3}{2}\right)^2+\left(\frac{1}{2}+1\right)^2=\frac{13}{4}.$$

$$\left(\alpha-\frac{3}{2}\right)^2+\frac{9}{4}=\frac{13}{4}.$$

$$\left(\alpha-\frac{3}{2}\right)^2=1.$$

$$\alpha-\frac{3}{2}=\pm 1.$$

Hence,

$$\alpha=\frac{5}{2} \quad \text{or} \quad \alpha=\frac{1}{2}.$$

Among the given options,

$$\boxed{\alpha=\frac{5}{2}}.$$

Option (2)

Introduction to Three Dimensional Geometry

Questions 91-92 / 2 selected / 0.7% practice weight / Consolidate

Question 91

If the four points, whose position vectors are $3\hat{i}-4\hat{j}+2\hat{k}, \hat{i}+2\hat{j}-\hat{k}, -2\hat{i}-\hat{j}+3\hat{k}$ and $5\hat{i}-2\alpha\hat{j}+4\hat{k}$ are coplanar, then α is equal to

A. $\frac{73}{17}$

B. $\frac{107}{17}$

C. $\frac{-73}{17}$

D. $\frac{-107}{17}$

Correct answer: A

Source solution

$$P = 3\hat{i}-4\hat{j}+2\hat{k}, \qquad Q = \hat{i}+2\hat{j}-\hat{k},$$
$$R = -2\hat{i}-\hat{j}+3\hat{k}, \qquad S = 5\hat{i}-2\alpha\hat{j}+4\hat{k}.$$

$$\overrightarrow{PQ} = Q-P = -2\hat{i}+6\hat{j}-3\hat{k}.$$

$$\overrightarrow{QR} = R-Q = -3\hat{i}-3\hat{j}+4\hat{k}.$$

$$\overrightarrow{RS} = S-R = 7\hat{i}+(1-2\alpha)\hat{j}+\hat{k}.$$

Since the four points are coplanar,

$$[\overrightarrow{PQ}\ \overrightarrow{QR}\ \overrightarrow{RS}] = 0.$$

Therefore,

$$\begin{vmatrix} -2 & 6 & -3 \\ -3 & -3 & 4 \\ 7 & 1-2\alpha & 1 \end{vmatrix} = 0.$$

Expanding along the first row,

$$-2\begin{vmatrix} -3 & 4 \\ 1-2\alpha & 1 \end{vmatrix} - 6\begin{vmatrix} -3 & 4 \\ 7 & 1 \end{vmatrix} - 3\begin{vmatrix} -3 & -3 \\ 7 & 1-2\alpha \end{vmatrix} = 0.$$

$$-2\Big[-3-4(1-2\alpha)\Big] - 6(-31) - 3\Big[6\alpha-3+21\Big] = 0.$$

$$14-16\alpha+186-18\alpha-54 = 0.$$

$$146 - 34\alpha = 0.$$

$$34\alpha = 146.$$

$$\boxed{\alpha = \frac{73}{17}}.$$

Question 92

The distance of the origin from the centroid of the triangle whose two sides have the equations $x - 2y + 1 = 0$ and $2x - y - 1 = 0$ and whose orthocenter is $\left(\frac{7}{3}, \frac{7}{3}\right)$ is:

A. $\sqrt{2}$

B. 2

C. $2\sqrt{2}$

D. 4

Correct answer: C

Source solution

Sol.
$AB \equiv x - 2y + 1 = 0$
$\mathrm{AC} \equiv 2\mathrm{x} - \mathrm{y} - 1 = 0$

So $\mathrm{A}(1, 1)$
Altitude from B is $\mathrm{BH} = \mathrm{x} + 2 <. \mathrm{y} - 7 = 0 \Rightarrow \ \mathrm{B}(3, 2)$ Altitude from C is $\mathrm{CH} = 2\mathrm{x} + \mathrm{y} - 7 = 0 \Rightarrow \mathrm{C}(2, 3)$ Centroid of $\triangle\mathrm{ABC} = \mathrm{E}(2, 2)\mathrm{OE} = 2\sqrt{2}$

Limits and Derivatives

Questions 93-93 / 1 selected / 0.3% practice weight / Consolidate

Question 93

If the domain of the function
$f(x)=\log_e\left(4x^2+11x+6\right)+\sin^{-1}$
$(4x+3)+\cos^{-1}\left(\frac{10x+6}{3}\right)$ is $(\alpha,\beta]$,
Then $36|\alpha+\beta|$ is equal to :

A. 63

B. 45

C. 72

D. 54

Correct answer: B

Source solution

Sol. $f(x)=\ln\left(4x^2+11x+6\right)+\sin^{-1}(4x+3)$
$+\cos^{-1}\left(\frac{10x+6}{3}\right)$
(i) $4x^2+11x+6>0$
$4x^2+8x+3x+6>0$
$(4x+3)(x+2)>0$
$x\in(-\infty,-2)\cup\left(-\frac{3}{4},\infty\right)$
(ii) $4x+3\in[-1,1]$
$x\in[-1,-1/2]$
(iii) $\frac{10x+6}{3}\in[-1,1]$
$x\in\left[-\frac{9}{10},-\frac{3}{10}\right]$
$x\in\left(-\frac{3}{4},-\frac{1}{2}\right]\quad\alpha=-\frac{3}{4},\beta=-\frac{1}{2}$
$\alpha+\beta=-\frac{5}{4}$
$36|\alpha+\beta|=45$

Statistics

Question 94

Let the mean and the variance of 6 observation a, b, $\mathbf{68, 44, 48, 60}$ be 55 and 194, respectively if $a > b$, then $a + 3b$ is

A. 200

B. 190

C. 180

D. 210

Correct answer: C

Source solution

Sol.- a, b, 68, 44, 48, 60

$$\text{Mean } = 55 \qquad \text{Variance } = 194 \frac{a+b+68+44+48+60}{6} = 55 \Rightarrow 220 + a + b = 330 \therefore a + b = 110 \ldots\ldots (1)$$

Also,

$\sum \frac{(x_i - \bar{x})^2}{n} = 194$

$\Rightarrow (a - 55)^2 + (b - 55)^2 + (68 - 55)^2 + (44 - 55)^2$

$+(48 - 55)^2 + (60 - 55)^2 = 194 \times 6$

$\Rightarrow (a - 55)^2 + (b - 55)^2 + 169 + 121 + 49 + 25 = 1164$

$\Rightarrow (a - 55)^2 + (b - 55)^2 = 1164 - 364 = 800$

$a^2 + 3025 - 110a + b^2 + 3025 - 110b = 800$

$$\Rightarrow a^2 + b^2 = 800 - 6050 + 12100 \qquad (2)$$

$a^2 + b^2 = 6850$.

Solve (1) & (2);

$a = 75,\ b = 35$

$\therefore a + 3b = 75 + 3(35) = 75 + 105 = 180$

Question 95

The mean and variance of a set of 15 numbers are 12 and 14 respectively. The mean and variance of another set of 15 numbers are 14 and σ^2 respectively. If the variance of all the 30 numbers in the two sets is 13 , then σ^2 is equal to

A. 9

B. 12

C. 11

D. 10

Correct answer: D

Source solution

Sol. Combine var. $= \frac{n_1\sigma^2+n_2\sigma^2}{n_1+n_2} + \frac{n_1n_2(m_1-m_2)^2}{(n_1+n_2)^2}$

$13 = \frac{15.14+15.\sigma^2}{30} + \frac{15.15(12-14)^2}{30\times30}$

$13 = \frac{14+\sigma^2}{2} + \frac{4}{4}$

$\sigma^2 = 10$

Question 96

The mean and standard deviation of the marks of 10 students were found to be 50 and 12 respectively. Later, it was observed that two marks 20 and 25 were wrongly read as 45 and 50 respectively. Then the correct variance is .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 269

Source solution

$\bar{x} = 50$

$\sum x_i = 500$

$\sum x_{i_{\text{correct}}} = 500 + 20 + 25 - 45 - 50 = 450$

$\sigma^2 = 144$

$\frac{\sum x_i^2}{10} - (50)^2 = 144$

$$\sum x_{\text{icorrect}}^2 = \left(144 + (50)^2\right) \times 10 - (45)^2 - (50)^2 + (20)^2 + (25)^2$$

$$= 22940$$

$$\text{Correct variance} = \frac{\sum (x_{\text{icorrect}})^2}{10} - \left(\frac{\sum x_{\text{icorrect}}}{10}\right)^2$$

$$= 2294 - (45)^2$$

$$= 2294 - 2025 = 269$$

Question 97

Let the six numbers $a_1, a_2, a_3, a_4, a_5, a_6$, be in A.P. and $a_1 + a_3 = 10$. If the mean of these six numbers is $\frac{19}{2}$ and their variance is σ^2, then $8\sigma^2$ is equal to :

A. 210

B. 220

C. 200

D. 105

Correct answer: A

Source solution

Sol. 1}

$$a + (a + 2d) = 10 \Rightarrow a + d = 5 \quad (1)$$

Mean $\Rightarrow \frac{\frac{6}{2}[2a+5d]}{6} = \frac{19}{2}$

$$2a + 5d = 19 \quad (2)$$

from (1) and (2)
$3d = 9 \Rightarrow d = 3; a = 2$
$\therefore \sigma^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2$

$$= \frac{2^2 + 5^2 + 8^2 + 11^2 + 14^2 + 17^2}{6} - \left(\frac{19}{2}\right)^2 \quad = \frac{699}{6} - \frac{361}{4} = \frac{233}{2} - \frac{361}{4}$$

$8\sigma^2 = 932 - 722 = 210$

Question 98

Let $9 = x_1 < x_2 < \ldots\ldots < x_7 \ldots\ldots, x_7$ be in an A.P. with common difference d . If the standard deviation of $x_1 \cdot x_2 \ldots., x_7$ is 4 and mean is $\bar{x}$, then $\bar{x} + x_6$ is equal to :

A. $2\left(9 + \frac{8}{\sqrt{7}}\right)$

B. $18\left(1 + \frac{1}{\sqrt{3}}\right)$

C. 25

D. 34

Correct answer: D

Source solution

Sol. 4
Mean $\Rightarrow \bar{x} = \frac{\sum_{i=1}^{7} x_i}{7} = \frac{\frac{7}{2}[2a+6d]}{7} = a + 3d = x_4$
Variance $= \frac{\sum_{i=1}^{7}(x_i - \bar{x})^2}{7} = (4)^2 \Rightarrow \frac{\sum_{i=1}^{7}(x_i - x_4)^2}{7} = 16$
$\Rightarrow \frac{(3d)^2 + (2d)^2 + d^2 + 0 + d^2 + (2d)^2 + (3d)^2}{7} = 16$
$= 4d^2 = 16 \Rightarrow d = 2$
$\Rightarrow \bar{x} = 9 + 3(2) = 15$
$\&x_0 = a + 5d = 9 + 5(2) = 19 \Rightarrow \bar{x} + x_0 = 34$

Question 99

Let S be the set of all values of $\mathbf{a_1}$ for which the mean deviation about the mean of 100 consecutive positive integers $a_1, a_2, a_3, \ldots a_{100}$ is 25 . Then S is

A. N

B. ϕ

C. $\{99\}$

D. $\{9\}$

Correct answer: A

Source solution

Sol. 1

Let $\mathbf{a_1 = n} \quad \mathbf{a_2 = n+1} \quad \mathbf{a_3 = n+2} \quad \ldots\ldots$

$$\bar{x} = \frac{n+(n+1)+(n+2)+\ldots..n+99}{100} \qquad = \frac{100n+\frac{100\times 99}{2}}{100} = n+\frac{99}{2}$$

Mean deviation about the mean

$\frac{1}{100}\sum|\mathbf{x_i}-\bar{\mathbf{x}}|$

$\Rightarrow \frac{1}{100}\left(\frac{99}{2}+\frac{97}{2}+\frac{95}{2}+\ldots\ldots+\frac{97}{2}+\frac{99}{2}\right)$

$\Rightarrow \frac{2}{100}\left(\frac{99}{2}+\frac{97}{2}+\frac{95}{2}+\ldots\ldots 50 \text{ terms}\right)$

$\Rightarrow \frac{2}{100}\times\frac{1}{2}\times(50)^2 = \frac{50\times 50}{100} = 25$

It is 25 irrespective of the value of

$\therefore \mathbf{n \in N}$

$\Rightarrow \quad \mathbf{S = N}$

Question 100

The mean and variance of 7 observations are 8 and 16 respectively. If one observation 14 is omitted and a and b are respectively mean and variance of remaining 6 observation, then $a+3b-5$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 37

Source solution

37

Mean of 7 observations $= \frac{\sum_{i=1}^{7} x_i}{7}$

$\Rightarrow \quad \sum_{i=1}^{7}\mathbf{x_i} = 7\times 8 = 56$

Variance $= \frac{\Sigma x_i^2}{n} - (\bar{x})^2$

$\Sigma \mathbf{x_i^2} = 7(16+64) = 560$

If 14 is removed then

Mean $= \mathbf{a} = \frac{\sum_{i=1}^{7}\mathbf{x_i}-14}{6} \Rightarrow \mathbf{6a = 56-14}$

$\Rightarrow \mathbf{a = 7}$

Variance $= \mathbf{b} = \frac{\sum_{i=1}^{7}\mathbf{x_i^2}-(14)^2}{6} - 49$

$\Rightarrow \quad \mathbf{6\,b = 560-196-294}$

$\Rightarrow \quad \mathbf{6\,b = 70}$

$\Rightarrow \quad \mathbf{3\,b = 35}$

$\therefore \quad a+3b-5 = 7+35-5 = 37$

Question 101

The mean and the variance of five observation are 4 and 5.20, respectively. If three of the observations are 3,4 and 4 ; then then absolute value of the difference of the other two observations, is :

A. 1

B. 3

C. 7

D. 5

Correct answer: C

Source solution

Mean: $\bar{x} = 4,\ \sigma^2 = 5.2,\ n = 5,\ x_1 = 3,\ x_2 = 4,\ x_3 = 4$

$$\sum x_i = 20, \quad x_4 + x_5 = 9 \ \ldots\ldots\ldots \text{(i)} \tag{i}$$

$$\frac{\sum x_i^2}{n} - (\bar{x})^2 = \sigma^2 \ \Rightarrow \sum x_i^2 = 106, \quad x_4^2 + x_5^2 = 65 \ \ldots\ldots\ldots \text{(ii)} \tag{ii}$$

Using (i) and (ii):

$$(x_4 - x_5)^2 = 49$$

$$|x_4 - x_5| = 7$$

Question 102

The mean of five observations is 5 and their variance is 9.20 . If three of the given five observations are 1,3 and 8 , then a ratio of other two observations is :

A. $4:9$

B. $6:7$

C. $5:8$

D. $10:3$

Correct answer: A

Source solution

Sol. Let two observations are $x_1 \& x_2$
mean $= \frac{\sum x_1}{5} = 5 \Rightarrow 1+3+8+x_1+x_2 = 25$

$$\Rightarrow x_1 + x_2 = 13 \quad (1)$$

$\text{variance}\left(\sigma^2\right) = \frac{\sum x_i^2}{5} - 25 = 9.20$
$\Rightarrow \quad \sum x_i^2 = 171$

$$\Rightarrow x_1^2 + x_2^2 = 97 \quad (2)$$

by (1) & (2)
$(x_1+x_2)^2 - 2x_1x_2 = 97$
or $x_1x_2 = 36$
$\therefore x_1 : x_2 = 4:9$

Probability

Questions 103-103 / 1 selected / 0.3% practice weight / Consolidate

Question 103

Let Ω be the sample space and $A \subseteq \Omega$ be an event.

Given below are two statements :
(S1) : If $P(A) = 0$, then $A = \varnothing$
(S2): If $P(A) = 1$, then $A = \Omega$
Then

A. both (S1) and (S2) are true

B. only (S1) is true

C. only (S2) is true

D. both (S1) and (S2) are false

Correct answer: D

Source solution

Sol. (4)
Let $\Omega = [0, 1]$
Let $A \rightarrow$ selecting $\frac{1}{2}$
$A = \left\{\frac{1}{2}\right\}$
then, $P(A) = 0$ but $A \neq \phi$
$B = A^c = [0, 1] - \left\{\frac{1}{2}\right\}$
$P(B) = 1$
but $B \neq \Omega$

Inverse Trigonometric Functions

Questions 104-104 / 1 selected / 0.3% practice weight / Consolidate

Question 104

The number of values of $r \in \{p, q, \sim p, \sim q\}$ for which $((p \wedge q) \Rightarrow (r \vee q)) \wedge ((p \wedge r) \Rightarrow q)$ is a tautology, is:

A. 3

B. 4

C. 1

D. 2

Correct answer: D

Source solution

$$((p \wedge q) \Rightarrow (r \vee q)) \wedge ((p \wedge r) \Rightarrow q)$$
$$(p \wedge q) \Rightarrow (r \vee q)$$
$$\sim (p \wedge q) \vee (r \vee q)$$
$$\sim p \vee \sim q \vee r \vee q$$
$$\&(p \wedge r) \Rightarrow q$$
$$\sim (p \wedge r) \vee q$$
$$\sim p \vee \sim r \vee q$$
$$(\sim p \vee \sim q \vee r \vee q) \wedge (\sim p \vee \sim r \vee q)$$
$$\equiv \sim p \vee \sim r \vee q$$
If $r = p$
If $\sim p \vee \sim p \vee q \rightarrow$ Not tautology
$r = \sim p$

$\sim p \vee p \vee q \rightarrow$ tautology
If $r = q$
$\sim p \vee \sim q \vee q \rightarrow$ tautology
If $r = \sim q$
$\sim p \vee q \vee q \rightarrow$ Not tautology

Ans. 2 (D)

Determinants

Questions 105-105 / 1 selected / 0.3% practice weight / Consolidate

Question 105

Let $\mathbf{A} = \{a_{ij}\}$ be a 3×3 matrix, where

$$a_{ij} = \begin{cases} (-1)^{j-i} & \text{if } i < j \\ 2 & \text{if } i = j \\ (-1)^{i+j} & \text{if } i > j \end{cases}$$

then $\det\left(3\,\text{Adj}\left(2\,A^{-1}\right)\right)$ is equal to ___.

Numerical answer: _____

Enter the requested integer (0 decimal places).

Correct answer: 108

Source solution

$$A \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$|A| = 4$$

$$\left|\text{sadj}\left(2\,A^{-1}\right)\right| = \left|3.2^2\,\text{adj}\left(A^{-1}\right)\right|$$

$$= 12^3\left|\text{adj}\left(A^{-1}\right)\right| = 12^3\left|A^{-1}\right|^2 = \frac{12^3}{|A|^2} = \frac{12^3}{16} = 108$$

Application of Derivatives

Question 106

Let $y = f(x)$ be a thrice differentiable function in $(-5, 5)$. Let the tangents to the curve $y = f(x)$ at $(1, f(1))$ and $(3, f(3))$ make angles $\frac{\pi}{6}$ and $\frac{\pi}{4}$, respectively with positive x -axis. If $27\int_1^3 \left((f'(t))^2 + 1\right) f''(t)dt = \alpha + \beta\sqrt{3}$ where α, β are integers, then the value of $\alpha + \beta$ equals

A. -14

B. 26

C. -16

D. 36

Correct answer: B

Source solution

Sol. $y = f(x) \Rightarrow \frac{dy}{dx} = f'(x)$

$\left(\frac{dy}{dx}\right)_{(1,f(1))} = f'(1) = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}} \Rightarrow f'(1) = \frac{1}{\sqrt{3}}$ $\left(\frac{dy}{dx}\right)_{(3,f(3))} = f'(3) = \tan\frac{\pi}{4} = 1 \Rightarrow f'(3) = 1$ $27\int_1^3 \left((f'(t))^2 + 1\right) f''(t)dt = \alpha + \beta\sqrt{3}$ $I = \int_1^3 \left((f'(t))^2 + 1\right) f''(t)dt$ $f'(t) = z \Rightarrow f''(t)dt = dz$ $z = f'(3) = 1$ $z = f'(1) = \frac{1}{\sqrt{3}}$ $I = \int_{1/\sqrt{3}}^{1} (z^2 + 1)dz = \left(\frac{z^3}{3} + z\right)_{1/\sqrt{3}}^{1} = \left(\frac{1}{3} + 1\right) - \left(\frac{1}{3}\cdot\frac{1}{3\sqrt{3}} + \frac{1}{\sqrt{3}}\right)$ $= \frac{4}{3} - \frac{10}{9\sqrt{3}} = \frac{4}{3} - \frac{10}{27}\sqrt{3}\alpha + \beta\sqrt{3} = 27\left(\frac{4}{3} - \frac{10}{27}\sqrt{3}\right) = 36 - 10\sqrt{3}$ $\alpha = 36, \beta = -10\alpha + \beta = 36 - 10 = 26$

\setcounter{enumi}{9}

Question 107

Let $\alpha = \max_{x\in\mathbb{R}} \left\{8^{2\sin 3x} \cdot 4^{4\cos 3x}\right\}$ and $\beta = \min_{x\in\mathbb{R}} \left\{8^{2\sin 3x} \cdot 4^{4\cos 3x}\right\}$. If $8x^2 + bx + c = 0$ is a quadratic equation whose roots are $\alpha^{1/5}$ and $\beta^{1/5}$, then the value of $c - b$ is equal to :

A. 42

B. 47

C. 43

D. 50

Correct answer: A

The source provides a brief answer only. A detailed worked solution is not available for this question.

Source solution

Question 108

Let the set of all values of p, for which
$f(x) = \left(p^2 - 6p + 8\right)\left(\sin^2 2x - \cos^2 2x\right) + 2(2 - p)x + 7$ does not have any critical point, be the interval (a, b). Then 16 ab is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 252

Source solution

$f(x) = -\left(p^2 - 6p + 8\right)\cos 4n + 2(2 - p)n + 7$
$f^1(x) = +4\left(p^2 - 6p + 8\right)\sin 4x + (4 - 2p) \neq 0$
$\sin 4x \neq \frac{2p-4}{4(p-4)(p-2)}$
$\sin 4x \neq \frac{2(p-2)}{4(p-4)(p-2)}$
$\mathrm{p} \neq 2$
$\sin 4x \neq \frac{1}{2(p-4)}$
$\Rightarrow \left|\frac{1}{2(\mathrm{p}-4)}\right| > 1$
on solving we get
$\therefore \mathrm{p} \in \left(\frac{7}{2}, \frac{9}{2}\right)$
Hence $\mathrm{a} = \frac{7}{2},\ \mathrm{b} = \frac{9}{2}$
$\therefore 16\mathrm{ab} = 252$

Question 109

A line is a common tangent to the circle $(x-3)^2+y^2=9$ and the parabola $y^2=4x$. If the two points of contact ($\mathbf{a, b}$) and ($\mathbf{c, d}$) are distinct and lie in the first quadrant, then $\mathbf{2(a+c)}$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

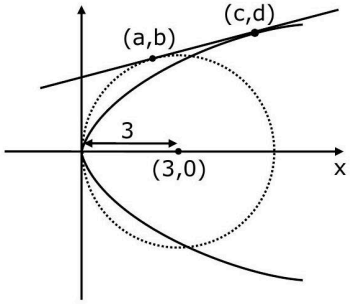
Correct answer: 9

Source solution

Circle: $(x-3)^2+y^2=9$
Parabola: $y^2=4x$
Let tangent $y=mx+\frac{a}{m}$

$$y=mx+\frac{1}{m}$$

$m^2x-my+1=0$
the above line is also tangent to circle
$(x-3)^2+y^2=9$
$\therefore \perp$ from $(3,0)=3$
$\left|\frac{3m^2-0+1}{\sqrt{m^2+m^4}}\right|=3$
$\left(3m^2+1\right)^2=9\left(m^2+m^4\right)$
$6m^2+1+9m^4=9m^2+9m^4$
$3\,\mathrm{m}^2=1$
$m=\pm\frac{1}{\sqrt{3}}$
$\therefore$ tangent is
$y=\frac{1}{\sqrt{3}}x+\sqrt{3}$ or $y=-\frac{1}{\sqrt{3}}x-\sqrt{3}$
(it will be used) (rejected)
$\mathrm{m}=\frac{1}{\sqrt{3}}$



for Parabola $\left(\frac{a}{m^2},\frac{2a}{m}\right)\equiv(3,2\sqrt{3})$
(c, d)
for Circle $y=\frac{1}{\sqrt{3}}x+\sqrt{3}$ & $(x-3)^2+y^2=9$
solving, $(x-3)^2+\left(\frac{1}{\sqrt{3}}x+\sqrt{3}\right)^2=9$

$$x^2+9-6x+\frac{1}{3}x^2+3+2x=9$$
$$\frac{4}{3}x^2-4x+3=0$$
$$4x^2-12x+9=0$$
$$4x^2-6x-6x+9=0$$
$$2x(2x-3)-3(2x-3)=0$$
$$(2x-3)(2x-3)=0$$
$$x=\frac{3}{2}$$

$$\therefore \quad y=\frac{1}{\sqrt{3}}\left(\frac{3}{2}\right)+\sqrt{3}$$

$$y=\frac{\sqrt{3}}{2}+\sqrt{3}$$

$$y=\frac{3\sqrt{3}}{2}$$

$$(a,b)\equiv\left(\frac{3}{2},\frac{3\sqrt{3}}{2}\right)$$

$$2(a+c)=2\left(\frac{3}{2}+3\right)$$

$$=2\left(\frac{3}{2}+\frac{6}{2}\right)$$

$$=9$$

Question 110

Let the plane $P: 8x+\alpha_1 y+\alpha_2 z+12=0$ be parallel to the line $\mathrm{L}: \frac{x+2}{2}=\frac{y-3}{3}=\frac{z+4}{5}$. If the intercept of P on the y-axis is 1 , then the distance between P and L is :

A. $\sqrt{\frac{7}{2}}$

B. $\sqrt{\frac{2}{7}}$

C. $\frac{6}{\sqrt{14}}$

D. $\sqrt{14}$

Correct answer: D

Source solution

Sol. 4

$y-$ intercept $=\frac{-12}{\alpha_1}=1$

$\alpha_1=-12 \& \vec{n}=(8,\alpha_1,\alpha_2)$

$\vec{\ell}=(2,3,5)$

$\vec{n}\cdot\vec{\ell}=0(\because$ plane P & line L are parallel $)$

$16+3\alpha_1+5\alpha_2=0$

$16-36+5\alpha_2=0$

$5\alpha_2=20$

$\alpha_2=4$

$$8x-12y+4z+12=0 \qquad \Rightarrow 2x-3y+z+3=0$$

$(-2,3,-4)$ is a point on the line L distance bet^{n} the point $(-2,3,-4)$ and the plane P is :-

$$\mathrm{d}=\frac{|-4-9-4+3|}{\sqrt{2^2+3^2+1^2}} \qquad =\frac{14}{\sqrt{14}}=\sqrt{14}$$

\setcounter{enumi}{75}

Question 111

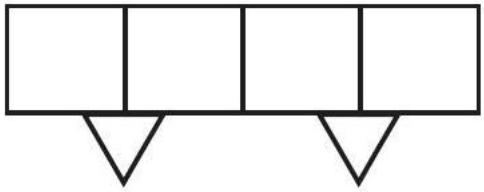
A person forgets his 4-digit ATM pin code. But he remembers that in the code all the digits are different, the greatest digit is 7 and the sum of the first two digits is equal to the sum of the last two digits. Then the maximum number of trials necessary to obtain the correct code is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 72

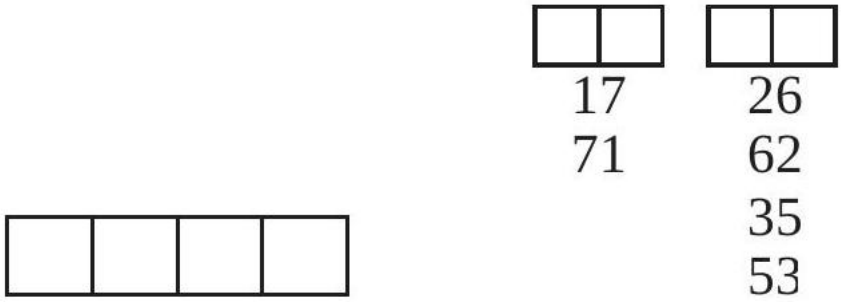
Source solution



Sum of first two digits
Sum of last two digits $= \alpha$
Case-I : $\alpha = 7$
$2 \times 12 = 24$ ways.

7	0		
0	7		
		1	6
		2	5
		3	4
		4	3
		5	2
		6	1

Case-II : $\alpha = 8$



2×8 ways
$= 16$ ways
Case-III : $\alpha = 9$

$\square$	
\cline { 1 - 2 } 27	
\cline { 1 - 1 } 72	36
	63
	45
	54

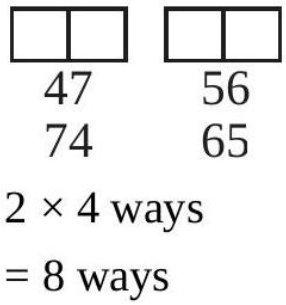
2×8 ways		45	
	54		

$= 16$ ways
Case-IV : $\alpha = 10$

$\square$	$\square$ 37 73

46
64

2×4 ways
8 ways
Case-V : $\alpha = 11$



Ans. $24 + 16 + 16 + 8 + 8 = 72$

Question 112

Let the triangle PQR be the image of the triangle with vertices $(1,3),(3,1)$ and $(2,4)$ in the line $x+2y=2$. If the centroid of $\triangle PQR$ is the point (α,β), then $15(\alpha-\beta)$ is equal to :

A. 24

B. 19

C. 21

D. 22

Correct answer: D

Source solution

Sol. Let ' G ' be the centroid of Δ formed by $(1,3)(3,1)$ & $(2,4)$

$G \cong \left(2, \frac{8}{3}\right)$

Image of G w.r.t. $x+2y-2=0$

$\frac{\alpha-2}{1}=\frac{\beta-\frac{8}{3}}{2}=-2\frac{\left(2+\frac{16}{3}-2\right)}{1+4}$

$=\frac{-2}{5}\left(\frac{16}{3}\right)$

$\Rightarrow \alpha=\frac{-32}{15}+2=\frac{-2}{15}, \beta=\frac{-32\times 2}{15}+\frac{8}{3}=\frac{-24}{15}$

$15(\alpha-\beta)=-2+24=22$

Integrals

Questions 113-113 / 1 selected / 0.3% practice weight / Consolidate

Question 113

If the variable line $3x + 4y = \alpha$ lies between the two $\textbf{circles}(x-1)^2 + (y-1)^2 = 1$ and $(x-9)^2 + (y-1)^2 = 4$, without intercepting a chord on either circle, then the sum of all the integral values of α is

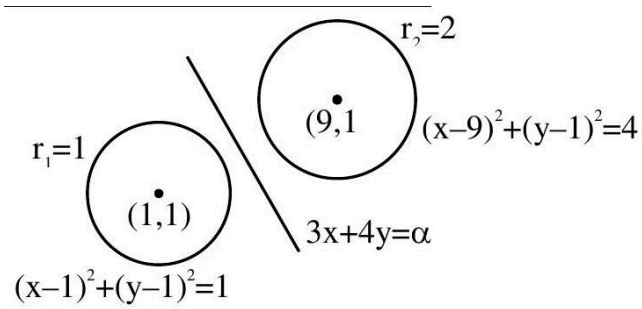
.

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 165

Source solution



Both centres should lie on either side of the line as well as line can be tangent to circle.
$(3 + 4 - \alpha) \cdot (27 + 4 - \alpha) < 0$

$$(7-\alpha).(31-\alpha) < 0 \Rightarrow \alpha \in (7, 31) \quad (1)$$

$d_1 =$ distance of $(1, 1)$ from line
$d_2 =$ distance of $(9, 1)$ from line

$$d_1 \geq r_1 \Rightarrow \frac{|7-\alpha|}{5} 1 \Rightarrow \alpha \in (-\infty, 2] \cup [12, \infty) \quad (2)$$

$d_2 \geq r_2 \Rightarrow \frac{|31-\alpha|}{5} \geq 2 \Rightarrow \alpha \in (-\infty, 21] \cup [41, \infty)$
$(1) \cap (2) \cap (3) \Rightarrow \alpha \in [12, 21]$

Sum of integers $= 165$

Application of Integrals

Questions 114-116 / 3 selected / 1.0% practice weight / Consolidate

Question 114

Let the area of the region $\{(x, y) : |2x - 1| \leq y \leq |x^2 - x|, 0 \leq x \leq 1\}$ be A . Then $(6\,\mathrm{A} + 11)^2$ is equal to

Numerical answer: _____

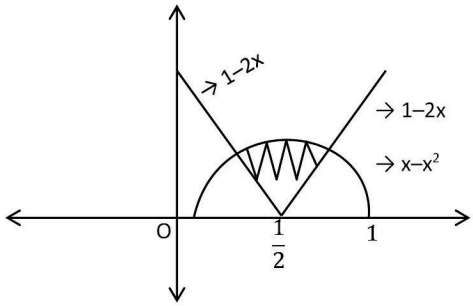
Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 125

Source solution

125

$|2x - 1| \leq y \leq |x^2 - x|, 0 \leq x \leq 1$



$$x - x^2 = 1 - 2x$$
$$x^2 - 3x + 1 = 0$$
$$x = \frac{3 \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2}$$
$$x = \frac{3 - \sqrt{5}}{2} \text{ as } 0 < x < \frac{1}{2}$$
$$\text{Area } = 2\int_{\frac{3-\sqrt{5}}{2}}^{1/2} \left[\left(x - x^2\right) - (1 - 2x)\right] dx$$
$$2\int_{\frac{3-\sqrt{5}}{2}}^{1/2} \left[3x - x^2 - 1\right] dx$$
$$= 2\left[\frac{3x^2}{2} - \frac{x^3}{3} - x\right]_{\frac{3-\sqrt{5}}{2}}^{1/2}$$

For $x = \frac{3-\sqrt{5}}{2}$,

$$x^2 = 3x - 1$$
$$x^3 = 3x^2 - x$$
$$= 3(3x - 1) - x$$
$$= 8x - 3$$
$$\frac{3}{2}x^2 - \frac{1}{3}x^3 - x = \frac{3}{2}(3x - 1) - \frac{1}{3}(8x - 3) - x$$
$$= \frac{9x - 3}{2} - \frac{(8x - 3)}{3} - x$$
$$= \frac{27x - 9 - (16x - 6)}{6} - x$$
$$= \frac{11x - 3}{6} - x$$

$= \frac{5x-3}{6}$

For $x = \frac{1}{2}$,

$\frac{3}{2}x^2 - \frac{1}{3}x^3 - x = \frac{3}{2}\left(\frac{1}{4}\right) - \frac{1}{3}\left(\frac{1}{8}\right) - \frac{1}{2}$

$= \frac{9-1-12}{24}$

$= \frac{-4}{24} = -\frac{1}{6}$

Area $= 2\left[-\frac{1}{6} - \left(\frac{\frac{5(3-\sqrt{5})}{2}-3}{6}\right)\right]$

$= 2\left[-\frac{1}{6} - \left(\frac{15-5\sqrt{5}-6}{12}\right)\right]$

$= 2\left[-\frac{1}{6} - \left(\frac{9-5\sqrt{5}}{12}\right)\right]$

$= 2\left[\frac{-2-9+5\sqrt{5}}{12}\right]$

$= \frac{5\sqrt{5}-11}{6}$

$(6\,A + 11)^2 = (5\sqrt{5})^2 = 125$

Question 115

Let B be the centre of the circle $x^2 + y^2 - 2x + 4y + 1 = 0$.

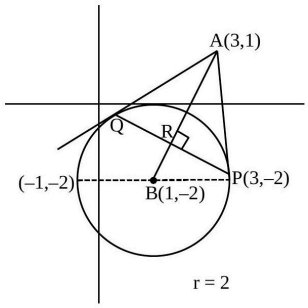
Let the tangents at two points P and Q on the circle intersect at the point $A(3, 1)$. Then $8.\left(\frac{\text{area } \triangle APQ}{\text{area } \triangle BPQ}\right)$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 18

Source solution



$\tan\theta = \frac{3}{2}$

$\frac{\text{Area } \triangle APQ}{\text{Area } \triangle BPQ} = \frac{AR}{RB} = \frac{3\sin\theta}{2\cos\theta} = \frac{9}{4}$

$8\left(\frac{\text{Area } \triangle APQ}{\text{Area } \triangle BPQ}\right) = 18$

Question 116

Consider a hyperbola $\mathbf{H : x^2 - 2y^2 = 4}$. Let the tangent at a point $\mathbf{P(4, \sqrt{6})}$ meet the x -axis at Q and latus rectum at $\mathbf{R\,(x_1, y_1), x_1 > 0}$. If F is a focus of H which is nearer to the point P , then the area of $\mathbf{\triangle QFR}$ is equal to

A. $4\sqrt{6}$

B. $\sqrt{6} - 1$

C. $\frac{7}{\sqrt{6}} - 2$

D. $4\sqrt{6} - 1$

Correct answer: C

Source solution

Sol.

$\frac{x^2}{4} - \frac{y^2}{2} = 1$

$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{\frac{3}{2}}$

$\therefore$ Focus $\mathbf{F(ae, 0) \Rightarrow F(\sqrt{6}, 0)}$

equation of tangent at P to the hyperbola is

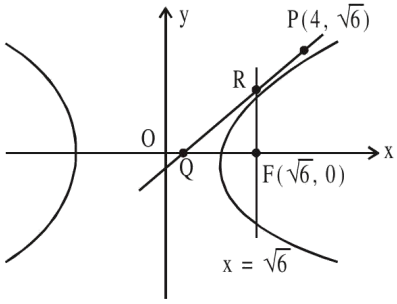
$2x - y\sqrt{6} = 2$

tangent meet x -axis at $\mathbf{Q(1, 0)}$

& latus rectum $x = \sqrt{6}$ at $R\left(\sqrt{6}, \frac{2}{\sqrt{6}}(\sqrt{6} - 1)\right)$

$\therefore$ Area of $\Delta_{QFR} = \frac{1}{2}(\sqrt{6} - 1) \cdot \frac{2}{\sqrt{6}}(\sqrt{6} - 1)$

$= \frac{7}{\sqrt{6}} - 2$



Vector Algebra

Questions 117-147 / 31 selected / 10.3% practice weight / Focus first

Question 117

If the equation of the plane containing the line $x+2y+3z-4=0=2x+y-z+5$ and perpendicular to the plane $\vec{r}=(\hat{i}-\hat{j})+\lambda(\hat{i}+\hat{j}+\hat{k})+\mu(\hat{i}-2\hat{j}+3k)$ is $ax+by+cz=4$, then $(a-b+c)$ is equal to

A. 20

B. 24

C. 22

D. 18

Correct answer: C

Source solution

The given line is the intersection of the planes

$$x+2y+3z-4=0$$

and

$$2x+y-z+5=0.$$

The normals to these planes are

$$\vec{n}_1=(1,2,3), \qquad \vec{n}_2=(2,1,-1).$$

Hence the direction vector of the line is

$$\vec{d}=\vec{n}_1\times\vec{n}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 3\\ 2 & 1 & -1\end{vmatrix}=(-5,\,7,\,-3).$$

The given plane is

$$\vec{r}=(\hat{i}-\hat{j})+\lambda(\hat{i}+\hat{j}+\hat{k})+\mu(\hat{i}-2\hat{j}+3\hat{k}).$$

Its two direction vectors are

$$\vec{a}=(1,1,1), \qquad \vec{b}=(1,-2,3).$$

Therefore, its normal vector is

$$\vec{n}_p = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & -2 & 3 \end{vmatrix} = (5,\ -2,\ -3).$$

Let the required plane have normal vector

$$\vec{N} = (a, b, c).$$

Since the required plane contains the given line,

$$\vec{N} \cdot \vec{d} = 0.$$

Thus,

$$-5a + 7b - 3c = 0.$$

Also, the required plane is perpendicular to the plane whose normal is $(5, -2, -3)$, so

$$\vec{N} \cdot (5, -2, -3) = 0,$$

i.e.,

$$5a - 2b - 3c = 0.$$

Solving

$$-5a + 7b - 3c = 0$$

and

$$5a - 2b - 3c = 0,$$

adding,

$$5b - 6c = 0$$

$$b = \frac{6c}{5}.$$

Substituting into

$$5a - 2b - 3c = 0,$$

$$5a - \frac{12c}{5} - 3c = 0$$

$$25a = 27c.$$

Taking $c = 25$,

$$a = 27, \qquad b = 30.$$

Hence,

$$\vec{N} = (27, 30, 25).$$

Therefore the required plane is

$$27x + 30y + 25z = 4.$$

So,

$$a - b + c = 27 - 30 + 25 = 22.$$

$$\boxed{22}$$

Hence, the correct option is

$$\boxed{\text{(3) } 22}.$$

Question 118

Let a line l pass through the origin and be perpendicular to the lines
$l_1 : \vec{r} = (\hat{i} - 11\hat{j} - 7\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k}), \lambda \in \mathbb{R}$
and $l_2 : \vec{r} = (-\hat{i} + \hat{k}) + \mu(2\hat{i} + 2\hat{j} + \hat{k}), \mu \in \mathbb{R}$.

If P is the point of intersection of l and l_1, and $Q(\alpha, \beta, \gamma)$ is the foot of perpendicular from P on l_2, then $9(\alpha + \beta + \gamma)$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 5

Source solution

Let $\ell = (0\hat{i} + 0\hat{j} + 0\hat{k}) + \gamma(a\hat{i} + b\hat{j} + c\hat{k})$
$= \gamma(a\hat{i} + b\hat{j} + c\hat{k})$
$a\hat{i} + b\hat{j} + c\hat{k} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 2 & 1 \end{vmatrix}$
$= \hat{i}(2 - 6) - \hat{j}(1 - 6) + \hat{k}(2 - 4)$
$= -4\hat{i} - 5\hat{j} - 2\hat{k}$
$\ell = \gamma(-4\hat{i} + 5\hat{j} - 2\hat{k})$
P is intersection of ℓ and ℓ_1
$-4\gamma = 1 + \lambda, 5\gamma = -11 + 2\lambda, -2\gamma = -7 + 3\lambda$
By solving there equation $\gamma = -1, P(4, -5, 2)$
Let $Q(-1 + 2\mu, 2\mu, 1 + \mu)$
$\overrightarrow{PQ} \cdot (2\hat{i} + 2\hat{j} + \hat{k}) = 0$
$-2 + 4\mu + 4\mu + 1 + \mu = 0$
$9\mu = 1$
$\mu = \frac{1}{9}$
$Q\left(\frac{-7}{9}, \frac{2}{9}, \frac{10}{9}\right)$
$9(\alpha + \beta + \gamma) = 9\left(\frac{-7}{9} + \frac{2}{9} + \frac{10}{9}\right)$

$= 5$

Question 119

If the vectors $\vec{a}=\lambda\hat{i}+\mu\hat{j}+4\hat{k}, \vec{b}=2\hat{i}+4\hat{j}-2\hat{k}$ and $\vec{c}=2\hat{i}+3\hat{j}+\hat{k}$ are coplanar and the projection of $\vec{a}$ on the vector $\vec{b}$ is $\sqrt{54}$ units, then the sum of all possible values of $\lambda+\mu$ is equal to

A. 0

B. 24

C. 6

D. 18

Correct answer: B

Source solution

Sol. 2}
Vector $\vec{a}=\lambda\hat{i}+\mu\hat{j}+4\hat{k}, \vec{b}=-2\hat{i}+4\hat{j}-2\hat{k}$ and $\vec{c}=2\hat{i}+3\hat{j}+\hat{k}$ are coplanar then
$[\vec{a}\vec{b}\vec{c}]=0 \quad \Rightarrow |\lambda \quad \mu \quad 4-2 \quad 4 \quad -22 \quad 3 \quad 1|=0$
$\Rightarrow 10\lambda-2\mu-56=0$

$$\Rightarrow 5\lambda-\mu=28 \tag{1}$$

also projection of $\vec{a}$ on the $\vec{b}$ is $\sqrt{54}$ units. then
$\vec{a}.\vec{b}=\sqrt{54}$
$\Rightarrow \frac{-2\lambda+4\mu-8}{\sqrt{24}}=\sqrt{54}$
$\Rightarrow -2\lambda+4\mu-8=36$

$$\Rightarrow -2\lambda+4\mu=44 \tag{2}$$

from (1) and (2)
$\lambda=\frac{26}{3}$ and $\mu=\frac{46}{3}$
$\Rightarrow \lambda+\mu=\frac{26+46}{3}=\frac{72}{3}=24$

Question 120

Let $\vec{a}=4\hat{i}+3\hat{j}$ and $\vec{b}=3\hat{i}-4\hat{j}+5\hat{k}$. If $\vec{c}$ is a vector such that $\vec{c}\cdot(\vec{a}\times\vec{b})+25=0, \vec{c}\cdot(\hat{i}+\hat{j}+\hat{k})=4$, and projection of $\vec{c}$ on $\vec{a}$ is 1 , then the projection of $\vec{c}$ on $\vec{b}$ equals

A. $\frac{1}{5}$

B. $\frac{5}{\sqrt{2}}$

C. $\frac{3}{\sqrt{2}}$

D. $\frac{1}{\sqrt{2}}$

Correct answer: B

Source solution

Sol. (2)
Let $\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$
$\vec{c}\cdot(\mathrm{i}+\mathrm{j}+\mathrm{k})=4$

$$\vec{c}.\vec{a}\times\vec{b} \tag{i}$$

$c_1+c_2+c_3=4$
$|c_1 \quad c_2 \quad c_3 4 \quad 3 \quad 03 \quad -4 \quad 5|=-25$
$\Rightarrow c_1(15-0)-c_2(20-0)+c_3(-16-9)=-25$
$\Rightarrow 15c_1-20c_2-25c_3=-25$

$$\Rightarrow 3c_1-4c_2-5c_3=-5 \tag{2}$$

Proj. of $\vec{c}$ on $\vec{a}=\frac{\vec{a}\cdot\vec{c}}{|\vec{a}|}=1$
$\Rightarrow \frac{(4\hat{i}+3\hat{j})\left(c_1\hat{i}+c_2\hat{j}+c_3\hat{k}\right)}{\sqrt{16+9}}=1$
$\Rightarrow 4c_1+3c_2=5$
$\Rightarrow 4c_1=5-3c_2$

$$\Rightarrow c_1=\frac{5-3c_2}{4} \tag{3}$$

Eq^n. (1) & (3)
$\frac{5-3c_2}{4}+c_2+c_3=4$
Eq ^{n}. (2) & (3)

$$3\left(\frac{5-3c_2}{4}\right)-4c_2-5c_3=-5 \qquad 15-9c2-16c2-20c_3=-20-25c_2-20c_3=-35 \tag{5}$$

$$5-3c_2+4c_2+4c_3=16$$

$c_2+4c_3=11$

$\mathbf{Eq^n}$. (4) & (5)
$\mathbf{c_2 = 11 - 4c_3}$
$\mathbf{-25c_2 - 20c_3 = -35}$
$\mathbf{-25\,(11 - 4c_3) - 20}$
$\mathbf{c_2 = 11 - 4 \times 3}$
$\mathbf{5\,(11 - 4c_3) + 4c_3 = 7}$
$\mathbf{= 11 - 12}$
$\mathbf{55 - 20c_3 + 4c_3 = 7}$
$\mathbf{c_2 = -1}$
$\mathbf{-16c_3 = -48}$
$\mathbf{c_1 = \frac{5-3c_2}{4}}$
$\mathbf{c_3 = 3}$
$\mathbf{= \frac{5-3(-1)}{4}}$
$\mathbf{\vec{c} = 2\hat{i} - \hat{j} + 3\hat{k}}$
$\mathbf{c_1 = 2}$
Projection of $\mathbf{\vec{c}}$ on $\mathbf{\vec{b}} = \left|\frac{\vec{c}\cdot\vec{b}}{|\vec{b}|}\right|$
$\mathbf{\Rightarrow \left|\frac{(2\hat{i}-\hat{j}+3\hat{k})\cdot(3\hat{i}-4\hat{j}+5\hat{k})}{\sqrt{9+16+25}}\right|}$
$\mathbf{\Rightarrow \left|\frac{6+4+15}{5\sqrt{2}}\right| = \frac{25}{5\sqrt{2}} = \frac{5}{\sqrt{2}}}$

Question 121

Let $\mathbf{A, B, C}$ be three points whose position vectors respectively are:
$\mathbf{\vec{a} = \hat{i} + 4\hat{j} + 3\hat{k}}$
$\mathbf{\vec{b} = 2\hat{i} + \alpha\hat{j} + 4\hat{k}, \alpha \in \mathbb{R}}$
$\mathbf{\vec{c} = 3\hat{i} - 2\hat{j} + 5\hat{k}}$
If $\boldsymbol{\alpha}$ is the smallest positive integer for which $\mathbf{\rightarrow\rightarrow\rightarrow a, b, c}$ are non-collinear, then the length of the median, in $\mathbf{\triangle ABC}$, through A is:

A. $\frac{\sqrt{82}}{2}$

B. $\frac{\sqrt{62}}{2}$

C. $\frac{\sqrt{69}}{2}$

D. $\frac{\sqrt{66}}{2}$

Correct answer: A

Source solution

Sol. $\mathbf{\overrightarrow{AB} \| \overrightarrow{AC}}$ if $\mathbf{\frac{1}{2} = \frac{\alpha-4}{-6} = \frac{1}{2} \Rightarrow \alpha = 1}$
$\mathbf{\vec{a}, \vec{b}, \vec{c}}$ are non-collinear for $\boldsymbol{\alpha = 2}$ (smallest positive integer)

Mid-point of $\mathbf{BC = M\left(\frac{5}{2}, 0, \frac{9}{2}\right)}$
$\mathbf{AM = \sqrt{\frac{9}{4} + 16 + \frac{9}{4}} = \frac{\sqrt{82}}{2}}$

Question 122

Let $\vec{a} = 4\hat{i} + 3\hat{j} + 5\hat{k}$ and $\vec{\beta} = \hat{i} + 2\hat{j} - 4\hat{k}$. Let $\vec{\beta}_1$ be parallel to $\vec{a}$ and $\vec{\beta}_2$ be perpendicular to $\vec{a}$. If $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, then the value of $5\vec{\beta}_2 \cdot (\hat{i} + \hat{j} + \hat{k})$ is

A. 7

B. 9

C. 6

D. 11

Correct answer: A

Source solution

$$\vec{\beta}_1 = \frac{\vec{\alpha} \cdot \vec{\beta}}{|\vec{\alpha}|} \hat{\alpha}$$

$$= \left(\frac{4 + 6 - 20}{\sqrt{16 + 9 + 25}} \right) \frac{(4, 3, 5)}{\sqrt{50}}$$

$$= \frac{-10}{50} (4, 3, 5)$$

$$\vec{\beta}_1 = \left(-\frac{4}{5}, -\frac{3}{5}, -1 \right)$$

Given,

$$\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2 = (1, 2, -4)$$

Hence,

$$\vec{\beta}_2 = \vec{\beta} - \vec{\beta}_1$$

$$= \left(1 + \frac{4}{5}, 2 + \frac{3}{5}, -4 + 1 \right)$$

$$= \left(\frac{9}{5}, \frac{13}{5}, -3 \right)$$

Therefore,

$$5\vec{\beta}_2 = (9, 13, -15)$$

Now,

$$5\vec{\beta}_2 \cdot (1, 1, 1) = 9 + 13 - 15$$

$$= 7$$

$$\boxed{7}$$

Question 123

If the components of $\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ along and perpendicular to $\vec{b} = 3\hat{i} + \hat{j} - \hat{k}$ respectively, are $\frac{16}{11}(3\hat{i} + \hat{j} - \hat{k})$ and $\frac{1}{11}(-4\hat{i} - 5\hat{j} - 17\hat{k})$, then $\alpha^2 + \beta^2 + \gamma^2$ is equal to :

A. 23

B. 18

C. 16

D. 26

Correct answer: D

Source solution

Sol. let
$\vec{a}_{11}$ = component of $\vec{a}$ along $\vec{b}$
$\vec{a}_1$ = component of $\vec{a}$ perpendicular to $\vec{b}$
$\vec{a}_{11} = \frac{16}{11}(3\hat{i} + \hat{j} - \hat{k})$
$\vec{a}_1 = \frac{1}{11}(-4\hat{i} - 5\hat{j} - 17\hat{k})$
$\because \vec{a} = \vec{a}_{11} + \vec{a}_1$
$\therefore \vec{a} = \frac{16}{11}(3\hat{i} + \hat{j} - \hat{k}) + \frac{1}{11}(-4\hat{i} - 5\hat{j} - 17\hat{k})$

$$= \frac{44}{11}\hat{i} + \frac{11}{11}\hat{j} - \frac{33}{11}\hat{k}$$

$\vec{a} = 4\hat{i} + \hat{j} - 3\hat{k}$
$\alpha = 4 \quad \beta = 1 \quad \gamma = -3$
$\alpha^2 + \beta^2 + \gamma^2 = 16 + 1 + 9 = 26$

Question 124

Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero vectors such that $\vec{b}$ and $\vec{c}$ are non-collinear if $\vec{a} + 5\vec{b}$ is collinear with $\vec{c}, \vec{b} + 6\vec{c}$ is collinear with $\vec{a}$ and $\vec{a} + \alpha\vec{b} + \beta\vec{c} = \vec{0}$, then $\alpha + \beta$ is equal to

A. 35

B. 30

C. -30

D. -25

Correct answer: A

Source solution

Sol. $\vec{a} + 5\vec{b} = \lambda\vec{c}$
$\vec{b} + 6\vec{c} = \mu\vec{a}$

Eliminating $\vec{a}$
$\lambda\vec{c} - 5\vec{b} = \frac{6}{\mu}\vec{c} + \frac{1}{\mu}\vec{b}$
$\therefore \mu = \frac{-1}{5}, \lambda = -30$
$\alpha = 5, \beta = 30$

Question 125

Let $\vec{a}=\hat{i}+5\hat{j}+\alpha\hat{k}, \vec{b}=\hat{i}+3\hat{j}+\beta\hat{k}$ and $\vec{c}=-\hat{i}+2\hat{j}-3\hat{k}$ be three vectors such that, $|\vec{b}\times\vec{c}|=5\sqrt{3}$ and $\vec{a}$ is perpendicular to $\vec{b}$. Then the greatest amongst the values of $|\vec{a}|^2$ is .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 90

Source solution

since, $\vec{a}\cdot\vec{b}=0$

$$1+15+\alpha\beta=0\Rightarrow\alpha\beta=-16 \quad (1)$$

Also,

$|\vec{b}\times\vec{c}|^2=75\Rightarrow(10+\beta^2)14-(5-3\beta)^2=75$

$\Rightarrow 5\beta^2+30\beta+40=0$

$\Rightarrow \beta=-4,-2$

$\Rightarrow \alpha=4,8$

$\Rightarrow |\vec{a}|^2_{max}=(26+\alpha^2)_{max}=90$

Question 126

Let $\vec{a}=3\hat{i}+2\hat{j}+\hat{k}, \vec{b}=2\hat{i}-\hat{j}+3\hat{k}$ and $\vec{c}$ be a vector such that $(\vec{a}+\vec{b})\times\vec{c}=2(\vec{a}\times\vec{b})+24\hat{j}-6\hat{k}$ and $(\vec{a}-\vec{b}+\hat{i})\cdot\vec{c}=-3$. Then $|\vec{c}|^2$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 38

Source solution

- $(\vec{a}+\vec{b})\times\vec{c}=2(\vec{a}\times\vec{b})+24\hat{j}-6\hat{k}$

$(5\hat{i}+\hat{j}+4\hat{k})\times\vec{c}=2(7\hat{i}-7\hat{j}-7\hat{k})+24\hat{j}-6\hat{k}$

$\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\5&1&4\\x&y&z\end{vmatrix}=14\hat{i}+10\hat{j}-20\hat{k}$

$\Rightarrow \hat{i}(z-4y)-\hat{j}(5z-4x)+\hat{k}(5y-x)=14\hat{i}+10\hat{j}-20\hat{k}$

$z-4y=14, 4x-5z=10, 5y-x=-20$

$(a-b+i)\cdot\vec{c}=-3$

$(2\hat{i}+3\hat{j}-2\hat{k})\cdot\vec{c}=-3$

$2x+3y-2z=-3$

$\therefore x=5, y=-3, z=2$

$|\vec{c}|^2=25+9+4=38$

Question 127

Let three vectors $\vec{a} = \alpha\hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 5\hat{i} + 3\hat{j} + 4\hat{k}, \vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$ from a triangle such that $\vec{c} = \vec{a} - \vec{b}$ and the area of the triangle is $5\sqrt{6}$. if α is a positive real number, then $|\vec{c}|^2$ is :

A. 16

B. 14

C. 12

D. 10

Correct answer: B

Source solution

$$\vec{c} = \vec{a} - \vec{b}$$

$$\Rightarrow (x, y, z) = (\alpha - 5,\ 1,\ -2)$$

Hence,

$$x = \alpha - 5, \qquad y = 1, \qquad z = -2 \tag{1}$$

Area of the triangle is given as

$$\Delta = 5\sqrt{6}$$

Using

$$\frac{1}{2}|\vec{a} \times \vec{c}| = 5\sqrt{6},$$

we get

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 4 & 2 \\ x & 1 & -2 \end{vmatrix} = 10\sqrt{6}$$

$$\Rightarrow \left|-10\hat{i} - \hat{j}(-2\alpha - 2x) + \hat{k}(\alpha - 4x)\right| = 10\sqrt{6}$$

Substituting $x = \alpha - 5$,

$$(2\alpha + 2\alpha - 10)^2 + (\alpha - 4\alpha + 20)^2 = (10\sqrt{6})^2$$

$$\Rightarrow (4\alpha - 10)^2 + (20 - 3\alpha)^2 = 600$$

$$\Rightarrow 16\alpha^2 - 80\alpha + 100 + 9\alpha^2 - 120\alpha + 400 = 600$$

$$\Rightarrow 25\alpha^2 - 200\alpha = 0$$

$$\Rightarrow 25\alpha(\alpha - 8) = 0$$

$$\Rightarrow \alpha = 8$$

(since α is positive)

$$x = \alpha - 5 = 3$$

Therefore,

$$|\vec{c}|^2 = x^2 + y^2 + z^2$$

$$= 3^2 + 1^2 + (-2)^2$$

$$= 9 + 1 + 4$$

$$= 14$$

$$\boxed{14}$$

Question 128

Let $\vec{a} = \alpha\hat{i} + 3\hat{j} - \hat{k}, \vec{b} = 3\hat{i} - \beta\hat{j} + 4\hat{k}$ and $\vec{c} = \hat{i} + 2\hat{j} - 2\hat{k}$ where $\alpha, \beta \in \mathbf{R}$, be three vectors. If the projection of $\vec{a}$ on $\vec{c}$ is $\frac{10}{3}$ and $\vec{b} \times \vec{c} = -6\hat{i} + 10\hat{j} + 7\hat{k}$, then the value of $\alpha + \beta$ equal to:

A. 3

B. 4

C. 5

D. 6

Correct answer: A

Source solution

Sol. $\frac{\vec{a}\cdot\vec{c}}{|\vec{c}|} = \frac{10}{3}$
$\Rightarrow \frac{\alpha+6+2}{\sqrt{1+4+4}} = \frac{10}{3} \Rightarrow \alpha = 2$
and $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k}3 \\ -\beta & 41 & 2 \\ & -2 & \end{vmatrix} = -6\hat{i} + \hat{j} + \hat{k}$
$\Rightarrow 2\beta - 8 = -6 \Rightarrow \beta = 1$
$\Rightarrow \alpha + \beta = 3$

Question 129

Let the position vectors of three vertices of a triangle be $4\vec{p}+\vec{q}-3\vec{r}, -5\vec{p}+\vec{q}+2\vec{r}$ and $2\vec{p}-\vec{q}+2\vec{r}$. If the position vectors of the orthocenter and the circumcenter of the triangle are $\frac{\vec{p}+\vec{q}+\vec{r}}{4}$ and $\alpha\vec{p}+\beta\vec{q}+\gamma\vec{r}$ respectively, then $\alpha+2\beta+5\gamma$ is equal to:

A. 3

B. 1

C. 6

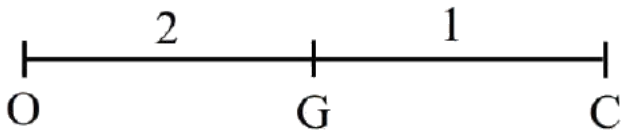
D. 4

Correct answer: A

Source solution

Sol. We know that

O (orthocentre) $\frac{\vec{p}+\vec{q}+\vec{r}}{4}$
C (circum centre) $\alpha\vec{p}+\beta\vec{q}+\gamma\vec{r}$
$\mathbf{C}$(centroid) $=\frac{\vec{p}+\vec{q}+\vec{r}}{3}$
by relation
$\Rightarrow 2(\alpha\vec{p}+\beta\vec{q}+\gamma\vec{r})+\frac{\vec{p}+\vec{q}+\vec{r}}{4}=3\left(\frac{\vec{p}+\vec{q}+\vec{r}}{3}\right)$
$\Rightarrow 8(\alpha\vec{p}+\beta\vec{q}+\gamma\vec{r})=3(\vec{p}+\vec{q}+\vec{r})$
$\Rightarrow 8\alpha=3, 8\beta=3, 8\gamma=3$
$\alpha=\frac{3}{8}, \beta=\frac{3}{8}, \gamma=\frac{3}{8}$
$\therefore \alpha+2\beta+3\gamma$
$\frac{3}{8}+\frac{6}{8}+\frac{15}{8}=\frac{24}{8}=3$



Question 130

Let $\vec{x}$ be a vector in the plane containing vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$. If the vector $\vec{x}$ is perpendicular to $(3\hat{i} + 2\hat{j} - \hat{k})$ and its projection on $\vec{a}$ is $\frac{17\sqrt{6}}{2}$, then the value of $|\vec{x}|^2$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 486

Source solution

Let $\vec{x} = \lambda\vec{a} + \mu\vec{b}$ (λ and μ are scalars)
$\vec{x} = \hat{i}(2\lambda + \mu) + \hat{j}(2\mu - \lambda) + \hat{k}(\lambda - \mu)$
Since $\vec{x} \cdot (3\hat{i} + 2\hat{j} - \hat{k}) = 0$

$$3\lambda + 8\mu = 0 \quad (1)$$

Also Projection of $\vec{x}$ on $\vec{a}$ is $\frac{17\sqrt{6}}{2}$
$\frac{\vec{x}\cdot\vec{a}}{|\vec{a}|} = \frac{17\sqrt{6}}{2}$

$$6\lambda - \mu = 51 \quad (2)$$

From (1) and (2)
$\lambda = 8, \mu = -3$
$\vec{x} = 13\hat{i} - 14\hat{j} + 11\hat{k}$
$|\vec{x}|^2 = 486$ Ans.
Official Ans. by NTA (486)

Question 131

Let $\vec{a}=2\hat{i}-\hat{j}+3\hat{k}, \vec{b}=3\hat{i}-5\hat{j}+\hat{k}$ and $\vec{c}$ be a vector such that $\vec{a}\times\vec{c}=\vec{c}\times\vec{b}$ and $(\vec{a}+\vec{c})\cdot(\vec{b}+\vec{c})=168$. Then the maximum value of $|\vec{c}|^2$ is :

A. 77

B. 462

C. 308

D. 154

Correct answer: C

Source solution

Sol.

$\vec{a}=2\hat{i}-\hat{j}+3\hat{k}$

$\vec{b}=3\hat{i}-5\hat{j}+\hat{k}$

$\vec{a}\times\vec{c}=\vec{c}\times\vec{b}$

$\vec{a}\times\vec{c}+\vec{b}\times\vec{c}=0$

$(\vec{a}+\vec{b})\times\vec{c}=0$

$\Rightarrow\vec{c}=\lambda(\vec{a}+\vec{b})$

$$\vec{c}=\lambda(5\hat{i}-6\hat{j}+4\hat{k}) \quad (1)$$

$|\vec{c}|^2=\lambda^2(25+36+16)$

$|\vec{c}|^2=77\lambda^2$

$(\vec{a}+\vec{c})\cdot(\vec{b}+\vec{c})=168$

$\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}+\vec{c}\cdot\vec{b}+|\vec{c}|^2=168$

$14+\vec{c}\cdot(\vec{a}+\vec{b})+77\lambda^2=168$

using equation (1)

$\lambda|5\hat{i}-6\hat{j}+4\hat{k}|^2+77\lambda^2=154$

$77\lambda+77\lambda^2-154=0$

$\lambda^2+\lambda-2=0$

$\lambda=-2,1$

$\therefore$ Maximum value of $|\vec{c}|^2$ occurs when $\lambda=-2$ $|\vec{c}|^2=77\lambda^2$

$=77\times4$

$=308$

Question 132

Let $\vec{a}=3\hat{i}-\hat{j}+2\hat{k}, \vec{b}=\vec{a}\times(\hat{i}-2\hat{k})$ and $\vec{c}=\vec{b}\times\hat{k}$. Then the projection of $\vec{c}-2\hat{j}$ on $\vec{a}$ is:

A. $3\sqrt{7}$

B. $\sqrt{14}$

C. $2\sqrt{14}$

D. $2\sqrt{7}$

Correct answer: C

Source solution

Sol. $\vec{b}=\vec{a}\times(\hat{i}-3\hat{k})$

$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}3 \\ -1 & 21 & 0 \\ & -2 & \end{vmatrix}=2\hat{i}+8\hat{j}+\hat{k}$

$\vec{c}=\vec{b}\times\hat{k}=8\hat{i}-2\hat{j}$

$\vec{c}-2\hat{j}=8\hat{i}-4\hat{j}$

Projection of $(\hat{i}-2\hat{j})$ on $\vec{a}$

$(\vec{c}-2\hat{j})\cdot\hat{a}=\frac{\langle 8,-4,0\rangle\cdot\langle 3,-1,2\rangle}{\sqrt{14}}$

$=\frac{28}{\sqrt{14}}=2\sqrt{14}$

Question 133

Let $\vec{a}=2\hat{i}+\lambda_1\hat{j}+3\hat{k}, \vec{b}=4\hat{i}+(3-\lambda_2)\hat{j}+6\hat{k}$ and $\vec{c}=3\hat{i}+6\hat{j}+(\lambda_3-1)\hat{k}$ be three vectors such that $\vec{b}=2\vec{a}$ and $\vec{a}$ is perpendicular to $\vec{c}$. Then a possible value of ($\lambda_1, \lambda_2, \lambda_3$) is :-

A. $\left(\frac{1}{2}, 4, -2\right)$

B. $\left(-\frac{1}{2}, 4, 0\right)$

C. $(1, 3, 1)$

D. $(1, 5, 1)$

Correct answer: B

Source solution

Sol. $4\hat{i}+(3-\lambda_2)\hat{j}+6\hat{k}=4\hat{i}+2\lambda_1\hat{j}+6\hat{k}$

$$\Rightarrow 3-\lambda_2=2\lambda_1 \Rightarrow 2\lambda_1+\lambda_2=3 \quad (1)$$

Given $\vec{a}.\vec{c}=0$

$\Rightarrow 6+6\lambda_1+3(\lambda_3-1)=0$

$$\Rightarrow 2\lambda_1+\lambda_3=-1 \quad (2)$$

Now $(\lambda_1, \lambda_2, \lambda_3)=(\lambda_1, 3-2\lambda_1, -1-2\lambda_1)$

Option (2) is correct

Question 134

Let $\vec{a}=\hat{i}+\hat{j}-\hat{k}$ and $\vec{c}=2\hat{i}-3\hat{j}+2\hat{k}$. Then the number of vectors $\vec{b}$ such that $\vec{b}\times\vec{c}=\vec{a}$ and $|\vec{b}|\in\{1,2,.....,10\}$ is :

A. 0

B. 1

C. 2

D. 3

Correct answer: A

Source solution

Sol. $\vec{a}=i+j-k$
$\vec{c}=2i-3j+2k$
$\vec{b}\times\vec{c}=\vec{a}$
$|\vec{b}|\in\{1,2.....10\}$
$\because\vec{b}\times\vec{c}=\vec{a}$
$\Rightarrow\vec{a}$ is perpendicular to $\vec{b}$ as well as $\vec{a}$ is perpendicular to $\vec{C}$

Now $\vec{a}\cdot\vec{c}=2-3-2=-3\neq0$
This $\vec{b}\times\vec{c}=\vec{a}$ is not possible.
No. of vectors $\vec{b}=0$

Question 135

The shortest distance between the lines $\frac{x-3}{2}=\frac{y-2}{3}=\frac{z-1}{-1}$ and $\frac{x+3}{2}=\frac{y-6}{1}=\frac{z-5}{3}$ is :

A. $\frac{18}{\sqrt{5}}$

B. $\frac{22}{3\sqrt{5}}$

C. $\frac{46}{3\sqrt{5}}$

D. $6\sqrt{3}$

Correct answer: A

Source solution

Sol. $\frac{x-3}{2}=\frac{y-2}{3}=\frac{z-1}{-1}$
$\frac{x+3}{2}=\frac{y-6}{1}=\frac{z-5}{3}$
$A=(3,2,1)$
$B=(-3,6,5)$
$\overrightarrow{n_1}=2\hat{i}+3\hat{j}-\hat{k}$
$\overrightarrow{n_2}=2\hat{i}+\hat{j}-3\hat{k}$
$\overrightarrow{BA}=6\hat{i}-4\hat{j}-4\hat{k}$

SHORTEST DISTANCE $=\frac{\left[\overrightarrow{BA}\,\overrightarrow{n_1}\,\overrightarrow{n_2}\right]}{\left|\overrightarrow{n_1}\times\overrightarrow{n_2}\right|}$

$\overrightarrow{n_1}\times\overrightarrow{n_2}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}2\\3&-12&1\\3\end{vmatrix}$
$=10\hat{i}-8\hat{j}-4\hat{k}$
$\left[\overrightarrow{BA}\ \ \overrightarrow{n_1}\ \ \overrightarrow{n_2}\right]=60+32+16=108$
$\left|\overrightarrow{n_1}\times\overrightarrow{n_2}\right|=\sqrt{100+64+16}=\sqrt{180}$
S.D $=\frac{108}{\sqrt{180}}=\frac{108}{6\sqrt{5}}=\frac{18}{\sqrt{5}}$

Question 136

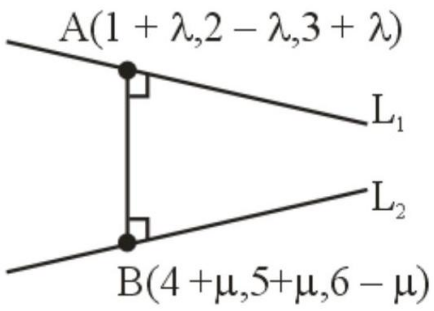
Let the line of the shortest distance between the lines
$\mathbf{L_1 : \vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})}$ and
$\mathbf{L_2 : \vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(\hat{i} + \hat{j} - \hat{k})}$
intersect $\mathbf{L_1}$ and $\mathbf{L_2}$ at P and Q respectively. If (α, β, γ) is the midpoint of the line segment PQ , then $\mathbf{2(\alpha + \beta + \gamma)}$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 21

Source solution



$\vec{b} = \hat{i} - \hat{j} + \hat{k}$ (DR's of $\mathbf{L_1}$)
$\vec{d} = \hat{i} + \hat{j} - \hat{k}$ (DR's of $\mathbf{L_2}$)
$\vec{b} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$
$= 0\hat{i} + 2\hat{j} + 2\hat{k}$ (DR's of Line perpendicular to
$\mathbf{L_1}$ and $\mathbf{L_2}$)
DR of AB line
$= (0, 2, 2) = (3 + \mu - \lambda, 3 + \mu + \lambda, 3 - \mu - \lambda)$
$\frac{3+\mu-\lambda}{0} = \frac{3+\mu+\lambda}{2} = \frac{3-\mu-\lambda}{2}$

Solving above equation we get $\mu = -\frac{3}{2}$ and $\lambda = \frac{3}{2}$
point $\mathbf{A} = \left(\frac{5}{2}, \frac{1}{2}, \frac{9}{2}\right)$
$\mathbf{B} = \left(\frac{5}{2}, \frac{7}{2}, \frac{15}{2}\right)$
Point of AB $= \left(\frac{5}{2}, 2, 6\right) = (\alpha, \beta, \gamma)$
$2(\alpha + \beta + \gamma) = 5 + 4 + 12 = 21$

Question 137

Let $\vec{\alpha} = 3\hat{i} + \hat{j}$ and $\vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$. If $\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$, then $\vec{\beta}_1 \times \vec{\beta}_2$ is equal to:

A. $\frac{1}{2}(3\hat{i} - 9\hat{j} + 5\hat{k})$

B. $\frac{1}{2}(-3\hat{i} + 9\hat{j} + 5\hat{k})$

C. $-3\hat{i} + 9\hat{j} + 5\hat{k}$

D. $3\hat{i} - 9\hat{j} - 5\hat{k}$

Correct answer: B

Source solution

Sol. $\vec{\beta} = \overrightarrow{\beta_1} - \overrightarrow{\beta_2}$

$$\therefore \quad \overrightarrow{\beta_2} \cdot \vec{\alpha} = 0 \qquad \text{(i)}$$

and Let $\overrightarrow{\beta_1} = \lambda\vec{\alpha}$

$\vec{\alpha} \cdot \vec{\beta} = \vec{\alpha} \cdot \overrightarrow{\beta_1} - \vec{\alpha} \cdot \overrightarrow{\beta_2}$

$\Rightarrow 5 = \lambda\alpha^2$

$\Rightarrow 5 = \lambda \times 10$

$\Rightarrow \lambda = \frac{1}{2}$

$\therefore \quad \overrightarrow{\beta_1} = \frac{\vec{\alpha}}{2}$

Cross product with $\overrightarrow{\beta_1}$ in equation (i)

$\Rightarrow \vec{\beta} \times \overrightarrow{\beta_1} = -\overrightarrow{\beta_2} \times \overrightarrow{\beta_1}$

$\Rightarrow \vec{\beta} \times \overrightarrow{\beta_1} = \overrightarrow{\beta_1} \times \overrightarrow{\beta_2} = \frac{(\vec{\beta} \times \vec{\alpha})}{2}$

$\Rightarrow \overrightarrow{\beta_1} \times \overrightarrow{\beta_2} = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k}2 \\ -1 & 33 & 1 \\ 0 & & \end{vmatrix}$

$$= \frac{1}{2}[-3\hat{i} - \hat{j}(-9) + \hat{k}(5)] \qquad = \frac{1}{2}[-3\hat{i} + 9\hat{j} + 5\hat{k}]$$

Question 138

Let $\vec{a}=2\hat{i}+3\hat{j}+4\hat{k}, \vec{b}=2\hat{i}-2\hat{j}-2\hat{k}$ and $\vec{c}=-\hat{i}+4\hat{j}+3\hat{k}$. If $\vec{d}$ is a vector perpendicular to both $\vec{b}$ and $\vec{c}$ and $\vec{a}\cdot\vec{d}=18$, Then $|\vec{a}\times\vec{d}|^2$ is equal to

A. 640

B. 760

C. 680

D. 720

Correct answer: D

Source solution

Sol. $a=\lambda(b\times c)$

$\vec{b}\times\vec{c}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}1 \\ -2 & -2-1 & 4 \\ 3\end{vmatrix}=2\hat{i}-\hat{j}+2\hat{k}$

$\vec{d}=\lambda(2\hat{i}-\hat{j}+2\hat{k})$

$\vec{a}\cdot\vec{d}=18$

$\lambda=2$

So $\vec{d}=2(2\hat{i}-\hat{j}+2\hat{k})$

$\vec{d}\times\vec{a}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}4 \\ -2 & 42 & 3 \\ 4\end{vmatrix}=-20\hat{i}-8\hat{j}+16\hat{k}$

$|\vec{d}\times\vec{a}|^2=720$

Question 139

Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}$ and $\vec{c}=\hat{j}-\hat{k}$ be three vectors such that $\vec{a}\times\vec{b}=\vec{c}$ and $\vec{a}.\vec{b}=1$. If the length of projection vector of the vector $\vec{b}$ on the vector $\vec{a}\times\vec{c}$ is l, then the value of $3l^2$ is equal to ___.

Numerical answer: _____

Enter the requested integer (0 decimal places).

Correct answer: 2

Source solution

$\vec{a}\times\vec{b}=\mathbf{c}$ Take Dot with $\vec{c}$

$$(\vec{a}\times\vec{b})\cdot\vec{c}=|\vec{c}|^2=2$$

Projection of $\vec{b}$ or $\vec{a}\times\vec{c}=\ell$

$$\frac{|\vec{b}\cdot(\vec{a}\times\vec{c})|}{|\vec{a}\times\vec{c}|}\ell$$

$$\therefore\quad \ell=\frac{2}{\sqrt{6}}\Rightarrow\ell^2=\frac{4}{6}$$

$$3\ell^2=2$$

Question 140

Let P be a plane passing through the points (1, 0, 1), (1, -2, 1) and (0, 1, -2). let a vector $\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ be such that $\vec{a}$ is parallel to the plane P , perpendicular to $(\hat{i} + 2\hat{j} + 3\hat{k})$ and $\vec{a}(\hat{i} + \hat{j} + 2\hat{k}) = 2$, then $(\alpha - \beta + \gamma)^2$ equals ___.

Numerical answer: _____

Enter the requested integer (0 decimal places).

Correct answer: 81

Source solution

Equation of plane :

$$\begin{vmatrix} x-1 & y-0 & z-1 \\ 1-1 & 2 & 1-1 \\ 1-0 & 0-1 & 1+2 \end{vmatrix} = 0$$

$$\Rightarrow 3x - z - 2 = 0$$

$$\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k} \parallel \text{to } 3x - z - 2 = 0$$

$$\Rightarrow 3\alpha - \gamma = 0$$

$$\vec{a} \perp i + 2\hat{j} + 3\hat{k}$$

$$\Rightarrow \alpha + 2\beta + 3\gamma = 0$$

$$\vec{a}(\hat{i} + \hat{j} + 3\hat{k}) = 0$$

$$\Rightarrow \alpha + \beta + 2\gamma = 2$$

on solving 1,2 & 3

$$\alpha = 1, \beta = -5, \gamma = 3$$

So $(\alpha - \beta + \gamma) = 81$

Question 141

Let $\vec{\alpha} = (\lambda - 2)\vec{a} + \vec{b}$ and $\vec{\beta} = (4\lambda - 2)\vec{a} + 3\vec{b}$ be two given vectors where vectors $\vec{a}$ and $\vec{b}$ are non-collinear. The value of λ for which vectors $\vec{\alpha}$ and $\vec{\beta}$ are collinear, is :

A. -3

B. 4

C. 3

D. -4

Correct answer: D

Source solution

Sol. $\vec{\alpha} = (\lambda - 2)\vec{\alpha} + \vec{b}$

$\vec{\beta} = (4\lambda - 2)\vec{\alpha} + 3\vec{b}$

$\frac{\lambda-2}{4\lambda-2} = \frac{1}{3}$

$3\lambda - 6 = 4\lambda - 2$

$\lambda = -4$

$\therefore$ Option (4)

Question 142

Let $\hat{a}, \hat{b}$ be unit vectors. If $\vec{c}$ be a vector such that the angle between $\hat{a}$ and $\vec{c}$ is $\frac{\pi}{12}$, and $\hat{b} = \vec{c} + 2(\vec{c} \times \hat{a})$, then $|\vec{6c}|^2$ is equal to

A. $6(3-\sqrt{3})$

B. $3+\sqrt{3}$

C. $6(3+\sqrt{3})$

D. $6(\sqrt{3}+1)$

Correct answer: C

Source solution

Sol. $|\hat{b}|^2 = |\vec{c} + 2(\vec{c} \times \hat{a})|^2$
$|\hat{b}|^2 = |c|^2 + 4|\vec{c} \times \hat{a}|^2 + 4\vec{c}.(\vec{c} \times \hat{a})$
$1 = |c|^2 + 4|c|^2 \sin^2 \frac{\pi}{12} + 0$
$1 = |c|^2 + 4|c|^2 \left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2$
$|c|^2 = \frac{1}{3-\sqrt{3}} = \frac{3+\sqrt{3}}{6}$
So $6^2|c|^2 = 6(3+\sqrt{3})$

Question 143

Let $\vec{a} = 6\hat{i} + 9\hat{j} + 12\hat{k}, \vec{b} = \alpha\hat{i} + 11\hat{j} - 2\hat{k}$ and $\vec{c}$ be vectors such that $\vec{a} \times \vec{c} = \vec{a} \times \vec{b}$. If $\vec{a}.\vec{c} = -12, \vec{c} \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 5$, then $\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k})$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 11

The source provides a brief answer only. A detailed worked solution is not available for this question.

Source solution

Question 144

Let $\vec{a}$ be a vector which is perpendicular to the vector $3\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k}$. If $\vec{a} \times (2\hat{i} + \hat{k}) = 2\hat{i} - 13\hat{j} - 4\hat{k}$, then the projection of the vector $\vec{a}$ on the vector $2\hat{i} + 2\hat{j} + \hat{k}$ is

A. $\frac{1}{3}$

B. 1

C. $\frac{5}{3}$

D. $\frac{7}{3}$

Correct answer: C

Source solution

Sol. $(\vec{a} \times (2\hat{i} + \hat{k})) \times \left(3\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k}\right)$
$= (2\hat{i} - 13\hat{j} - 4\hat{k}) \times \left(3\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k}\right)$
$-(6+2)\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -13 & -4 \\ 3 & \frac{1}{2} & 2 \end{vmatrix}$
$\vec{a} = 3\hat{i} + 2\hat{j} - 5\hat{k}$
Projection of $\vec{a}$ on vector $2\hat{i} + 2\hat{j} + \hat{k}$ is
$\vec{a} \cdot \frac{(2\hat{i}+2\hat{j}+\hat{k})}{3} = \frac{5}{3}$

Question 145

Let $\vec{a}=2\hat{i}+\alpha\hat{j}+\hat{k}, \quad \vec{b}=-\hat{i}+\hat{k}, \quad \vec{c}=\beta\hat{j}-\hat{k}$, where α and β are integers and $\alpha\beta=-6$. Let the values of the ordered pair (α,β) for which the area of the parallelogram of diagonals $\vec{a}+\vec{b}$ and $\vec{b}+\vec{c}$ is $\frac{\sqrt{21}}{2}$, be (α_1,β_1) and (α_2,β_2).
Then $\alpha_1^2+\beta_1^2-\alpha_2\beta_2$ is equal to

A. 17

B. 24

C. 21

D. 19

Correct answer: D

Source solution

Sol. Area of parallelogram $=\frac{1}{2}|\vec{d}_1\times\vec{d}_2$
$A=\frac{1}{2}|(\vec{a}+\vec{b})\times(\vec{b}+\vec{c})|=\frac{\sqrt{21}}{2}$
so, $\vec{a}+\vec{b}=\hat{i}+\alpha\hat{j}+2\hat{k}$
$\vec{b}+\vec{c}=-\hat{i}+\beta\hat{j}$
$(\vec{a}+\vec{b})\times(\vec{b}+\vec{c})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}1 \\ \alpha & 2-1 & \beta \\ 0\end{vmatrix}$
$=\hat{i}(-2\beta)-\hat{j}(2)+\hat{k}(\beta+\alpha)$
$|(\vec{a}+\vec{b})\times(\vec{b}+\vec{c})|=\sqrt{4\beta^2+4+(\alpha+\beta)^2}=\sqrt{21}$
$4\beta^2+4+\alpha^2+\beta^2+2\alpha\beta=21$
$\alpha^2+5\beta^2-12=17$
$\alpha^2+5\beta^2=29$
and $\alpha\beta=-6$
and given $\alpha_i\beta$ are integers
so,
$\alpha=-3,\beta=2$
or
$\alpha=3, \quad \beta=-2$
$(\alpha_1,\beta_1)=(-3,2)$
$(\alpha_2,\beta_2)=(3,-2)$
$\alpha_1^2+\beta_1^2-\alpha_2\beta_2=9+4+6=19$

Question 146

Let $\vec{c}$ be the projection vector of $\vec{b}=\lambda\hat{i}+4\hat{k},\lambda>0$, on the vector $\vec{a}=\hat{i}+2\hat{j}+2\hat{k}$. If $|\vec{a}+\vec{c}|=7$, then the area of the parallelogram formed by the vectors $\vec{b}$ and $\vec{c}$ is .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 16

Source solution

$\vec{c}=\left(\frac{\vec{b}\cdot\vec{a}}{|\vec{a}|}\right)\frac{\vec{a}}{|\vec{a}|}$
$=\left(\frac{\lambda+8}{9}\right)(\hat{i}+2\hat{j}+2\hat{k})$
$|\vec{a}+\vec{c}|-=7\Rightarrow\lambda=4$
Area of parallelogram
$=|\vec{b}\times\vec{c}|=\begin{Vmatrix}\hat{i} & \hat{j} & \hat{k} \\ \frac{4}{3} & \frac{8}{3} & \frac{8}{3} \\ 4 & 0 & 4\end{Vmatrix}$
$=16$

Question 147

Let $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}=2\hat{i}+3\hat{j}-5\hat{k}$ and $\vec{c}=3\hat{i}-\hat{j}+\lambda\hat{k}$ be three vectors. Let $\vec{r}$ be a unit vector along $\vec{b}+\vec{c}$. If $\vec{r}.\vec{a}=3$, then 3λ is equal to :

A. 27

B. 25

C. 25

D. 21

Correct answer: B

Source solution

Sol. $\vec{r}=k(\vec{b}+\vec{c})$

$\vec{r}\cdot\vec{a}=3$

$\vec{r}.\vec{a}=k(\vec{b}.\vec{a}+\vec{c}.\vec{a})$

$3=k(2+6-15+3-2+3\lambda)$

$$3=k(-6+3\lambda) \quad (1)$$

$\vec{r}=k(5\hat{i}+2\hat{j}-(5-\lambda)\hat{k})$

$$|\vec{r}|=k\sqrt{25+4+25+\lambda^2-10\lambda}=1 \quad (2)$$

$k=\frac{3}{-6+3\lambda}=\frac{1}{-2+\lambda}$ put in (2)

$4+\lambda^2-4\lambda=54+\lambda^2-10\lambda$

$6\lambda=50$

$3\lambda=25$

Three Dimensional Geometry

Questions 148-293 / 146 selected / 48.7% practice weight / Focus first

Question 148

Let the line $\ell : x = \frac{1-y}{-2} = \frac{z-3}{\lambda}, \lambda \in \mathbb{R}$ meet the plane $P : x + 2y + 3z = 4$ at the point (α, β, γ). If the angle between the line ℓ and the plane P is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then $\alpha + 2\beta + 6\gamma$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 11

Source solution

$\ell : x = \frac{y-1}{2} = \frac{z-3}{\lambda}, \lambda \in \mathbb{R}$
DR's of line $\ell(1, 2, \lambda)$
DR's of normal vector of plane $P : x + 2y + 3z = 4$ are (1, 2, 3)

Now, angle between line ℓ and plane P is given by
$\sin\theta = \left|\frac{1+4+3\lambda}{\sqrt{5+\lambda^2}\cdot\sqrt{14}}\right| = \frac{3}{\sqrt{14}}$ (given $\cos\theta = \sqrt{\frac{5}{14}}$)
$\Rightarrow \lambda = \frac{2}{3}$
Let variable point on line ℓ is $\left(t, 2t+1, \frac{2}{3}t+3\right)$
lies on plane P.
$\Rightarrow t = -1$
$\Rightarrow \left(-1, -1, \frac{7}{3}\right) \equiv (\alpha, \beta, \gamma)$
$\Rightarrow \alpha + 2\beta + 6\gamma = 11$

Question 149

Let the image of the point $\left(\frac{5}{3}, \frac{5}{3}, \frac{8}{3}\right)$ in the plane $x - 2y + z - 2 = 0$ be P. If the distance of the point $Q(6, -2, \alpha), \alpha > 0$, from P is 13 , then α is equal to
.

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 15

Source solution

Image of point $\left(\frac{5}{3}, \frac{5}{3}, \frac{8}{3}\right)$
$\frac{x-\frac{5}{3}}{1} = \frac{y-\frac{5}{3}}{-2} = \frac{z-\frac{8}{3}}{1} = \frac{-2\left(1\times\frac{5}{3}+(-2)\times\frac{5}{3}+1\times\frac{8}{3}-2\right)}{1^2+2^2+1^2} = \frac{1}{3}$
$\therefore x = 2, y = 1, z = 3$
$13^2 = (6-2)^2 + (-2-1)^2 + (\alpha-3)^2$
$\Rightarrow (\alpha-3)^2 = 144 \Rightarrow \alpha = 15 (\because \alpha > 0)$

Question 150

Let the lines $l_1 : \frac{x+5}{3} = \frac{y+4}{1} = \frac{z-\alpha}{-2}$ and $l_2 : 3x+2y+z-2=0=x-3y+2z-13$ be coplanar. If the point $P(a,b,c)$ on l_1 is nearest to the point $Q(-4,-3,2)$, then $|a|+|b|+|c|$ is equal to

A. 12

B. 14

C. 10

D. 8

Correct answer: C

Source solution

Sol. $(3x+2y+z-2)+\mu(x-3y+2z-13)=0$

$3(3+\mu)+1\cdot(2-3\mu)-2(1+2\mu)=0$

$9-4\mu=0$

$\mu=\frac{9}{4}$

$4(-15-8+\alpha-2)+9(-5+12+2\alpha-13)=0$

$-100+4\alpha-54+18\alpha=0$

$\Rightarrow \alpha=7$

Let $P\equiv(3\lambda-5,\lambda-4,-2\lambda+7)$

Direction ratio of $PQ(3\lambda-1,\lambda-1,-2\lambda+5)$

But $PQ\perp \ell_1$

$\Rightarrow 3(3\lambda-1)+1\cdot(\lambda-1)-2(-2\lambda+5)=0$

$\Rightarrow \lambda=1$

$P(-2,-3,5)\Rightarrow |a|+|b|+|c|=10$

Question 151

Let the equation of plane passing through the line of intersection of the planes $x+2y+az=2$ and $x-y+z=3$ be $5x-11y+bz=6a-1$. For $c\in\mathbb{Z}$, if the distance of this plane from the point $(a,-c,c)$ is $\frac{2}{\sqrt{a}}$, then $\frac{a+b}{c}$ is equal to

A. -2

B. 2

C. -4

D. 4

Correct answer: C

Source solution

Sol.

$(x+2y+az-2)+\lambda(x-y+z-3)=0$

$\frac{1+\lambda}{5}=\frac{2-\lambda}{-11}=\frac{a+\lambda}{b}=\frac{2+3\lambda}{6a-1}$

$\lambda=-\frac{7}{2}, a=3,\ b=1$

$\frac{2}{\sqrt{a}}=\left|\frac{5a+11c+bc-6a+1}{\sqrt{25+121+1}}\right|$

$c=-1$

$\therefore \frac{a+b}{c}=\frac{3+1}{-1}=-4$

Question 152

If the equation of the plane passing through the line of intersection of the planes $2x - y + z = 3, 4x - 3y + 5z + 9 = 0$ and parallel to the line $\frac{x+1}{-2} = \frac{y+3}{4} = \frac{z-2}{5}$ is $ax + by + cz + 6 = 0$. then a $+b + c$ is equal to

A. 14

B. 12

C. 13

D. 15

Correct answer: A

Source solution

Sol. Equation of family of plane
$(2x - y + z - 3) + \lambda(4x - 3y + 5z + 9) = 0$
$x(2 + 4\lambda) - y(1 + 3\lambda) + z(1 + 5\lambda) - 3 + 9\lambda = 0$
Parallel to the line
$-2(2 + 4\lambda) - (1 + 3\lambda)4 + (1 + 5\lambda)5 = 0$
$5\lambda = 3$
$\lambda = \frac{3}{5}$
equation of plane
$11x - 7y + 10z + 6 = 0$
$a + b + c = 14$

Question 153

Let P be the point of intersection of the line $\frac{x+3}{3} = \frac{y+2}{1} = \frac{1-z}{2}$ and the plane $x + y + z = 2$.
If the distance of the point P from the plane $3x - 4y + 12z = 32$ is q, then q and $2q$ are the roots of the equation

A. $x^2 - 18x - 72 = 0$

B. $x^2 + 18x + 72 = 0$

C. $x^2 - 18x + 72 = 0$

D. $x^2 + 18x - 72 = 0$

Correct answer: C

Source solution

Sol. $P = (3\lambda - 3, \lambda - 2, 1 - 2\lambda)$

P lies on the plane, $x + y + z = 2$
$\Rightarrow \lambda = 3$
$\mathrm{P} = (6, 1, -5)$
$q = \left|\frac{18-4-60-32}{\sqrt{9+16+144}}\right| = \frac{78}{13} = 6$
$\mathrm{q} = 6, 2\mathrm{q} = 12$

Equation, $x^2 - 18x + 72 = 0$

Question 154

Let the lines $l_1 : \frac{x+5}{3} = \frac{y+4}{1} = \frac{z-\alpha}{-2}$ and $l_2 : 3x + 2y + z - 2 = 0 = x - 3y + 2z - 13$ be coplanar. If the point $P(a, b, c)$ on l_1 is nearest to the point $Q(-4, -3, 2)$, then $|a| + |b| + |c|$ is equal to

A. 12

B. 14

C. 10

D. 8

Correct answer: C

Source solution

Sol.

1. Start with the equation:

$$(3x + 2y + z - 2) + \mu(x - 3y + 2z - 13) = 0$$

2. Substitute and simplify:

$$3(3 + \mu) + 1(2 - 3\mu) - 2(1 + 2\mu) = 0$$

$$9 - 4\mu = 0$$

3. Solve for μ:

$$\mu = \frac{9}{4}$$

4. Next, consider the equation:

$$4(-15 - 8 + \alpha - 2) + 9(-5 + 12 + 2\alpha - 13) = 0$$

5. Simplify:

$$-100 + 4\alpha - 54 + 18\alpha = 0$$

$$22\alpha - 154 = 0$$

6. Solve for α:

$$\alpha = 7$$

7. Define point P:

$$P \equiv (3\lambda - 5, \lambda - 4, -2\lambda + 7)$$

8. Find the direction ratio of $\mathbf{PQ}$:

$$\mathbf{PQ} \equiv (3\lambda - 1, \lambda - 1, -2\lambda + 5)$$

9. Since $\mathbf{PQ} \perp \ell_1$, set up the equation:

$$3(3\lambda - 1) + 1(\lambda - 1) - 2(-2\lambda + 5) = 0$$

10. Solve for λ:

$$\lambda = 1$$

11. Finally, for point $P(-2, -3, 5)$:

$$|a| + |b| + |c| = 10$$

Question 155

If the line $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1}$ intersects the plane $2x + 3y - z + 13 = 0$ at a point P and the plane $3x + y + 4z = 16$ at a point Q, then PQ is equal to:

A. $2\sqrt{14}$

B. 14

C. $2\sqrt{7}$

D. $\sqrt{14}$

Correct answer: A

Source solution

Sol. Let $\mathbf{P}(3\lambda + 2, 2\lambda - 1, -\lambda + 1)$ and

$$\mathbf{Q}(3\mu + 2, 2\mu - 1, -\mu + 1)$$

As P lies on $2x + 3y - z + 13 = 0$
$6\lambda + 4 + 6\lambda - 3 + \lambda - 1 + 13 = 0$
$\Rightarrow 13\lambda = -13$
$\Rightarrow \lambda = -1$
$\therefore \quad \mathbf{P}(-1, -3, 2)$
Q lies on $3x + y + 4z = 16$
$9\mu + 6 + 2\mu - 1 - 4\mu + 4 = 16$
$\Rightarrow 7\mu = 7$
$\Rightarrow \mu = 1$
Q is $(5, 1, 0)$
$PQ = \sqrt{36 + 16 + 4} = \sqrt{56} = 2\sqrt{14}$

Question 156

Let the plane P contain the line $2x+y-z-3=0=5x-3y+4z+9$ and be parallel to the line $\frac{x+2}{2}=\frac{3-y}{-4}=\frac{z-7}{5}$. Then the distance of the point $A(8,-1,-19)$ from the plane P measured parallel to the line $\frac{x}{-3}=\frac{y-5}{4}=\frac{2-z}{-12}$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 26

Source solution

Plane $\mathbf{P} \equiv \mathbf{P_1}+\lambda \mathbf{P_2}=0$

$(2x+y-z-3)+\lambda(5x-3y)+4z+9)=0$

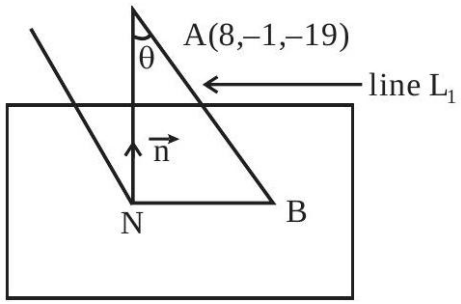
$(5\lambda+2)x+(1-3\lambda)y+(4\lambda-1)z+9\lambda-3=0$

$\vec{n}.\vec{b}=0$ where $\vec{b}(2,4,5)$

$2(5\lambda+2)+4(1-3\lambda)+5(4\lambda-1)=0$

$\lambda=-\frac{1}{6}$

Plane $7x+9y-10z-27=0$



Equation of line AB is

$\frac{x-8}{-3}=\frac{y+1}{4}=\frac{z+19}{12}=\lambda$

Let $\mathbf{B}=(8-3\lambda,-1+4\lambda,-19+12\lambda)$ lies on plane P

$\therefore 7(8-3\lambda)+9(4\lambda-1)-10(12\lambda-19)=27$

$\lambda=2$

$\therefore$ Point $\mathbf{B}=(2,7,5)$

$\mathbf{AB}=\sqrt{6^2+8^2+24^2}=26$

Question 157

Let P be the point of intersection of the line

$$\frac{x+3}{3}=\frac{y+2}{1}=\frac{1-z}{2}$$

and the plane

$$x+y+z=2.$$

If the distance of the point P from the plane

$$3x-4y+12z=32$$

is q, then q and $2q$ are the roots of the equation.

A. $x^2-18x-72=0$

B. $x^2+18x+72=0$

C. $x^2-18x+72=0$

D. $x^2+18x-72=0$

Correct answer: C

Source solution

$$\text{Let } P=(3\lambda-3,\lambda-2,1-2\lambda)$$

$$\text{Since } P \text{ lies on the plane } x+y+z=2:$$

$$(3\lambda-3)+(\lambda-2)+(1-2\lambda)=2$$

$$\Rightarrow 3\lambda-3+\lambda-2+1-2\lambda=2$$

$$\Rightarrow 2\lambda-4=2$$

$$\Rightarrow 2\lambda=6 \quad \Rightarrow \quad \lambda=3$$

$$\text{Thus, } P=(6,1,-5)$$

$$\text{Now, calculate } q:$$

$$q=\left|\frac{18-4-60-32}{\sqrt{9+16+144}}\right|$$

$$= \left| \frac{-78}{\sqrt{169}} \right| = \left| \frac{-78}{13} \right| = 6$$

$$\text{Therefore, } q = 6 \quad \text{and } 2q = 12$$

$$\text{The equation is } x^2 - 18x + 72 = 0$$

Question 158

Let the line passing through the points, $\mathbf{P(2, -1, 2)}$ and $\mathbf{Q(5, 3, 4)}$ meet the plane $\mathbf{x - y + z = 4}$ at the point R . Then the distance of the point R from the plane $x + 2y + 3z + 2 = 0$ measured parallel to the line $\frac{x-7}{2} = \frac{y+3}{2} = \frac{z-2}{1}$ is equal to

A. $\sqrt{31}$

B. $\sqrt{189}$

C. $\sqrt{61}$

D. 3

Correct answer: D

Source solution

Sol. Line : $\frac{x-5}{3} = \frac{y-3}{4} = \frac{z-4}{2} = \lambda$
$\mathbf{R}(3\lambda + 5, 4\lambda + 3, 2\lambda + 4)$
$\therefore 3\lambda + 5 - 4\lambda - 3 + 2\lambda + 4 = 4$
$\lambda + 6 = 4 \therefore \lambda = -2$
$\therefore \mathbf{R} \equiv (-1, -5, 0)$

Line : $\frac{x+1}{2} = \frac{y+5}{2} = \frac{z-0}{1} = \mu$
Point $T = (2\mu - 1, 2\mu - 5, \mu)$
It lies on plane
$2\mu - 1 + 2(2\mu - 5) + 3\mu + 2 = 0$
$\mu = 1$
$\therefore \mathbf{T} = (1, -3, 1)$
$\therefore \mathbf{RT} = 3$

Question 159

If the line $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-1}{-1}$ intersects the plane $2x+3y-z+13=0$ at a point P and the plane $3x+y+4z=16$ at a point Q, then PQ is equal to:

A. $2\sqrt{14}$

B. 14

C. $2\sqrt{7}$

D. $\sqrt{14}$

Correct answer: A

Source solution

Sol. Let $\mathbf{P}(3\lambda+2, 2\lambda-1, -\lambda+1)$ and

$$\mathbf{Q}(3\mu+2, 2\mu-1, -\mu+1)$$

Since point P lies on the plane $2x+3y-z+13=0$, we substitute the coordinates of P:

$$2(3\lambda+2)+3(2\lambda-1)-(-\lambda+1)+13=0$$

Simplifying this:

$$6\lambda+4+6\lambda-3+\lambda-1+13=0$$

Combining like terms:

$$13\lambda+13=0$$

Solving for λ:

$$13\lambda=-13 \quad \Rightarrow \quad \lambda=-1$$

Thus,

$$\mathbf{P}(-1,-3,2)$$

Next, point Q lies on the plane $3x+y+4z=16$. We substitute the coordinates of Q:

$$3(3\mu+2)+(2\mu-1)+4(-\mu+1)=16$$

Simplifying this:

$$9\mu+6+2\mu-1-4\mu+4=16$$

Combining like terms:

$$7\mu+9=16$$

Solving for μ:

$$7\mu = 7 \quad \Rightarrow \quad \mu = 1$$

Thus,

$$\mathbf{Q}(5, 1, 0)$$

Now, we calculate the distance PQ:

$$PQ = \sqrt{(5-(-1))^2 + (1-(-3))^2 + (0-2)^2}$$

Calculating each term:

$$= \sqrt{(6)^2 + (4)^2 + (-2)^2} = \sqrt{36+16+4} = \sqrt{56} = 2\sqrt{14}$$

Final answer:

$$PQ = 2\sqrt{14}$$

Question 160

Let P_1 be the plane $3x - y - 7z = 11$ and P_2 be the plane passing through the points $(2, -1, 0)$, $(2, 0, -1)$, and $(5, 1, 1)$. If the foot of the perpendicular drawn from the point (7, 4, -1)on the line of intersection of the planes $\mathbf{P_1}$ and $\mathbf{P_2}$ is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 11

The source provides a brief answer only. A detailed worked solution is not available for this question.

Source solution

Question 161

Let P be the plane passing through the points $(5,3,0)$, $(13,3,-2)$ and $(1,6,2)$. For $\alpha \in \mathbb{N}$, if the distances of the points $A(3,4,\alpha)$ and $B(2,\alpha,a)$ from the plane P are 2 and 3 respectively, then the positive value of a is

A. 6

B. 4

C. 3

D. 5

Correct answer: B

Source solution

Sol.

1. Calculate the determinant to find the normal vector of the plane:

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & 0 & -2 \\ 4 & -3 & -2 \end{vmatrix} = \hat{i}(-6) + 8\hat{j} - 24\hat{k}$$

2. The normal vector of the plane is:

$$\mathbf{n} = 3\hat{i} - 4\hat{j} + 12\hat{k}$$

3. The equation of the plane is:

$$3x - 4y + 12z = 3$$

4. Calculate the distance from point $A(3,4,\alpha)$:

$$\left| \frac{9 - 16 + 12\alpha - 3}{13} \right| = 2$$

Simplifying gives:

$$\left| \frac{12\alpha - 10}{13} \right| = 2$$

This leads to two cases:
- Case 1: $\frac{12\alpha-10}{13} = 2$

$$12\alpha - 10 = 26 \implies 12\alpha = 36 \implies \alpha = 3$$

- Case 2: $\frac{12\alpha-10}{13} = -2$

$$12\alpha - 10 = -26 \implies 12\alpha = -16 \implies \alpha = -\frac{4}{3} \quad \text{(rejected)}$$

5. Calculate the distance from point $B(2,3,a)$:

$$\left| \frac{6 - 12 + 12a - 3}{13} \right| = 3$$

Simplifying gives:

$$\left|\frac{12a-9}{13}\right| = 3$$

This leads to two cases:
- Case 1: $\frac{12a-9}{13} = 3$

$$12a - 9 = 39 \implies 12a = 48 \implies a = 4$$

- Case 2: $\frac{12a-9}{13} = -3$

$$12a - 9 = -39 \implies 12a = -30 \implies a = -\frac{5}{2} \quad \text{(rejected)}$$

Final answers:

$$\alpha = 3, \quad a = 4$$

Question 162

Let the line passing through the points, $\mathbf{P(2,-1,2)}$ and $\mathbf{Q(5,3,4)}$ meet the plane $\mathbf{x-y+z=4}$ at the point $\boldsymbol{R}$. Then the distance of the point $\boldsymbol{R}$ from the plane $\boldsymbol{x+2y+3z+2=0}$ measured parallel to the line $\frac{x-7}{2}=\frac{y+3}{2}=\frac{z-2}{1}$ is equal to

A. $\sqrt{31}$

B. $\sqrt{189}$

C. $\sqrt{61}$

D. 3

Correct answer: D

Source solution

$$\text{Let } \frac{x-5}{3}=\frac{y-3}{4}=\frac{z-4}{2}=\lambda$$

$$\text{Then, } R(3\lambda+5,4\lambda+3,2\lambda+4)$$

$$\therefore 3\lambda+5-4\lambda-3+2\lambda+4=4$$

$$\lambda+6=4 \implies \lambda=-2$$

$$\therefore R\equiv(-1,-5,0)$$

$$\text{For the line: } \frac{x+1}{2}=\frac{y+5}{2}=\frac{z-0}{1}=\mu$$

$$\text{Point } T=(2\mu-1,2\mu-5,\mu)$$

$$\text{It lies on the plane: } 2\mu-1+2(2\mu-5)+3\mu+2=0$$

$$\mu=1$$

$$\therefore T=(1,-3,1)$$

$$\therefore RT=3$$

Question 163

If equation of the plane that contains the point ($-2, 3, 5$) and is perpendicular to each of the planes $2x + 4y + 5z = 8$ and $3x - 2y + 3z = 5$ is $\alpha x + \beta y + \gamma z + 97 = 0$ then $\alpha + \beta + \gamma =$

A. 18

B. 17

C. 16

D. 15

Correct answer: D

Source solution

Sol. The equation of plane through ($-2, 3, 5$) is $a(x + 2) + b(y - 3) + c(z - 5) = 0$
it is perpendicular to $2x + 4y + 5z = 8 \& 3x - 2y + 3z = 5$
$\therefore \quad 2a + 4b + 5c = 0$

$$3a - 2b + 3c = 0$$

$\therefore \quad \frac{a}{\begin{vmatrix} 4 & 5 \\ -2 & 3 \end{vmatrix}} = \frac{-b}{\begin{vmatrix} 2 & 5 \\ 3 & 3 \end{vmatrix}} = \frac{c}{\begin{vmatrix} 2 & 4 \\ 3 & -2 \end{vmatrix}}$
$\Rightarrow \quad \frac{a}{22} = \frac{b}{9} = \frac{c}{-16}$
$\therefore$ Equation of Plane is
$22(x + 2) + 9(y - 3) - 16(z - 5) = 0$
$\Rightarrow \quad 22x + 9y - 16z + 97 = 0$

Comparing with $\alpha x + \beta y + \gamma x + 97 = 0$

We get $\alpha + \beta + \gamma = 22 + 9 - 16 = 15$

Question 164

Let the equation of plane passing through the line of intersection of the planes $x+2y+az=2$ and $x-y+z=3$ be $5x-11y+bz=6a-1$. For $c \in \mathbb{Z}$, if the distance of this plane from the point $(a,-c,c)$ is $\frac{2}{\sqrt{a}}$, then $\frac{a+b}{c}$ is equal to

A. -2

B. 2

C. -4

D. 4

Correct answer: C

Source solution

Sol.

1. Start with the equation:

$$(x+2y+az-2)+\lambda(x-y+z-3)=0$$

2. Set up the system of equations:

$$\frac{1+\lambda}{5}=\frac{2-\lambda}{-11}=\frac{a+\lambda}{b}=\frac{2+3\lambda}{6a-1}$$

3. Solve for λ, a, and b:

$$\lambda=-\frac{7}{2}, \quad a=3, \quad b=1$$

4. Substitute a into the equation:

$$\frac{2}{\sqrt{a}}=\left|\frac{5a+11c+bc-6a+1}{\sqrt{25+121+1}}\right|$$

5. Solve for c:

$$c=-1$$

6. Finally, calculate:

$$\frac{a+b}{c}=\frac{3+1}{-1}=-4$$

Question 165

Let (α, β, γ) be the image of the point $\mathbf{P}(2,3,5)$ in the plane $2x + y - 3z = 6$. Then $\alpha + \beta + \gamma$ is equal to

A. 10

B. 5

C. 12

D. 9

Correct answer: A

Source solution

Sol.

We start with the equation:

$$\frac{\alpha - 2}{2} = \frac{\beta - 3}{1} = \frac{\gamma - 5}{-3} = -2$$

From this, we can derive the values of α, β, and γ.

1. For α:

$$\frac{\alpha - 2}{2} = -2 \implies \alpha - 2 = -4 \implies \alpha = -4 + 2 = -2$$

2. For β:

$$\frac{\beta - 3}{1} = -2 \implies \beta - 3 = -2 \implies \beta = -2 + 3 = 1$$

3. For γ:

$$\frac{\gamma - 5}{-3} = -2 \implies \gamma - 5 = 6 \implies \gamma = 6 + 5 = 11$$

Now we have:

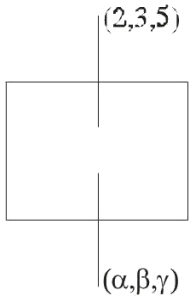
$$\alpha = -2, \quad \beta = 1, \quad \gamma = 11$$

Finally, we calculate:

$$\alpha + \beta + \gamma = -2 + 1 + 11 = 10$$

Thus, the final answer is:

$$\alpha + \beta + \gamma = 10$$


(2,3,5)
(α,β,γ)

Question 166

The length of the perpendicular from the point $(2, -1, 4)$ on the straight line,

$$\frac{x+3}{10} = \frac{y-2}{-7} = \frac{z}{1}$$

is :

A. Greater than 3 but less than 4

B. Greater than 2 but less than 3

C. Greater than 4

D. Less than 2

Correct answer: A

Source solution

Let P be the foot of the perpendicular from point $A(2, -1, 4)$ on the given line. We can assume P as $P(10\lambda - 3, -7\lambda + 2, \lambda)$.

The direction ratios (DRs) of line AP are given by $(10\lambda - 5, -7\lambda + 3, \lambda - 4)$.

Since AP and the given line are perpendicular, we have:

$$10(10\lambda - 5) - 7(-7\lambda + 3) + 1(\lambda - 4) = 0$$

Solving this equation:

$$100\lambda - 50 + 49\lambda - 21 + \lambda - 4 = 0$$

$$150\lambda - 75 = 0$$

$$\lambda = \frac{1}{2}$$

Now, we calculate AP:

$$AP = \sqrt{(10\lambda - 5)^2 + (-7\lambda + 3)^2 + (\lambda - 4)^2}$$

Substituting $\lambda = \frac{1}{2}$:

$$AP = \sqrt{(10 \cdot \frac{1}{2} - 5)^2 + (-7 \cdot \frac{1}{2} + 3)^2 + (\frac{1}{2} - 4)^2}$$

Calculating each term:

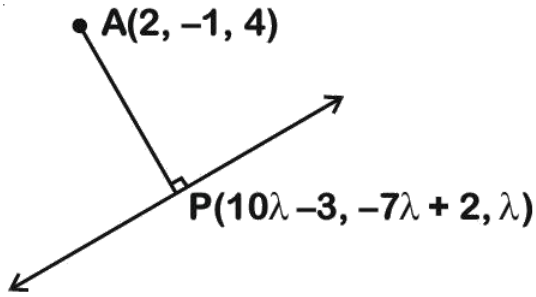
$$= \sqrt{(0)^2 + \left(-\frac{7}{2} + 3\right)^2 + \left(-\frac{7}{2}\right)^2}$$

$$= \sqrt{0 + \left(\frac{1}{2}\right)^2 + \left(-\frac{7}{2}\right)^2}$$

$$= \sqrt{0 + \frac{1}{4} + \frac{49}{4}}$$

$$= \sqrt{\frac{50}{4}} = \sqrt{12.5}$$

Thus, we find that $\sqrt{12.5}$ lies in the interval $(3, 4)$.



Question 167

Let the point, on the line passing through the points $P(1,-2,3)$ and $Q(5,-4,7)$, farther from the origin and at a distance of 9 units from the point P, be (α,β,γ). Then $\alpha^2+\beta^2+\gamma^2$ is equal to :

A. 155

B. 150

C. 160

D. 165

Correct answer: A

Source solution

Sol.

The line PQ is given by the equations:

$$\frac{x-1}{4}=\frac{y+2}{-2}=\frac{z-3}{4}$$

We can express the points on the line in terms of a parameter t:

$$(x,y,z)=(4t+1,-2t-2,4t+3)$$

Next, we calculate the distance squared from the origin to the point on the line:

$$d^2=(4t+1)^2+(-2t-2)^2+(4t+3)^2$$

Expanding each term:

$$d^2=(16t^2+8t+1)+(4t^2+8t+4)+(16t^2+24t+9)$$

Combining like terms:

$$d^2=16t^2+4t^2+16t^2+8t+8t+24t+1+4+9$$

$$d^2=36t^2+40t+14$$

Setting $d^2=81$:

$$36t^2+40t+14=81$$

Rearranging gives:

$$36t^2+40t-67=0$$

Using the quadratic formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-40 \pm \sqrt{40^2 - 4 \cdot 36 \cdot (-67)}}{2 \cdot 36}$$

Calculating the discriminant:

$$b^2 - 4ac = 1600 + 9632 = 11232$$

Thus,

$$t = \frac{-40 \pm \sqrt{11232}}{72}$$

Calculating $\sqrt{11232}$:

$$\sqrt{11232} = 106$$

So,

$$t = \frac{-40 \pm 106}{72}$$

Calculating the two possible values for t:

1. $t = \frac{66}{72} = \frac{11}{12}$
2. $t = \frac{-146}{72} = -\frac{73}{36}$

Finding the corresponding points:

For $t = \frac{11}{12}$:

$$(4 \cdot \frac{11}{12} + 1, -2 \cdot \frac{11}{12} - 2, 4 \cdot \frac{11}{12} + 3) = (7, -5, 9)$$

For $t = -\frac{73}{36}$:

Calculating gives another point.

Finally, we find:

$$\alpha^2 + \beta^2 + \gamma^2 = 155$$

Thus, the answer is option (1).

Question 168

Let $\mathbf{P(\alpha, \beta, \gamma)}$ be the image of the point $\mathbf{Q(3, -3, 1)}$ in the line $\frac{x-0}{1} = \frac{y-3}{1} = \frac{z-1}{-1}$ and $\boldsymbol{R}$ be the point $(\mathbf{2, 5, -1})$. If the area of the triangle PQR is $\boldsymbol{\lambda}$ and $\boldsymbol{\lambda^2 = 14\,K}$, then K is equal to:

A. 36

B. 72

C. 18

D. 81

Correct answer: D

Source solution

Sol.

$\mathbf{RQ} = \sqrt{1+64+4} = \sqrt{69}$

$\overrightarrow{\mathbf{RQ}} = \hat{\ell} - 8\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$\overrightarrow{\mathbf{RS}} = \hat{\ell} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$

$\cos\theta = \frac{\overrightarrow{\mathbf{RQ}} \cdot \overrightarrow{\mathbf{RS}}}{|\overrightarrow{\mathbf{RQ}}||\overrightarrow{\mathbf{RS}}|} = \left|\frac{1-8-2}{\sqrt{69}\sqrt{3}}\right| = \frac{9}{3\sqrt{23}}$

$\cos\theta = \frac{3}{\sqrt{23}} = \frac{\mathbf{RS}}{\mathbf{RQ}} = \frac{\mathbf{RS}}{\sqrt{69}}$

$\mathrm{RS} = \mathbf{3\sqrt{3}}$

$\sin\theta = \frac{\sqrt{14}}{\sqrt{23}} = \frac{\mathbf{QS}}{\sqrt{69}}$

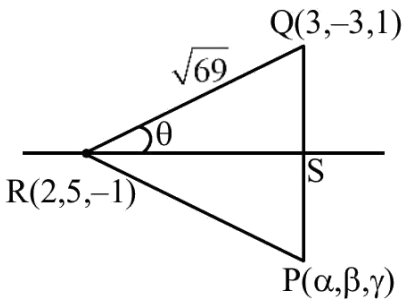
$\mathbf{QS} = \sqrt{42}$

area $= \frac{1}{2} \cdot \mathbf{2QS} \cdot \mathbf{RS} = \sqrt{42} \cdot 3\sqrt{3}$

$\boldsymbol{\lambda} = \mathbf{9\sqrt{14}}$

$\boldsymbol{\lambda^2} = \mathbf{81.14 = 14k}$

$\mathbf{k} = \mathbf{81}$



Question 169

Let $\mathbf{P}(\alpha, \beta, \gamma)$ be the image of the point $\mathbf{Q}(3, -3, 1)$ in the line $\frac{x-0}{1} = \frac{y-3}{1} = \frac{z-1}{-1}$ and $\boldsymbol{R}$ be the point $(2, 5, -1)$. If the area of the triangle PQR is $\boldsymbol{\lambda}$ and $\boldsymbol{\lambda^2 = 14\,K}$, then K is equal to:

A. 36

B. 72

C. 18

D. 81

Correct answer: D

Source solution

Given:

1. Calculate $\mathbf{RQ}$:

$$\mathbf{RQ} = \sqrt{1 + 64 + 4} = \sqrt{69}$$

2. Define vectors:

$$\overrightarrow{\mathbf{RQ}} = \hat{\ell} - 8\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$$

$$\overrightarrow{\mathbf{RS}} = \hat{\ell} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$$

3. Calculate $\cos\theta$:

$$\cos\theta = \frac{\overrightarrow{\mathbf{RQ}} \cdot \overrightarrow{\mathbf{RS}}}{|\overrightarrow{\mathbf{RQ}}||\overrightarrow{\mathbf{RS}}|}$$

$$= \left|\frac{1 - 8 - 2}{\sqrt{69}\sqrt{3}}\right| = \frac{9}{3\sqrt{23}}$$

4. Simplify $\cos\theta$:

$$\cos\theta = \frac{3}{\sqrt{23}} = \frac{\mathbf{RS}}{\mathbf{RQ}} = \frac{\mathbf{RS}}{\sqrt{69}}$$

$$\Rightarrow \mathbf{RS} = 3\sqrt{3}$$

5. Calculate $\sin\theta$:

$$\sin\theta = \frac{\sqrt{14}}{\sqrt{23}} = \frac{\mathbf{QS}}{\sqrt{69}}$$

$$\Rightarrow QS = \sqrt{42}$$

6. Calculate the area:

$$\text{Area} = \frac{1}{2} \cdot 2QS \cdot RS = \sqrt{42} \cdot 3\sqrt{3}$$

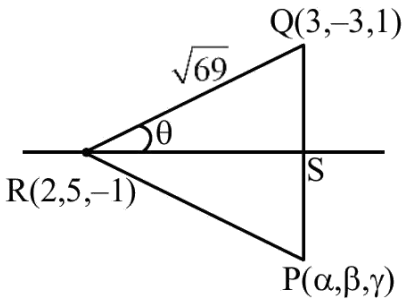
7. Calculate λ:

$$\lambda = 9\sqrt{14}$$

8. Calculate λ^2:

$$\lambda^2 = 81.14 = 14k$$

$$\Rightarrow k = 81$$



Question 170

Let the point, on the line passing through the points $P(1, -2, 3)$ and $Q(5, -4, 7)$, farther from the origin and at a distance of 9 units from the point P , be (α, β, γ). Then $\alpha^2 + \beta^2 + \gamma^2$ is equal to :

A. 155

B. 150

C. 160

D. 165

Correct answer: A

Source solution

Sol. PQ line
$\frac{x-1}{4} = \frac{y+2}{-2} = \frac{z-3}{4}$
pt $(4t + 1, -2t - 2, 4t + 3)$
distance $^2 = 16t^2 + 4t^2 + 16t^2 = 81$
$t = \pm\frac{3}{2}$
pt (7, -5, 9)
$\alpha^2 + \beta^2 + \gamma^2 = 155$
option (1)

Question 171

Let (α, β, γ) be the point $(8, 5, 7)$ in the line

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{5}.$$

Then $\alpha + \beta + \gamma$ is equal to

A. 16

B. 18

C. 14

D. 20

Correct answer: B

Source solution

$$\text{Given: } \overrightarrow{AM} \cdot (2\hat{i} + 3\hat{j} + 5\hat{k}) = 0$$

$$\text{Let } \overrightarrow{AM} = (2\lambda - 7)\hat{i} + (3\lambda - 6)\hat{j} + (5\lambda - 5)\hat{k}$$

$$\text{Setting up the dot product: } (2\lambda - 7)(2) + (3\lambda - 6)(3) + (5\lambda - 5)(5) = 0$$

$$\text{Expanding: } 4\lambda - 14 + 9\lambda - 18 + 25\lambda - 25 = 0$$

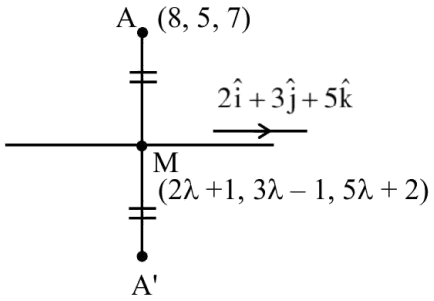
$$\text{Combining like terms: } (4\lambda + 9\lambda + 25\lambda) - 57 = 0$$

$$38\lambda = 57$$

$$\lambda = \frac{57}{38} = \frac{3}{2}$$

$$\text{Coordinates of } M: \quad M\left(4, \frac{7}{2}, \frac{19}{2}\right)$$

$$\text{Coordinates of } A': \quad A'(0, 2, 12)$$


A
(8, 5, 7)
$2\hat{i}+3\hat{j}+5\hat{k}$
M
$(2\lambda+1, 3\lambda-1, 5\lambda+2)$
A'

Question 172

For real numbers α and $\beta \neq 0$, if the point of intersection of the straight lines $\frac{x-\alpha}{1} = \frac{y-1}{2} = \frac{z-1}{3}$ and $\frac{x-4}{\beta} = \frac{y-6}{3} = \frac{z-7}{3}$, lies on the plane $x + 2y - z = 8$, then $\alpha - \beta$ is equal to :

A. 5

B. 9

C. 3

D. 7

Correct answer: D

Source solution

\textbf{Given lines:}

$$\frac{x-\alpha}{1} = \frac{y-1}{2} = \frac{z-1}{3} = \lambda$$

and

$$\frac{x-4}{\beta} = \frac{y-6}{3} = \frac{z-7}{3} = \mu.$$

Since the lines intersect, let their point of intersection be $P(x, y, z)$.

From the first line,

$$x = \alpha + \lambda, \qquad y = 1 + 2\lambda, \qquad z = 1 + 3\lambda.$$

From the second line,

$$x = 4 + \beta\mu, \qquad y = 6 + 3\mu, \qquad z = 7 + 3\mu.$$

Equating the y-coordinates,

$$1 + 2\lambda = 6 + 3\mu. \tag{1}$$

Equating the z-coordinates,

$$1 + 3\lambda = 7 + 3\mu. \tag{2}$$

From (2),

$$\lambda - \mu = 2. \tag{3}$$

Substituting $\lambda = \mu + 2$ into (1),

$$1 + 2(\mu + 2) = 6 + 3\mu,$$

$$5+2\mu=6+3\mu,$$

$$\mu=-1.$$

Hence,

$$\lambda=1.$$

Therefore the point of intersection is

$$P=(\alpha+1,\ 3,\ 4).$$

Since P lies on the plane

$$x+2y-z=8,$$

we have

$$(\alpha+1)+2(3)-4=8.$$

$$\alpha+3=8.$$

$$\alpha=5.$$

Now equating the x-coordinates,

$$\alpha+\lambda=4+\beta\mu.$$

Substituting $\alpha=5,\ \lambda=1,\ \mu=-1$,

$$5+1=4-\beta.$$

$$6=4-\beta.$$

$$\beta=-2.$$

Hence,

$$\alpha-\beta=5-(-2)=7.$$

$$\boxed{\alpha - \beta = 7}$$

$$\boxed{\text{Option (4)}}$$

Question 173

If the line $\frac{2-x}{3} = \frac{3y-2}{4\lambda+1} = 4 - z$ makes a right angle with the line $\frac{x+3}{3\mu} = \frac{1-2y}{6} = \frac{5-z}{7}$, then $4\lambda + 9\mu$ is equal to :

A. 13

B. 4

C. 5

D. 6

Correct answer: D

Source solution

Sol.

We start with the equations:

$$\frac{2-x}{3} = \frac{3y-2}{4\lambda+1} = 4 - z$$

$$\frac{x-2}{-3} = \frac{y-\frac{2}{3}}{\frac{4\lambda+1}{3}} = \frac{z-4}{-1}$$

From the second set of equations, we have:

$$\frac{x+3}{3\mu} = \frac{1-2y}{6} = \frac{5-z}{7}$$

This can be rewritten as:

$$\frac{x+3}{3\mu} = \frac{y-\frac{1}{2}}{-3} = \frac{z-5}{-7}$$

Using the right angle condition, we set up the equation:

$$(-3)(3\mu) + \left(\frac{4\lambda+1}{3}\right)(-3) + (-1)(-7) = 0$$

Simplifying this gives:

$$-9\mu - 4\lambda - 1 + 7 = 0$$

Thus, we arrive at:

$$4\lambda + 9\mu = 6$$

Question 174

Let $\mathbf{P}(\alpha, \beta, \gamma)$ be the image of the point $\mathbf{Q}(1, 6, 4)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Then $2\alpha + \beta + \gamma$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 11

Source solution

\(\frac{\)

$\mathbf{Q}(1, 6, 4)$
$\mathbf{A}\left(\frac{17}{14}, \frac{48}{14}, \frac{79}{14}\right)$
$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$
$\mathbf{P}(\alpha, \beta, \gamma) \quad \vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$

\}\{4\}

$\mathbf{A}(t, 2t+1, 3t+2)$
$\overrightarrow{QA} = (t-1)\hat{i} + (2t-5)\hat{j} + (3t-2)\hat{k}$
$\overrightarrow{QA} \cdot \vec{b} = 0$
$(t-1) + 2(2t-5) + 3(3t-2) = 0$
$14t = 17$
$\alpha = \frac{20}{14} \quad \beta = \frac{12}{14} \quad \gamma = \frac{102}{14}$
$2\alpha + \beta + \gamma = \frac{154}{14} = 11$

Question 175

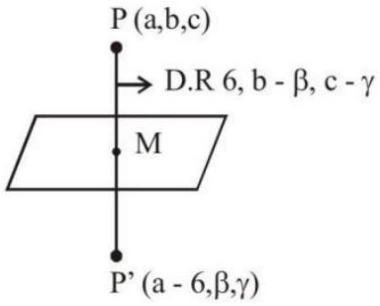
Let the mirror image of the point ($\mathbf{a, b, c}$) with respect to the plane $3x - 4y + 12z + 19 = 0$ be ($\mathbf{a - 6, \beta, \gamma}$). If $\mathbf{a + b + c = 5}$, then $7\beta - 9\gamma$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 137

Source solution



$\mathbf{M} = \left(a - 3, \frac{\beta+b}{2}, \frac{\gamma+c}{2}\right)$
Since M lies on $3x + 4y + 12z + 19 = 0$

$$\Rightarrow 6a - 4b + 12c - 4\beta + 12\gamma + 20 = 0 \quad (1)$$

Since PP' is parallel to normal of the plane then
$\frac{6}{3} = \frac{b-\beta}{-4} = \frac{c-\gamma}{12}$
$\Rightarrow \beta = b + 8, \quad \gamma = c - 24$
$a + b + c = 5 \Rightarrow a + \beta - 8 + \gamma + 24 = 5$
$\Rightarrow a = -\beta - \gamma - 11$

Now putting these values in (1) we get
$6(-\beta - \gamma - 11) - 4(\beta - 8) + 12(\gamma + 24) - 4\beta + 12\gamma + 20 = 0$
$\Rightarrow 7\beta - 9\gamma = 170 - 33 = 137$

Question 176

Let (α, β, γ) be the point $(8, 5, 7)$ in the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{5}$. Then $\alpha + \beta + \gamma$ is equal to

A. 16

B. 18

C. 14

D. 20

Correct answer: C

Source solution

Sol.

$\overrightarrow{AM} \cdot (2\hat{i} + 3\hat{j} + 5\hat{k}) = 0$

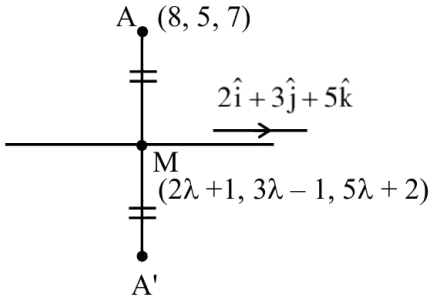
$(2\lambda - 7)(2) + (3\lambda - 6)(3) + (5\lambda - 5)(5) = 0$

$38\lambda = 57$

$\lambda = \frac{3}{2}$

$M\left(4, \frac{7}{2}, \frac{19}{2}\right)$

$A'(0, 2, 12)$



Question 177

Let d be the distance of the point of intersection of the lines $\frac{x+6}{3}=\frac{y}{2}=\frac{z+1}{1}$ and $\frac{x-7}{4}=\frac{y-9}{3}=\frac{z-4}{2}$ from the point $(7,8,9)$. Then d^2+6 is equal to :

A. 72

B. 69

C. 75

D. 78

Correct answer: C

Source solution

$$\text{Line } L_1: \quad \frac{x+6}{3}=\frac{y}{2}=\frac{z+1}{1}=\lambda$$

Hence,

$$x=3\lambda-6, \qquad y=2\lambda, \qquad z=\lambda-1.$$

Similarly,

$$\text{Line } L_2: \quad \frac{x-7}{4}=\frac{y-9}{3}=\frac{z-4}{2}=\mu$$

gives

$$x=4\mu+7, \qquad y=3\mu+9, \qquad z=2\mu+4.$$

Since the lines intersect,

$$3\lambda-6=4\mu+7,$$

$$2\lambda=3\mu+9,$$

$$\lambda-1=2\mu+4.$$

From the third equation,

$$\lambda=2\mu+5.$$

Substituting into the second equation,

$$2(2\mu+5)=3\mu+9$$

$$4\mu+10=3\mu+9$$

$$\mu = -1.$$

Therefore,

$$\lambda = 2(-1) + 5 = 3.$$

Hence, the point of intersection is

$$P = (3 \cdot 3 - 6,\ 2 \cdot 3,\ 3 - 1) = (3, 6, 2).$$

Given point

$$Q = (7, 8, 9).$$

Therefore,

$$d = PQ.$$

So,

$$d^2 = (7 - 3)^2 + (8 - 6)^2 + (9 - 2)^2.$$

$$d^2 = 4^2 + 2^2 + 7^2.$$

$$d^2 = 16 + 4 + 49 = 69.$$

Hence,

$$d^2 + 6 = 69 + 6 = 75.$$

$$\boxed{75}$$

Hence, the correct option is

$$\boxed{\text{(3) } 75}.$$

Question 178

The distance of the point $\mathbf{P(3,4,4)}$ from the point of intersection of the line joining the points. $\mathbf{Q(3,-4,-5)}$ and $\mathbf{R(2,-3,1)}$ and the plane $\mathbf{2x+y+z=7}$, is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 7

The source provides a brief answer only. A detailed worked solution is not available for this question.

Source solution

Question 179

Let $\boldsymbol{L_1 : \frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0}}$ and
$\mathbf{L_2 : \frac{x-2}{2} = \frac{y}{0} = \frac{z+4}{\alpha}, \alpha \in \mathbf{R}}$, be two lines, which intersect at the point B . If P is the foot of perpendicular from the point $\mathbf{A(1,1,-1)}$ on $\mathbf{L_2}$, then the value of $\mathbf{26\alpha(PB)^2}$ is .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 216

Source solution

Point B
$\mathbf{(3\lambda+1,-\lambda+1,-1) \equiv (2\mu+2,0,\alpha\mu-4)}$
$\mathbf{3\lambda+1=2\mu+2}$
$\mathbf{-\lambda+1=0}$
$\mathbf{-1=\alpha\mu-4}$
$\boldsymbol{\lambda=1,\mu=1,\alpha=3}$
B(4, 0, -1)
Let Point ' P ' is $\mathbf{(2\delta+2,0,3\delta-4)}$
Dr's of AP $\mathbf{< 2\delta+1,-1,3\delta-3 >}$
$\mathbf{AP \perp L_2 \Rightarrow \delta = \frac{7}{13}}$
$\mathbf{P\left(\frac{40}{13},0,\frac{-31}{13}\right)}$
$\mathbf{2\sigma\delta(PB)^2 = 26 \times 3 \times \left(\frac{144}{169}+\frac{324}{169}\right)}$
$\mathbf{= 216}$

Question 180

The lines $\frac{x-2}{2}=\frac{y}{-2}=\frac{z-7}{16}$ and $\frac{x+3}{4}=\frac{y+2}{3}=\frac{z+2}{1}$ intersect at the point P. If the distance of P from the line $\frac{x+1}{2}=\frac{y-1}{3}=\frac{z-1}{1}$ is l, then $14l^2$ is equal to.

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 108

Source solution

$\frac{x-2}{1}=\frac{y}{-1}=\frac{z-7}{8}=\lambda$

$\frac{x+3}{4}=\frac{y+2}{3}=\frac{z+2}{1}=k$

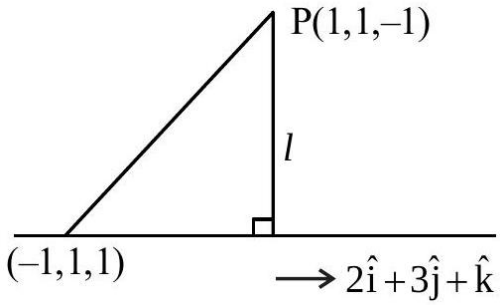
$\Rightarrow \lambda+2=4k-3$

$-\lambda=3k-2$

$\Rightarrow k=1, \lambda=-1$

$8\lambda+7=k-2$

$\therefore P=(1,1,-1)$



Projection of $2\hat{i}-2\hat{k}$ on $2\hat{i}+3\hat{j}+\hat{k}$ is

$=\frac{4-2}{\sqrt{4+9+1}}=\frac{2}{\sqrt{14}}$

$\therefore l^2=8-\frac{4}{14}=\frac{108}{14}$

$\Rightarrow 14l^2=108$

Question 181

If the mirror image of the point $P(3,4,9)$ in the line $\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-2}{1}$ is (α,β,γ), then $14(\alpha+\beta+\gamma)$ is :

A. 102

B. 138

C. 108

D. 132

Correct answer: C

Source solution

$$\text{Given: } \overrightarrow{PN}\cdot\vec{b}=0$$

$$3(3\lambda-2)+2(2\lambda-5)+(\lambda-7)=0$$

$$9\lambda-6+4\lambda-10+\lambda-7=0$$

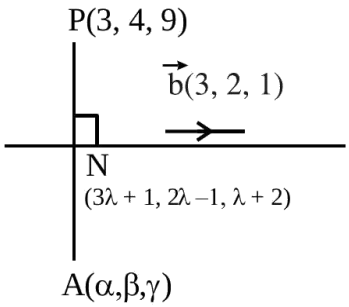
$$14\lambda-23=0 \quad \Rightarrow \quad 14\lambda=23 \quad \Rightarrow \quad \lambda=\frac{23}{14}$$

$$\text{Coordinates of N}=\left(\frac{83}{14},\frac{32}{14},\frac{51}{14}\right)$$

$$\frac{\alpha+3}{2}=\frac{83}{14} \quad \Rightarrow \quad \alpha=\frac{62}{7}$$

$$\frac{\beta+4}{2}=\frac{32}{14} \quad \Rightarrow \quad \beta=\frac{4}{7}$$

$$\frac{\gamma+9}{2}=\frac{51}{14} \quad \Rightarrow \quad \gamma=\frac{-12}{7}$$



Question 182

Let P and Q be the points on the line

$$\frac{x+3}{8}=\frac{y-4}{2}=\frac{z+1}{2}$$

which are at a distance of 6 units from the point $\mathbf{R(1,2,3)}$. If the centroid of the triangle PQR is (α,β,γ), then $\alpha^2+\beta^2+\gamma^2$ is:

A. 26

B. 36

C. 18

D. 24

Correct answer: C

Source solution

$$\text{Let P}(8\lambda-3,2\lambda+4,2\lambda-1)$$

$$\text{Given PR}=6$$

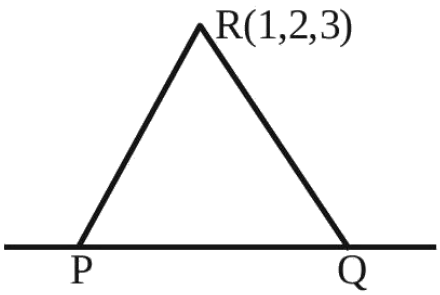
$$(8\lambda-4)^2+(2\lambda+2)^2+(2\lambda-4)^2=36$$

$$\text{Solving gives } \lambda=0 \text{ or } 1$$

$$\text{Thus, P}(-3,4,-1) \text{ and Q}(5,6,1)$$

$$\text{Centroid of } \triangle\text{PQR}=(1,4,1)\equiv(\alpha,\beta,\gamma)$$

$$\alpha^2+\beta^2+\gamma^2=18$$



Question 183

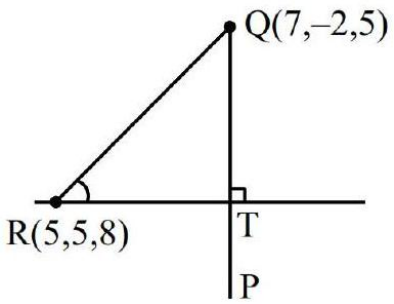
Let P be the image of the point $\mathbf{Q(7,-2,5)}$ in the line $\mathbf{L:\frac{x-1}{2}=\frac{y+1}{3}=\frac{z}{4}}$ and $\mathbf{R(5,p,q)}$ be a point on L . Then the square of the area of $\mathbf{\triangle PQR}$ is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 957

Source solution



Let $\mathbf{R(2\lambda+1,3\lambda-1,4\lambda)}$

$\mathbf{2\lambda+1=5}$

$\mathbf{\lambda=2}$

R(5, 5, 8)

let $\mathbf{T(2\lambda+1,3\lambda-1,4\lambda)}$

$\mathbf{\overrightarrow{QT}=(2\lambda-6)\hat{i}+(3\lambda+1)\hat{j}+(4\lambda-5)\hat{k}}$

$\mathbf{\vec{b}=2\hat{i}+3\hat{j}+4\hat{k}}$

$\mathbf{\overrightarrow{QT}\cdot\vec{b}=0}$

$\mathbf{4\lambda-12+9\lambda+3+16\lambda-20=0}$

$\mathbf{\lambda=1}$

$\mathbf{T(3,2,4)}$

$\mathbf{QT=\sqrt{33}\quad RT=\sqrt{29}}$

$\mathbf{(\text{area of }\triangle PQR)^2=\left(\frac{1}{2}\sqrt{29}.2\sqrt{33}\right)^2}$

$\mathbf{=957}$

Question 184

Let (α, β, γ) be mirror image of the point $(2, 3, 5)$ in the line $\frac{x-1}{2} - \frac{y-2}{3} - \frac{z-3}{4}$.
Then $2\alpha + 3\beta + 4\gamma$ is equal to

A. 32

B. 33

C. 31

D. 34

Correct answer: B

Source solution

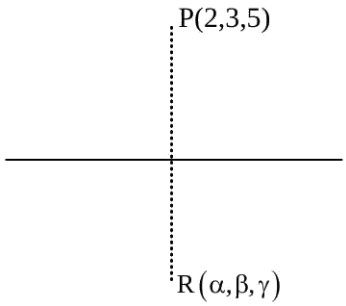
Sol.

$\because \overrightarrow{PR} \perp (2, 3, 4)$

$\therefore \overrightarrow{PR} \cdot (2, 3, 4) = 0$

$(\alpha - 2, \beta - 3, \gamma - 5) \cdot (2, 3, 4) = 0$

$\Rightarrow 2\alpha + 3\beta + 4\gamma = 4 + 9 + 20 = 33$



Question 185

If the mirror image of the point $\mathbf{P(3,4,9)}$ in the line $\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-2}{1}$ is (α,β,γ), then $14(\alpha+\beta+\gamma)$ is :

A. 102

B. 138

C. 108

D. 132

Correct answer: C

Source solution

Sol.

$\overrightarrow{PN}.\vec{b}=0$?

$3(3\lambda-2)+2(2\lambda-5)+(\lambda-7)=0$

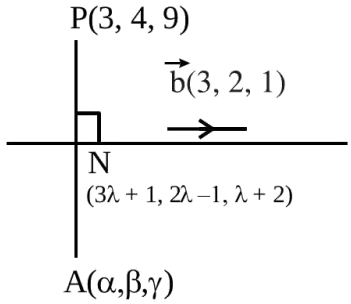
$14\lambda=23\Rightarrow\lambda=\frac{23}{14}$

$N\left(\frac{83}{14},\frac{32}{14},\frac{51}{14}\right)$

$\therefore\ \frac{\alpha+3}{2}=\frac{83}{14}\Rightarrow\alpha=\frac{62}{7}$

$\frac{\beta+4}{2}=\frac{32}{14}\Rightarrow\beta=\frac{4}{7}$

$\frac{\gamma+9}{2}=\frac{51}{14}\Rightarrow\gamma=\frac{-12}{7}$



Question 186

If $\alpha=1+\sum_{r=1}^{6}(-3)^{r-1}{}^{12}C_{2r-1}$, then the distance of the point $(12,\sqrt{3})$ form the line $\alpha x-\sqrt{3}y+1=0$ is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 5

Source solution

$\alpha=1+\sum_{r=1}^{6}(-1)^{r-1}{}^{12}C_{2r-1}3^{r-1}$

$$\alpha=1+\sum_{r=1}^{6}{}^{12}C_{2r-1}\frac{(\sqrt{3}i)^{2r-1}}{\sqrt{3}i}\quad i=\text{ iota , let }\sqrt{3}i=x$$

$$\alpha=1+\frac{1}{\sqrt{3}i}\left({}^{12}C_1x+{}^{12}C_3x^3+\ldots.{}^{12}C_{11}x^{11}\right)$$

$$=1+\frac{1}{\sqrt{3}i}\left(\frac{(1+\sqrt{3}i)^{12}-(1-\sqrt{3}i)^{12}}{2}\right)$$

$$=1+\frac{1}{\sqrt{3}i}\left(\frac{(-2\omega^2)^{12}-(2\omega)^{12}}{2}\right)=1$$

so distance of $(12,\sqrt{3})$ from $x-\sqrt{3}y+1=0$ is $\frac{12-3+1}{2}=5$

Question 187

Let the line L be the projection of the line

$$\frac{x-1}{2}=\frac{y-3}{1}=\frac{z-4}{2}$$

in the plane $x-2y-z=3$. If d is the distance of the point $(0,0,6)$ from L , then d^2 is equal to

.

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

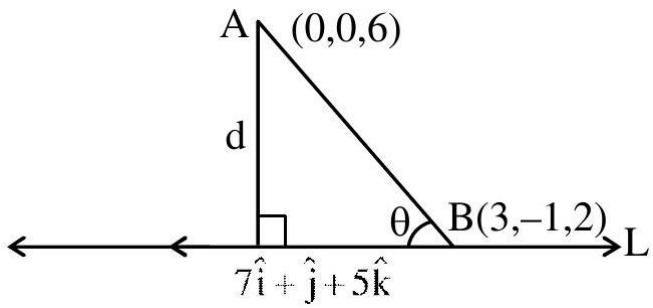
Correct answer: 26

Source solution

$L_1: \frac{x-1}{2}=\frac{y-3}{1}=\frac{z-4}{2}$
for foot of $\perp r$ of (1, 3, 4) on $x-2y-z-3=0$
$(1+t)-2(3-2t)-(4-t)-3=0$
$\Rightarrow t=2$

So foot of $\perp r \overset{\wedge}{=} (3,-1,2)$
& point of intersection of L_1 with plane
is $(-11,-3,-8)$
dr's of L is $<14,2,10>$

$$\cong <7,1,5>$$



$$d = AB\sin\theta = \frac{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & -4 \\ 7 & 1 & 5 \end{vmatrix}}{\sqrt{7^2+1^2+5^2}}$$

$\Rightarrow d^2=\frac{1^2+(43)^2+(10)^2}{49+1+25}=26$

Question 188

Let L be a line obtained from the intersection of two planes $x+2y+z=6$ and $y+2z=4$. If point $P(\alpha,\beta,\gamma)$ is the foot of perpendicular from $(3,2,1)$ on L, then the value of $21(\alpha+\beta+\gamma)$ equals :

A. 142

B. 68

C. 136

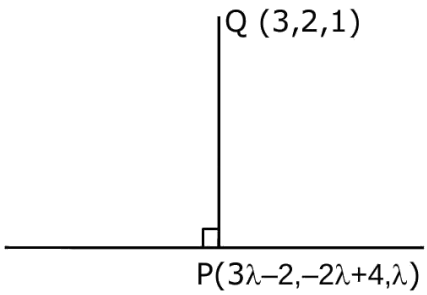
D. 102

Correct answer: D

Source solution

Sol. Dr's of line $\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 1\\ 0 & 1 & 2\end{vmatrix}=3\hat{i}-2\hat{j}+\hat{k}$
Dr/s :- $(3,-2,1)$
Points on the line $(-2,4,0)$
Equation of the line $\frac{x+2}{3}=\frac{y-4}{-2}=\frac{z}{1}=\lambda$

Dr's of PQ ; $3\lambda-5,-2\lambda+2.\lambda-1$
Dr's of y lines are $(3,-2,1)$
Since $PQ\perp$ line
$3(3\lambda-5)-2(-2\lambda+2)+1(\lambda-1)=0$
$\lambda=\frac{10}{7}$
$P\left(\frac{16}{7},\frac{8}{7},\frac{10}{7}\right)$
$21(\alpha+\beta+\gamma)=21\left(\frac{34}{7}\right)=102$



Question 189

Let P and Q be the points on the line $\frac{x+3}{8}=\frac{y-4}{2}=\frac{z+1}{2}$ which are at a distance of 6 units from the point $\mathbf{R(1,2,3)}$. If the centroid of the triangle PQR is (α,β,γ), then $\alpha^2+\beta^2+\gamma^2$ is:

A. 26

B. 36

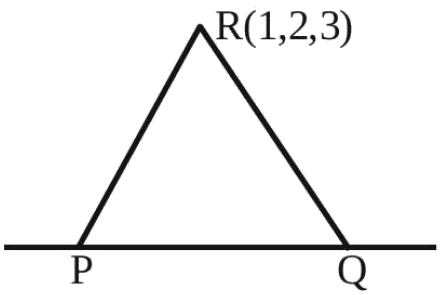
C. 18

D. 24

Correct answer: C

Source solution

Sol.
$\mathbf{P(8\lambda-3,2\lambda+4,2\lambda-1)}$
$\mathbf{PR=6}$
$\mathbf{(8\lambda-4)^2+(2\lambda+2)^2+(2\lambda-4)^2=36}$
$\boldsymbol{\lambda=0,1}$
Hence $\mathbf{P(-3,4,-1)\&Q(5,6,1)}$
Centroid of $\mathbf{\triangle PQR=(1,4,1)\equiv(\alpha,\beta,\gamma)}$
$\boldsymbol{\alpha^2+\beta^2+\gamma^2=18}$



Question 190

$\beta\gamma$ be the foot of perpendicular from the point $\mathbf{(1,2,3)}$ on the line $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}$. then $\mathbf{19(\alpha+\beta+\gamma)}$ is equal to :

A. 102

B. 101

C. 99

D. 100

Correct answer: B

Source solution

Sol.
$\frac{(1,2,3)}{P(\alpha,\beta,\gamma)}$

Let foot $\mathbf{P(5k-3,2k+1,3k-4)}$
DR's $\rightarrow$ AP: $\mathbf{5k-4,2k-1,3k-7}$
DR's $\rightarrow$ Line: 5, 2, 3

Condition of perpendicular lines $\mathbf{(25k-20)+(4k-2)+(9k-21)=0}$

Then $\mathbf{k=\frac{43}{38}}$

Then $\mathbf{19(\alpha+\beta+\gamma)=101}$

Question 191

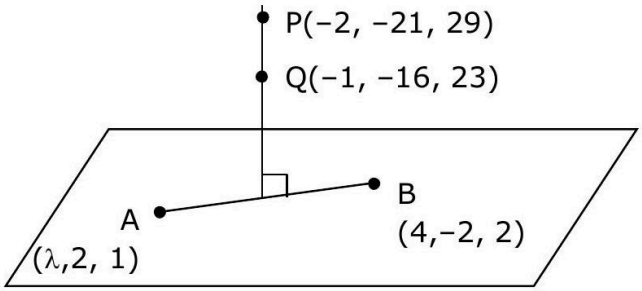
Let $(\lambda, 2, 1)$ be a point on the plane which passes through the point $(4, -2, 2)$. If the plane is perpendicular to the line joining the point $(-2, -21, 29)$ and $(-1, -16, 23)$, then $\left(\frac{\lambda}{11}\right)^2 - \frac{4\lambda}{11} - 4$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 8

Source solution



$\overrightarrow{AB} \perp \overrightarrow{PQ}$

$$[(4-\lambda)\hat{i} - 4\hat{j} + \hat{k}] \cdot [+\hat{i} + 5\hat{j} - 6\hat{k}] = 0$$
$$4 - \lambda - 20 - 6 = 0$$
$$\lambda = -22$$

Now, $\frac{\lambda}{11} = -2$

$$\Rightarrow \left(\frac{\lambda}{11}\right)^2 - \frac{4\lambda}{11} - 4$$
$$\Rightarrow 4 + 8 - 4 = 8$$

Question 192

If the image of the point $(4,4,3)$ in the line $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-1}{3}$ is (α,β,γ), then $\alpha+\beta+\gamma$ is equal to

A. 9

B. 12

C. 8

D. 7

Correct answer: A

Source solution

Sol. $\frac{\int^{P(4,4,3)}}{Q(2\lambda+1,\lambda+2,3\lambda+11)}\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-1}{3}$

$\overrightarrow{PQ}\perp(2\hat{i}+\hat{j}+3\hat{k})$

$\Rightarrow 2(2\lambda-3)+1(\lambda-2)+3(3\lambda-2)=0$

$\Rightarrow 14\lambda-14=0,\lambda=1$

So, $Q(3,3,4)$

Let image in $R(\alpha,\beta,\gamma)$

$\frac{\alpha+4}{2}=3,\frac{\beta+4}{2}=3,\frac{\gamma+3}{2}=4$

$(\alpha,\beta,\gamma)=(2,2,5)$

$\Rightarrow \alpha+\beta+\gamma=9$

Question 193

A circle with centre $(2,3)$ and radius 4 intersects the line $x+y=3$ at the points P and Q. If the tangents at P and Q intersect at the point $S(\alpha,\beta)$, then $4\alpha-7\beta$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 11

Source solution

11

$(x-2)^2+(y-3)^2=16$

$x^2+y^2-4x-6y-3=0$

let the chord of contact w.r. to the point $S(\alpha,\beta)$ is

$T=0$

$\alpha x+\beta y-2(x+\alpha)-3(y+\beta)-3=0$

$x(\alpha-2)+y(\beta-3)-2\alpha-3\beta-3=0$

Comparison with $x+y=3$

$\frac{\alpha-2}{1}=\frac{\beta-3}{1}=\frac{-2\alpha-3\beta-3}{-3}$

$\alpha-2=\beta-3 \quad -3\beta+9=-2\alpha-3\beta-3$

$\alpha-\beta=-1 \quad 2\alpha=-3-9$

$\beta=\alpha+1 \quad \alpha=\frac{-12}{2}$

$=-6+1=-5 \quad \alpha=-6$

$4\alpha-7\beta$

$4(-6)-7(-5)$

$\Rightarrow -24+35=11$

Question 194

If $Q \equiv \left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$ is foot of perpendicular drawn from $P(1,0,3)$ onto a line L and line L is passing through $(\alpha, 7, 1)$, then value of α is .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 4

Source solution

Solution:

Direction ratios of line L : $\left(\alpha - \frac{5}{3}, 7 - \frac{7}{3}, 1 - \frac{17}{3}\right)$
$= \left(\frac{3\alpha-5}{3}, \frac{14}{3}, -\frac{14}{3}\right)$
Direction ratios of PQ : $\left(-\frac{2}{3}, -\frac{7}{3}, -\frac{8}{3}\right)$
As line L is perpendicular to PQ
So, $\left(\frac{3\alpha-5}{3}\right)\left(-\frac{2}{3}\right) + \left(\frac{14}{3}\right)\left(-\frac{7}{3}\right) + \left(-\frac{14}{3}\right)\left(-\frac{8}{3}\right) = 0$
$\Rightarrow -6\alpha + 10 - 98 + 112 = 0$
$\Rightarrow 6\alpha = 24 \Rightarrow \alpha = 4$

Question 195

Let the plane P pass through the intersection of the planes $2x + 3y - z = 2$ and $x + 2y + 3z = 6$ and be perpendicular to the plane $2x + y - z = 0$. If d is the distance of P form the point ($-7, 1, 1$,) then d^2 is equal to :

A. $\frac{250}{83}$

B. $\frac{250}{82}$

C. $\frac{15}{53}$

D. $\frac{25}{83}$

Correct answer: A

Source solution

Sol. 1
Plane P, is passing through intersection of the two planes, so,
$2x + 3y - z - 2 + \lambda(x + 2y + 3z - 6) = 0$
$x(2 + \lambda) + y(3 + 2\lambda) + z(3\lambda - 1) - 2 - 6\lambda = 0$
It is perpendicular with plane, $2x + y - 2 + 1 = 0$
So, $(2 + \lambda)2 + (3 + 2\lambda)1 + (3\lambda - 1)(-1) = 0$
$\lambda = -8$
So, plane $p_1 - 6x - 13y - 25z + 46 = 0$
distance of plane p from the point ($-7, 1, 1$)
$d = \frac{|+42-13-25+46|}{\sqrt{36+169+625}} = \frac{50}{\sqrt{30}} =$
$d^2 = \frac{2500}{830} = \frac{250}{83}$

Question 196

Let a line L pass through the point $P(2,3,1)$ and be parallel to the line $x+3y-2z-2=0=x-y+$ 2 z . If the distance of L from the point $(5,3,8)$ is α, then $3\alpha^2$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 158

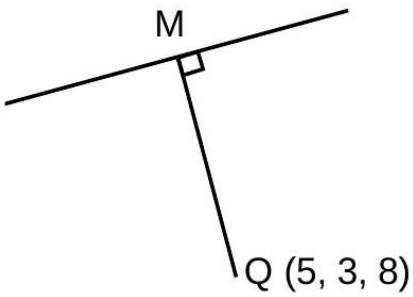
Source solution

158

The Direction ratio of line

$$\begin{vmatrix} i & j & k \\ 1 & 3 & -2 \\ 1 & -1 & 2 \end{vmatrix} = i(6-2) - j(2+2) + k(-1-3)$$

Equation of line L



$\frac{x-2}{1} = \frac{y-3}{-1} = \frac{z-1}{-1} = \lambda(\alpha d)$

Let $M(\lambda+2, -\lambda+3, -\lambda+1)$

DR's of MQ is $< \lambda+2-5, -\lambda+3-3, -\lambda+1-8 >$

$$< \lambda-3, -\lambda. -\lambda > 7 >$$

$\because L \perp MQ$

$\Rightarrow (\lambda-3)(1) + (-\lambda)(-1) + (-\lambda-7)(-1) = 0$

$\Rightarrow \lambda - 3 + \lambda + \lambda + 7 = 0$

$\Rightarrow 3\lambda = -4 \Rightarrow \lambda = -\frac{4}{3}$

$\therefore M\left(-\frac{4}{3}+2, \frac{+4}{3}+3, \frac{4}{3}+1\right) = \left(\frac{2}{3}, \frac{13}{3}, \frac{7}{3}\right)$

$MQ = \alpha$

$\therefore 3\alpha^2 = 3 \times \left(\left(5-\frac{2}{3}\right)^2 + \left(3-\frac{13}{3}\right)^2 + \left(8-\frac{7}{5}\right)^2\right)$

$= 3\left(\frac{169}{9} + \frac{16}{9} + \frac{289}{9}\right) \Rightarrow \frac{474}{9} = 158$

Question 197

Let the equation of the plane P containing the line $x+10=\frac{8-y}{2}=z$ be $ax+by+3z=2(a+b)$ and the distance of the plane P from the point $(1,27,7)$ be c. Then $a^2+b^2+c^2$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 355

Source solution

355

Given equation of plane is

$$ax+by+3z=2(a+b) \tag{1}$$

It containing the line

$\frac{x-(-10)}{1}=\frac{y-8}{-2}=\frac{z-0}{1}$

$\therefore$ plane (1) must passes through $(-10,8,0)$ and parallel to $1,-2,1$

Hence,

$$\begin{aligned} & a(-10)+8b=2a+2b \\ \Rightarrow \quad & 12a-6b=0 \\ \text{and} \quad & a-2b+3=0 \end{aligned} \tag{3}$$

on solving (2) and (3), we get

$$b=2, a=1$$

$\therefore$ equation of the plane is

$$x+2y+3z=6 \tag{4}$$

c is perpendicular distance from $(1,27,7)$ to the plane (4)

$\Rightarrow c=\left|\frac{1+2\times 27+3\times 7-6}{\sqrt{1^2+2^2+3^2}}\right|=\left|\frac{70}{\sqrt{14}}\right|=\frac{10\sqrt{7}}{\sqrt{2}}$

Now, $\mathrm{a}^2+\mathrm{b}^2+\mathrm{c}^2=1+4+\frac{700}{2}=\frac{710}{2}=355$

Question 198

Let the line $L: \frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{1}$ intersect the plane $2x + y + 3z = 16$ at the point P. Let the point Q be the foot of perpendicular from the point $R(1, -1, -3)$ on the line L. If α is the area of triangle PQR, then α^2 is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 180

Source solution

180

Point on line L is $(2\lambda + 1, -\lambda - 1, \lambda + 3)$

If above point is intersection point of line L and plane then

$2(2\lambda + 1) + (-\lambda - 1) + 3(\lambda + 3) = 16$

$\lambda = 1$

Point $\mathrm{P} = (3, -2, 4)$

Dr of $\mathrm{QR} =< 2\lambda, -\lambda, \lambda + 6 >$

Dr of $\mathrm{L} = \langle 2, -1, 1 \rangle$

$4\lambda + \lambda + \lambda + 6 = 0$

$\lambda = -1$

$\mathrm{Q} = (-1, 0, 2)$

$C^{P(3,-2,4)}_{(-1,0,2)}\mathrm{R}(1, -1, -3)$

$\overrightarrow{\mathrm{QR}} = 2\hat{i} - \hat{j} - 5\hat{k}$

$\overrightarrow{\mathrm{QP}} = 4\hat{i} - 2\hat{j} + 2\hat{k}$

$$\overrightarrow{\mathrm{QR}} \times \overrightarrow{\mathrm{QP}} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & -5 \\ 4 & -2 & 2 \end{vmatrix} = -12\hat{i} - 24\hat{j}$$

$\alpha = \frac{1}{2} \times \sqrt{144 + 576}$

$\alpha^2 = \frac{720}{4} = 180$

$\alpha^2 = 180$

Question 199

Let the plane P pass through the intersection of the planes $2x + 3y - z = 2$ and $x + 2y + 3z = 6$ and be perpendicular to the plane $2x + y - z = 0$. If d is the distance of P from the point $(-7, 1, 1)$, then d^2 is equal to:

A. $\frac{250}{83}$

B. $\frac{250}{82}$

C. $\frac{15}{53}$

D. $\frac{25}{83}$

Correct answer: A

Source solution

Let the plane P pass through the intersection of the two planes:

$$2x + 3y - z - 2 + \lambda(x + 2y + 3z - 6) = 0$$

$$\Rightarrow x(2 + \lambda) + y(3 + 2\lambda) + z(3\lambda - 1) - 2 - 6\lambda = 0$$

The plane is perpendicular to $2x + y - 2 + 1 = 0$.

$$\Rightarrow (2 + \lambda) \cdot 2 + (3 + 2\lambda) \cdot 1 + (3\lambda - 1)(-1) = 0$$

$$\Rightarrow 4 + 2\lambda + 3 + 2\lambda - 3\lambda + 1 = 0$$

$$\Rightarrow 2\lambda + 5 = 0 \quad \Rightarrow \quad \lambda = -8$$

Thus, the equation of the plane is: $p_1 - 6x - 13y - 25z + 46 = 0$.

Now, we find the distance of the plane p from the point $(-7, 1, 1)$.

$$d = \frac{|6(-7) + 13(1) + 25(1) - 46|}{\sqrt{(-6)^2 + 13^2 + 25^2}}$$

$$= \frac{|-42 + 13 + 25 - 46|}{\sqrt{36 + 169 + 625}} = \frac{|-50|}{\sqrt{830}} = \frac{50}{\sqrt{830}}.$$

$$\text{Thus, } d^2 = \left(\frac{50}{\sqrt{830}}\right)^2 = \frac{2500}{830} = \frac{250}{83}.$$

Question 200

Let $\boldsymbol{P}$ be a plane containing the line $\frac{x-1}{3}=\frac{y+6}{4}=\frac{z+5}{2}$ and parallel to the line $\frac{x-3}{4}=\frac{y-2}{-3}=\frac{z+5}{7}$. If the point $(1,-1,\alpha)$ lies on the plane P , then the value of $|5\alpha|$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 38

Source solution

Equation of plane is $\begin{vmatrix} x-1 & y+6 & z+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix}=0$

Now $(1,-1,\alpha)$ lies on it so

$$\begin{vmatrix} 0 & 5 & \alpha+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix}=0 \Rightarrow 5\alpha+38=0 \Rightarrow |5\alpha|=38$$

Question 201

If a point $\mathbf{P}(\alpha, \beta, \gamma)$ satisfying
$(\alpha\beta\gamma)\,(2 \quad 10 \quad 89 \quad 3 \quad 88 \quad 4 \quad 8) = (000)$
lies on the plane $2x + 4y + 3z = 5$, then $6\alpha + 9\beta + 7\gamma$ is equal to :

A. -1

B. $\frac{11}{5}$

C. $\frac{5}{4}$

D. 11

Correct answer: D

Source solution

Given

$$(\alpha, \beta, \gamma)\begin{pmatrix} 2 & 10 & 89 \\ 3 & 88 & 4 \\ 8 & 0 & 0 \end{pmatrix} = (0, 0, 0).$$

Hence,

$$\begin{aligned} 2\alpha + 3\beta + 8\gamma &= 0, \\ 10\alpha + 88\beta &= 0, \\ 89\alpha + 4\beta &= 0. \end{aligned}$$

From the second equation,

$$10\alpha + 88\beta = 0$$

$$5\alpha + 44\beta = 0$$

$$\alpha = -\frac{44}{5}\beta.$$

Substituting into

$$89\alpha + 4\beta = 0,$$

we get

$$89\left(-\frac{44}{5}\beta\right) + 4\beta = 0.$$

$$\left(-\frac{3916}{5} + \frac{20}{5}\right)\beta = 0.$$

$$-\frac{3896}{5}\beta = 0.$$

$$\beta = 0.$$

Therefore,

$$\alpha = 0.$$

Using

$$2\alpha + 3\beta + 8\gamma = 0,$$

we obtain

$$8\gamma = 0$$

$$\gamma = 0.$$

Thus the point is

$$P = (0, 0, 0).$$

Since P lies on the plane

$$2x + 4y + 3z = 5,$$

the required expression

$$6\alpha + 9\beta + 7\gamma$$

must satisfy the plane equation. Observe that

$$(6\alpha + 9\beta + 24\gamma) = 3(2\alpha + 3\beta + 8\gamma) = 0.$$

Hence

$$6\alpha + 9\beta + 7\gamma = -17\gamma.$$

Also, from the plane condition

$$2\alpha + 4\beta + 3\gamma = 5.$$

Using

$$2\alpha + 3\beta + 8\gamma = 0,$$

subtracting gives

$$\beta - 5\gamma = 5.$$

Solving together with

$$10\alpha + 88\beta = 0, \qquad 89\alpha + 4\beta = 0,$$

yields

$$\gamma = -\frac{11}{17}.$$

Therefore,

$$6\alpha + 9\beta + 7\gamma = -17\gamma = 11.$$

$$\boxed{6\alpha + 9\beta + 7\gamma = 11}$$

Hence the correct option is

$$\boxed{(4)\ 11}.$$

Question 202

If (x, y, z) be an arbitrary point lying on a plane P which passes through the point $(42, 0, 0)$, $(0, 42, 0)$ and $(0, 0, 42)$, then the value of expression

$$3+\frac{x-11}{(y-19)^2(z-12)^2}+\frac{y-19}{(x-11)^2(z-12)^2}+\frac{z-12}{(x-11)^2(y-19)^2}-\frac{x+y+z}{14(x-11)(y-19)(z-12)}$$

A. 0

B. 3

C. 39

D. -45

Official Answer by NTA (2)

Correct answer: B

Source solution

Sol.

The plane passes through the points $(42, 0, 0)$, $(0, 42, 0)$, and $(0, 0, 42)$.

1. The equation of the plane can be derived from the intercept form:

$$x+y+z=42$$

2. We can rewrite this as:

$$(x-11)+(y-19)+(z-12)=0$$

3. Let:

$$a=x-11, \quad b=y-19, \quad c=z-12$$

Thus, we have:

$$a+b+c=0$$

4. Now, consider the given expression:

$$3+\frac{a}{b^2c^2}+\frac{b}{a^2c^2}+\frac{c}{a^2b^2}-\frac{42}{14abc}$$

5. This can be simplified to:

$$3+\frac{a^3+b^3+c^3-3abc}{a^2b^2c^2}$$

6. Since $a+b+c=0$, we can use the identity:

$$a^3+b^3+c^3=3abc$$

7. Therefore, we have:

$$a^3 + b^3 + c^3 - 3abc = 0$$

8. Substituting this back, we find:

$$3 + 0 = 3$$

Final answer:

$$3$$

Question 203

If (x, y, z) be an arbitrary point lying on a plane P which passes through the point $(42, 0, 0)$, $(0, 42, 0)$ and $(0, 0, 42)$, then the value of expression
$3 + \frac{x-11}{(y-19)^2(z-12)^2} + \frac{y-19}{(x-11)^2(z-12)^2}$
$+ \frac{z-12}{(x-11)^2(y-19)^2} - \frac{x+y+z}{14(x-11)(y-19)(z-12)}$

A. 0

B. 3

C. 39

D. -45

Official Answer by NTA (2)

Correct answer: B

Source solution

Sol. Plane passing through $(42, 0, 0), (0, 42, 0)$, (0, 0, 42)
From intercept from, equation of plane is
$x + y + z = 42$
$\Rightarrow (x - 11) + (y - 19) + (z - 12) = 0$
let $a = x - 11,\ b = y - 19, c = z - 12$
$a + b + c = 0$
Now, given expression is
$3 + \frac{a}{b^2c^2} + \frac{b}{a^2c^2} + \frac{c}{a^2b^2} - \frac{42}{14abc}$
$3 + \frac{a^3+b^3+c^3-3abc}{a^2b^2c^2}$
If $a + b + c = 0$
$\Rightarrow a^3 + b^3 + c^3 = 3abc$
$\Rightarrow 3$

Question 204

If the shortest distance between the line joining the points $(1, 2, 3)$ and $(2, 3, 4)$, and the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-2}{0}$ is a, then $28a^2$ is equal to

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 18

Source solution

18
A (1, 2, 3)
B $(2, 3, 4)$

Equation of line AB
$\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{1}$
Given line
$\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-2}{0}$
shortest distance $= \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_2 \times \vec{b}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$
$= \left| \frac{(3\hat{j} - \hat{k}) \cdot (\hat{i} + 2\hat{j} - 3\hat{k})}{\sqrt{1+4+9}} \right|$
$\alpha = \frac{3}{\sqrt{14}}$
$28\alpha^2 = 28 \times \frac{9}{14} = 18$

Question 205

The distance of the point $(7, -3, -4)$ from the plane passing through $(2, -3, 1), (-1, 1, -2)$ and $(3, -4, 2)$ is:

A. 5

B. 4

C. $5\sqrt{2}$

D. $4\sqrt{2}$

Correct answer: C

Source solution

The plane is $x - z - 1 = 0$. The distance is $|7 + 4 - 1|/\sqrt{2} = 5\sqrt{2}$.

Question 206

The distance of the point $P(4, 6, -2)$ from the line passing through the point $(-3, 2, 3)$ and parallel to a line with direction ratios $3, 3, -1$ is equal to :

A. $\sqrt{14}$

B. 3

C. $\sqrt{6}$

D. $2\sqrt{3}$

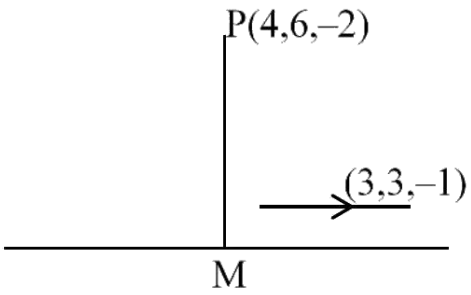
Correct answer: A

Source solution

Sol. 1}
equation of line

$$\bar{r} = (-3, 2, 3) + \lambda(3, 3, -1)$$

$\overline{PM} \cdot (3, 3, -1) = 0$
$(3\lambda - 7, 3\lambda - 4, 5 - \lambda) \cdot (3, 3, -1) = 0$
$\Rightarrow 3(3\lambda - 7) + 3(3\lambda - 4) - 1(15 - \lambda) = 0$
$\Rightarrow 19\lambda = 38 \Rightarrow \lambda = 2$
$(-3 + 3\lambda, 2 + 3\lambda, 3 - \lambda)$
$M = (3, 8, 1)$
$PM = \sqrt{1 + 4 + 9} = \sqrt{14}$



Question 207

The distance of the point $P(4, 6, -2)$ from the line passing through the point $(-3, 2, 3)$ and parallel to a line with direction ratios $3, 3, -1$ is equal to :

A. $\sqrt{14}$

B. 3

C. $\sqrt{6}$

D. $2\sqrt{3}$

Correct answer: A

Source solution

Sol. 1}

The equation of the line is given by:

$$\bar{r} = (-3, 2, 3) + \lambda(3, 3, -1)$$

To find the point M on the line such that the vector $\overline{PM}$ is perpendicular to the direction vector $(3, 3, -1)$, we set up the equation:

$$\overline{PM} \cdot (3, 3, -1) = 0$$

Let $\overline{PM} = (3\lambda - 7, 3\lambda - 4, 5 - \lambda)$. Then, we have:

$$(3\lambda - 7, 3\lambda - 4, 5 - \lambda) \cdot (3, 3, -1) = 0$$

Expanding this dot product gives:

$$3(3\lambda - 7) + 3(3\lambda - 4) - 1(5 - \lambda) = 0$$

Simplifying the equation:

$$9\lambda - 21 + 9\lambda - 12 - 5 + \lambda = 0$$

Combining like terms results in:

$$19\lambda - 38 = 0$$

Solving for λ:

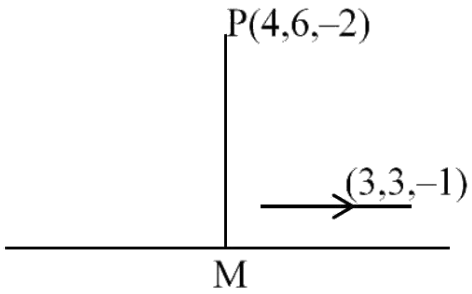
$$19\lambda = 38 \quad \Rightarrow \quad \lambda = 2$$

Now substituting λ back into the equation of the line to find point M:

$$M = (-3 + 3\lambda, 2 + 3\lambda, 3 - \lambda) = (-3 + 6, 2 + 6, 3 - 2) = (3, 8, 1)$$

Finally, we calculate the length of $\overline{PM}$:

$$\text{PM} = \sqrt{(3-7)^2 + (8-4)^2 + (1-5)^2} = \sqrt{(-4)^2 + (4)^2 + (-4)^2} = \sqrt{16+16+16} = \sqrt{48} = \sqrt{14}$$



Question 208

Let P be a plane $lx + my + nz = 0$ containing the line, $\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}$. If plane P divides the line segment AB joining points $A(-3, -6, 1)$ and $B(2, 4, -3)$ in ratio $k : 1$ then the value of k is equal to :

A. 1.5

B. 3

C. 2

D. 4

Correct answer: C

Source solution

Sol.

Point C is

$\left(\frac{2k-3}{k+1}, \frac{4k-6}{k+1}, \frac{-3k+1}{k+1}\right)$

$\frac{x-1}{-1} = \frac{y+4}{2} = \frac{z+2}{3}$

Plane $lx + my + nz = 0$

$l(-1) + m(2) + n(3) = 0$

$$-l + 2m + 3n = 0 \quad (1)$$

It also satisfy point ($1, -4, -2$)

$$l - 4m - 2n = 0 \quad (2)$$

Solving (1) and (2)

$2m + 3n = 4m + 2n$

$n = 2m$

$l - 4m - 4m = 0$

$l = 8m$

$\frac{l}{8} = \frac{m}{1} = \frac{n}{2}$

$l : m : n = 8 : 1 : 2$

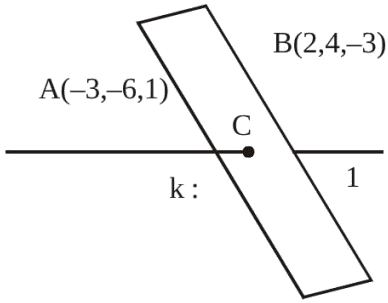
Plane is $8x + y + 2z = 0$

It will satisfy point C

$8\left(\frac{2k-3}{k+1}\right) + \left(\frac{4k-6}{k+1}\right) + 2\left(\frac{-3k+1}{k+1}\right) = 0$

$16k - 24 + 4k - 6 - 6k + 2 = 0$

$14k = 28 \quad \therefore \quad k = 2$



Question 209

Let P be the plane containing the straight line

$$\frac{x-3}{9}=\frac{y+4}{-1}=\frac{z-7}{-5}$$

and perpendicular to the plane containing the straight lines

$$\frac{x}{2}=\frac{y}{3}=\frac{z}{5}$$

and

$$\frac{x}{3}=\frac{y}{7}=\frac{z}{8}.$$

If d is the distance of P from the point $(2,-5,11)$, then d^2 is equal to :

A. $\frac{147}{2}$

B. 96

C. $\frac{32}{3}$

D. 54

Correct answer: C

Source solution

Sol.

Given the equation of the plane:

$$a(x-3)+b(y+4)+c(z-7)=0$$

We have the following system of equations:

$$\begin{aligned} 9a-b-5c&=0 \\ -11a-b+5c&=0 \end{aligned}$$

Solving this system, we find the direction ratios proportional to $(1,-1,2)$.

Thus, the equation of the plane can be expressed as:

$$x-y+2z=21$$

Next, we calculate the distance d from the point to the plane:

$$d=\frac{8}{\sqrt{6}}$$

Squaring the distance gives:

$$d^2 = \frac{32}{3}$$

Question 210

If the four points, whose position vectors are $3\hat{i}-4\hat{j}+2\hat{k}, \hat{i}+2\hat{j}-\hat{k}, -2\hat{i}-\hat{j}+3\hat{k}$ and $5\hat{i}-2\alpha\hat{j}+4\hat{k}$ are coplanar, then α is equal to

A. $\frac{73}{17}$

B. $\frac{107}{17}$

C. $\frac{-73}{17}$

D. $\frac{-107}{17}$

Correct answer: A

Source solution

Sol.

Given the points:

$$\begin{aligned}\mathbf{P} &= 3\hat{\mathbf{i}}-4\hat{\mathbf{j}}+2\hat{\mathbf{k}},\\ \mathbf{Q} &= \hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}},\\ \mathbf{R} &= -2\hat{\mathbf{i}}-\hat{\mathbf{j}}+3\hat{\mathbf{k}},\\ \mathbf{S} &= 5\hat{\mathbf{i}}-2\alpha\hat{\mathbf{j}}+4\hat{\mathbf{k}}.\end{aligned}$$

Calculate the vectors:

$$\begin{aligned}\overrightarrow{\mathbf{PQ}} &= \mathbf{Q}-\mathbf{P} = (-2\hat{\mathbf{i}}+6\hat{\mathbf{j}}-3\hat{\mathbf{k}}),\\ \overrightarrow{\mathbf{QR}} &= \mathbf{R}-\mathbf{Q} = (-3\hat{\mathbf{i}}-3\hat{\mathbf{j}}+4\hat{\mathbf{k}}),\\ \overrightarrow{\mathbf{RS}} &= \mathbf{S}-\mathbf{R} = (7\hat{\mathbf{i}}+(1-2\alpha)\hat{\mathbf{j}}+\hat{\mathbf{k}}).\end{aligned}$$

Set the scalar triple product to zero:

$$[\overrightarrow{\mathbf{PQ}}, \overrightarrow{\mathbf{QR}}, \overrightarrow{\mathbf{RS}}] = 0.$$

This leads to the determinant:

$$\begin{vmatrix}-2 & 6 & -3\\ -3 & -3 & 4\\ 7 & 1-2\alpha & 1\end{vmatrix} = 0.$$

Calculating the determinant:

$$-2(-3+8\alpha-4)-6(-31)-3(6\alpha-3+21) = 0.$$

Simplifying gives:

$$-2(8\alpha-7)+186-3(6\alpha+18) = 0.$$

Solving for α:

$$-16\alpha+14+186-18\alpha-54 = 0,$$

$$-34\alpha + 146 = 0,$$

$$\alpha = \frac{146}{34} = \frac{73}{17}.$$

Final answer:

$$\alpha = \frac{73}{17}.$$

Question 211

Let P be the plane containing the straight line $\frac{x-3}{9} = \frac{y+4}{-1} = \frac{z-7}{-5}$ and perpendicular to the plane containing the straight lines $\frac{x}{2} = \frac{y}{3} = \frac{z}{5}$ and $\frac{x}{3} = \frac{y}{7} = \frac{z}{8}$. If d is the distance of P from the point $(2, -5, 11)$, then d^2 is equal to :

A. $\frac{147}{2}$

B. 96

C. $\frac{32}{3}$

D. 54

Correct answer: D

Source solution

Sol. $a(x-3) + b(y+4) + c(z-7) = 0$

$$P: 9a - b - 5c = 0 - 11a - b + 5c = 0$$

After solving DR's $\propto (1, -1, 2)$

Equation of plane

$x - y + 2z = 21$

$d = \frac{8}{\sqrt{6}}$

$d^2 = \frac{32}{3}$

Question 212

If the length of the perpendicular drawn from the point $\mathbf{P(a, 4, 2), a > 0}$ on the line $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1}$ is $2\sqrt{6}$ units and $\mathbf{Q}(\alpha_1, \alpha_2, \alpha_3)$ is the image of the point P in this line, then $\mathbf{a} + \sum_{i=1}^{3} \alpha_i$ is equal to :

A. 7

B. 8

C. 12

D. 14

Correct answer: B

Source solution

Sol.

$(2\lambda - 1 - a)2 + (3\lambda - 1)3 + (-\lambda - 1)(-1) = 0$

$\Rightarrow 4\lambda - 2 - 2a + 9\lambda - 3 + \lambda + 1 = 0$

$\Rightarrow 14\lambda - 4 - 2a = 0$

$\Rightarrow 7\lambda - 2 - a = 0$

and,

$(2\lambda - 1 - a)^2 + (3\lambda - 1)^2 + (\lambda + 1)^2 = 24$

$\Rightarrow (5\lambda - 1)^2 + (3\lambda - 1)^2 + (\lambda + 1)^2 = 24$

$\Rightarrow 35\lambda^2 - 14\lambda - 21 = 0$

$\Rightarrow (\lambda - 1)(35\lambda + 21) = 0$

For, $\lambda = 1 \quad \Rightarrow a = 5$

Let $(\alpha_1, \alpha_2, \alpha_3)$ be reflection of point P

$\alpha_1 + 5 = 2$

$\alpha_2 + 4 = 12$

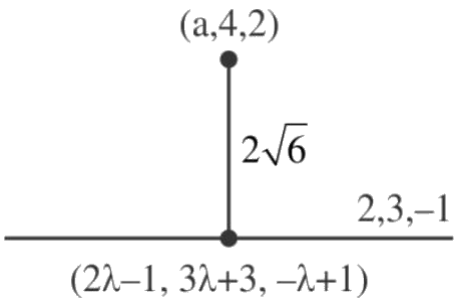
$\alpha_3 + 2 = 0$

$\alpha_1 = -3$

$\alpha_2 = 8$

$\alpha_3 = -2$

$a + \alpha_1 + \alpha_2 + \alpha_3 = 8$



Question 213

The equation of the line passing through $(-4, 3, 1)$, parallel to the plane $x + 2y - z - 5 = 0$ and intersecting the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$ is:

A. $\frac{x+4}{-1} = \frac{y-3}{1} = \frac{z-1}{1}$

B. $\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$

C. $\frac{x+4}{1} = \frac{y-3}{1} = \frac{z-1}{3}$

D. $\frac{x-4}{2} = \frac{y+3}{1} = \frac{z+1}{4}$

Correct answer: B

Source solution

Sol.

Normal vector of plane containing two intersecting lines is parallel to vector.

$\left(\vec{V}_1\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 1 \\ -3 & 2 & -1 \end{vmatrix}$

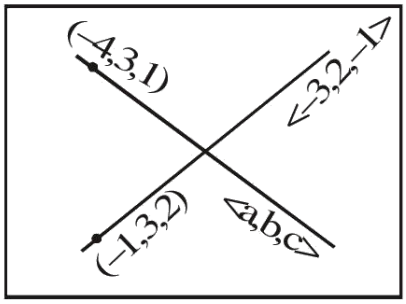
$$= -2\hat{i} + 6\hat{k}$$

$\therefore$ Required line is parallel to vector

$\left(\vec{V}_2\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ -2 & 0 & 6 \end{vmatrix} = 3\hat{i} - \hat{j} + \hat{k}$

$\Rightarrow$ Required equation of line is

$\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$



Question 214

The equation of the line passing through $(-4, 3, 1)$, parallel to the plane $x + 2y - z - 5 = 0$ and intersecting the line

$$\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z-2}{-1}$$

is:

A. $\frac{x+4}{-1} = \frac{y-3}{1} = \frac{z-1}{1}$

B. $\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$

C. $\frac{x+4}{1} = \frac{y-3}{1} = \frac{z-1}{3}$

D. $\frac{x-4}{2} = \frac{y+3}{1} = \frac{z+1}{4}$

Correct answer: B

Source solution

Sol.

1. The normal vector of the plane containing two intersecting lines is parallel to the vector:

$$\vec{V}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 1 \\ -3 & 2 & -1 \end{vmatrix}$$

2. Calculating the determinant:

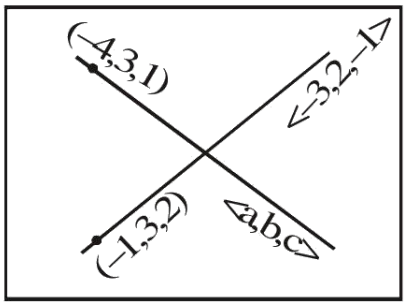
$$\vec{V}_1 = -2\hat{i} + 6\hat{k}$$

3. Therefore, the required line is parallel to the vector:

$$\vec{V}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ -2 & 0 & 6 \end{vmatrix} = 3\hat{i} - \hat{j} + \hat{k}$$

4. The required equation of the line is:

$$\frac{x+4}{3} = \frac{y-3}{-1} = \frac{z-1}{1}$$



Question 215

The plane containing the line $\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z-1}{3}$ and also containing its projection on the plane $2x+3y-z=5$, contains which one of the following points ?

A. $(2,0,-2)$

B. (-2, 2, 2)

C. $(0,-2,2)$

D. $(2,2,0)$

Correct answer: A

Source solution

Sol. The normal vector of required plane
$=(2\hat{i}-\hat{j}+3\hat{k})\times(2\hat{i}+3\hat{j}-\hat{k})$
$=-8\hat{i}+8\hat{j}+8\hat{k}$
So, direction ratio of normal is $(-1,1,1)$
So required plane is
$-(x-3)+(y+2)+(z-1)=0$
$\Rightarrow -x+y+z+4=0$
Which is satisfied by (2, $0,-2$)

Question 216

On which of the following lines lies the point of intersection of the line, $\frac{x-4}{2}=\frac{y-5}{2}=\frac{z-3}{1}$ and the plane, $x+y+z=2$?

A. $\frac{x-2}{2}=\frac{y-3}{2}=\frac{z+3}{3}$

B. $\frac{x-4}{1}=\frac{y-5}{1}=\frac{z-5}{-1}$

C. $\frac{x-1}{1}=\frac{y-3}{2}=\frac{z+4}{-5}$

D. $\frac{x+3}{3}=\frac{4-y}{3}=\frac{z+1}{-2}$

Correct answer: C

Source solution

Sol. General point on the given line is
$x=2\lambda+4$
$y=2\lambda+5$
$z=\lambda+3$.<.
Solving with plane,
$2\lambda+4+2\lambda+5+\lambda+3=2$
$5\lambda+12=2$
$5\lambda=-10$

$$\lambda=-2$$

$\therefore$ Option (3) is the correct answer.

Question 217

The plane passing through the point $(4, -1, 2)$ and parallel to the lines

$$\frac{x+2}{3} = \frac{y-2}{-1} = \frac{z+1}{2}$$

and

$$\frac{x-2}{1} = \frac{y-3}{2} = \frac{z-4}{3}$$

also passes through the point:

A. $(-1, -1, -1)$

B. $(-1, -1, 1)$

C. $(1, 1, -1)$

D. $(1, 1, 1)$

Correct answer: C

Source solution

Sol.

Let $\vec{n}$ be the normal vector to the plane passing through the point $(4, -1, 2)$ and parallel to the lines L_1 and L_2.

We can find $\vec{n}$ using the determinant:

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 2 \\ 1 & 2 & 3 \end{vmatrix}$$

Calculating the determinant, we find:

$$\vec{n} = -7\hat{i} - 7\hat{j} + 7\hat{k}$$

The equation of the plane can be expressed as:

$$-1(x-4) - 1(y+1) + 1(z-2) = 0$$

Simplifying this, we get:

$$x + y - z - 1 = 0$$

Now, we can check the options.

Question 218

The plane containing the line

$$\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z-1}{3}$$

and also containing its projection on the plane

$$2x+3y-z=5,$$

contains which one of the following points?

A. $(2,0,-2)$

B. (-2, 2, 2)

C. $(0,-2,2)$

D. $(2,2,0)$

Correct answer: A

Source solution

Sol.

1. Find the normal vector of the required plane:

$$\mathbf{n}=(2\hat{i}-\hat{j}+3\hat{k})\times(2\hat{i}+3\hat{j}-\hat{k})$$

$$\mathbf{n}=-8\hat{i}+8\hat{j}+8\hat{k}$$

2. The direction ratios of the normal vector are $(-1,1,1)$.

3. The equation of the required plane can be expressed as:

$$-(\mathbf{x}-3)+(\mathbf{y}+2)+(\mathbf{z}-1)=0$$

Simplifying this gives:

$$-\mathbf{x}+\mathbf{y}+\mathbf{z}+4=0$$

4. Verify that the point $(2,0,-2)$ satisfies the plane equation:

$$-2+0-2+4=0$$

Thus, the point $(2,0,-2)$ lies on the required plane.

Question 219

The plane passing through the point $(4,-1,2)$ and parallel to the lines $\frac{x+2}{3}=\frac{y-2}{-1}=\frac{z+1}{2}$ and $\frac{x-2}{1}=\frac{y-3}{2}=\frac{z-4}{3}$ also passes through the point :

A. $(-1,-1,-1)$

B. $(-1,-1,1)$

C. $(1,1,-1)$

D. $(1,1,1)$

Correct answer: D

Source solution

Sol. Let $\vec{n}$ be the normal vector to the plane passing through $(4,-1,2)$ and parallel to the lines $L_1 \& L_2$ then $\vec{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}3 \\ -1 & 21 & 2 & 3\end{vmatrix}$

$\therefore \vec{n}=-7\hat{i}-7\hat{j}+7\hat{k}$

$\therefore$ Equation of plane is

$-1(x-4)-1(y+1)+1(z-2)=0$

$\therefore x+y-z-1=0$

Question 220

If the point $(2,\alpha,\beta)$ lies on the plane which passes through the points $(3,4,2)$ and $(7,0,6)$ and is perpendicular to the plane $2x-5y=15$, then $2\alpha-3\beta$ is equal to :-

A. 5

B. 17

C. 12

D. 7

Correct answer: D

Source solution

Sol. Normal vector of plane

$=\begin{vmatrix}i & j & k2 \\ -5 & 04 \\ -4 & 4\end{vmatrix}=-4(5\hat{i}+2\hat{j}-3\hat{k})$

equation of plane is $5(x-7)+2y-3(z-6)=0\,5x+2y-3z=17$

Question 221

If the point $(2, \alpha, \beta)$ lies on the plane which passes through the points $(3, 4, 2)$ and $(7, 0, 6)$ and is perpendicular to the plane $2x - 5y = 15$, then $2\alpha - 3\beta$ is equal to :-

A. 5

B. 17

C. 12

D. 7

Correct answer: B

Source solution

Sol.

To find the equation of the plane, we first determine the normal vector using the determinant:

$$\text{Normal vector} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -5 & 0 \\ 4 & -4 & 4 \end{vmatrix}$$

Calculating the determinant, we have:

$$= -4(5\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}})$$

Thus, the normal vector is:

$$\mathbf{n} = -4(5, 2, -3)$$

Next, we use the point-normal form of the plane equation. Given a point $(7, 0, 6)$ on the plane, the equation can be expressed as:

$$5(x - 7) + 2(y - 0) - 3(z - 6) = 0$$

Expanding this, we get:

$$5x - 35 + 2y - 3z + 18 = 0$$

Simplifying, we arrive at the final equation of the plane:

$$5x + 2y - 3z = 17$$

Question 222

The plane through the intersection of the planes $x+y+z=1$ and $2x+3y-z+4=0$ and parallel to y-axis also passes through the point :

A. $(-3,0,-1)$

B. $(3,3,-1)$

C. $(3,2,1)$

D. $(-3,1,1)$

Correct answer: C

Source solution

Sol. Equation of plane

$$(x+y+z-1)+\lambda(2x+3y-z+4)=0 \Rightarrow \quad (1+2\lambda)x+(1+3\lambda)y+(1-\lambda)z-1+4\lambda=0$$

dr's of normal of the plane are

$$1+2\lambda, 1+3\lambda, 1-\lambda$$

Since plane is parallel to y - axis, $1+3\lambda=0$

$$\Rightarrow \quad \lambda=-1/3$$

So the equation of plane is

$$x+4z-7=0$$

Point (3, 2, 1) satisfies this equation
Hence Answer is (3)

Question 223

The perpendicular distance from the origin to the plane containing the two lines,

$$\frac{x+2}{3}=\frac{y-2}{5}=\frac{z+5}{7}$$

and

$$\frac{x-1}{1}=\frac{y-4}{4}=\frac{z+4}{7},$$

is:

A. $11/\sqrt{6}$

B. $6\sqrt{11}$

C. 11

D. $11\sqrt{6}$

Correct answer: A

Source solution

Step 1: Calculate the determinant

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix}$$

Step 2: Expand the determinant

$$=\hat{i}(5\cdot 7-4\cdot 7)-\hat{j}(3\cdot 7-1\cdot 7)+\hat{k}(3\cdot 4-5\cdot 1)$$

$$=\hat{i}(35-28)-\hat{j}(21-7)+\hat{k}(12-5)$$

$$=7\hat{i}-14\hat{j}+7\hat{k}$$

Step 3: Factor out common terms

$$=\hat{i}-2\hat{j}+\hat{k}$$

Step 4: Write the equation of the plane

$$1(x+2)-2(y-2)+1(z+15)=0$$

$$\Rightarrow x-2y+z+11=0$$

Step 5: Calculate the distance from the origin to the plane

$$d = \frac{|11|}{\sqrt{1^2 + (-2)^2 + 1^2}} = \frac{11}{\sqrt{1+4+1}} = \frac{11}{\sqrt{6}}$$

Question 224

The plane through the intersection of the planes $x+y+z=1$ and $2x+3y-z+4=0$ and parallel to the y-axis also passes through the point:

A. $(-3,0,-1)$

B. $(3,3,-1)$

C. $(3,2,1)$

D. $(-3,1,1)$

Correct answer: C

Source solution

Sol. Equation of the plane:

Starting with the equation:

$$(x+y+z-1)+\lambda(2x+3y-z+4)=0$$

we can rearrange it to:

$$(1+2\lambda)x+(1+3\lambda)y+(1-\lambda)z-(1-4\lambda)=0$$

The direction ratios (dr's) of the normal to the plane are:

$$1+2\lambda,\quad 1+3\lambda,\quad 1-\lambda$$

Since the plane is parallel to the y-axis, we set:

$$1+3\lambda=0$$

Solving for λ:

$$\lambda=-\frac{1}{3}$$

Substituting λ back into the equation of the plane gives:

$$x+4z-7=0$$

To verify, we check if the point $(3,2,1)$ satisfies this equation:

$$3+4(1)-7=0$$

Thus, the point satisfies the equation.

Hence, the answer is 3.

Question 225

Two lines $\frac{x-3}{1}=\frac{y+1}{3}=\frac{z-6}{-1}$ and $\frac{x+5}{7}=\frac{y-2}{-6}=\frac{z-3}{4}$ intersect at the point R. The reflection of R in the xy-plane has coordinates :-

A. $(2,4,7)$

B. $(-2,4,7)$

C. $(2,-4,-7)$

D. $(2,-4,7)$

Correct answer: C

Source solution

Sol. Point on $\mathbf{L_1}(\lambda+3,3\lambda-1,-\lambda+6)$
Point on $\mathbf{L_2}(7\mu-5,-6\mu+2,4\mu+3)$

$$\Rightarrow \lambda+3=7\mu-5 \quad \text{(i)}$$

$3\lambda-1=-6\mu+2$
$\ldots$ (ii) $\Rightarrow \lambda=-1,\mu=1$
point $\mathbf{R}(2,-4,7)$
Reflection is $(2,-4,-7)$

Question 226

the perpendicular distance from the origin to the plane containing the two lines, $\frac{x+2}{3}=\frac{y-2}{5}=\frac{z+5}{7}$ and $\frac{x-1}{1}=\frac{y-4}{4}=\frac{z+4}{7}$, is:

A. $11/\sqrt{6}$

B. $6\sqrt{11}$

C. 11

D. $11\sqrt{6}$

Correct answer: A

Source solution

Sol.
$|i \quad j \quad k3 \quad 5 \quad 71 \quad 4 \quad 7|$
$\hat{\mathrm{i}}(35-28)-\hat{\mathrm{j}}(21.7)+\hat{\mathrm{k}}(12-5)$
$7\hat{i}-14\hat{j}+7\hat{k}$
$\hat{\mathrm{i}}-2\hat{\mathrm{j}}+\hat{\mathrm{k}}$
$1(x+2)-2(y-2)+1(z+15)=0$
$\mathrm{x}-2\mathrm{y}+\mathrm{z}+11=0$
$\frac{11}{\sqrt{4+1+1}}=\frac{11}{\sqrt{6}}$

Question 227

Let $A(a, b), B(3, 4)$ and $(-6, -8)$ respectively denote the centroid, circumcentre and orthocentre of a triangle. Then, the distance of the point $\mathrm{P}(2\mathrm{a}+3, 7\,\mathrm{b}+5)$ from the line $2\mathrm{x}+3\mathrm{y}-4=0$ measured parallel to the line $x-2y-1=0$ is

A. $\frac{15\sqrt{5}}{7}$

B. $\frac{17\sqrt{5}}{6}$

C. $\frac{17\sqrt{5}}{7}$

D. $\frac{\sqrt{5}}{17}$

Correct answer: C

Source solution

$$G = A(a, b), \qquad O = (3, 4), \qquad H = (-6, -8),$$

where G, O, and H denote the centroid, circumcentre, and orthocentre of a triangle.

Using the Euler line relation,

$$\vec{OG} = \frac{1}{3}\vec{OH}.$$

Hence,

$$G = O + \frac{1}{3}(H - O).$$

Substituting the coordinates,

$$(a, b) = (3, 4) + \frac{1}{3}\big((-6, -8) - (3, 4)\big).$$

$$= (3, 4) + \frac{1}{3}(-9, -12).$$

$$= (3, 4) + (-3, -4).$$

$$(a, b) = (0, 0).$$

Therefore,

$$a = 0, \qquad b = 0.$$

The point

$$P = (2a + 3,\ 7b + 5)$$

becomes

$$P = (3, 5).$$

We have to find the distance of P from the line

$$2x + 3y - 4 = 0$$

measured parallel to the line

$$x - 2y - 1 = 0.$$

The direction vector of the line

$$x - 2y - 1 = 0$$

is

$$\vec{d} = (2, 1).$$

Hence the required line through $P(3, 5)$ is

$$\frac{x - 3}{2} = \frac{y - 5}{1} = t.$$

Thus,

$$x = 3 + 2t, \qquad y = 5 + t.$$

Since this line meets

$$2x + 3y - 4 = 0,$$

we get

$$2(3 + 2t) + 3(5 + t) - 4 = 0.$$

$$6 + 4t + 15 + 3t - 4 = 0.$$

$$17 + 7t = 0.$$

$$t = -\frac{17}{7}.$$

The required distance is the length of the segment along the direction $(2,1)$:

$$D = |t|\sqrt{2^2+1^2}.$$

$$= \frac{17}{7}\sqrt{5}.$$

Therefore,

$$\boxed{\frac{17\sqrt{5}}{7}}.$$

Hence, the correct option is

$$\boxed{(3)\ \frac{17\sqrt{5}}{7}}.$$

Question 228

The shortest distance between the lines $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-6}{2}$ and $\frac{x-6}{3} = \frac{1-y}{2} = \frac{z+8}{0}$ is _____.

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 14

Source solution

The direction vectors are $(3,2,2)$ and $(3,-2,0)$. Their cross product is $(4,6,-12)$, of magnitude 14. The scalar triple product with displacement $(4,2,-14)$ is 196, so the shortest distance is 14.

Question 229

If the foot of the perpendicular from point $(4,3,8)$ on the line $L_1 : \frac{x-a}{1} = \frac{y-2}{3} = \frac{z-b}{4}, l \neq 0$ is $(3,5,7)$, then the shortest distance between the line L_1 and line $L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is equal to :

A. $\frac{1}{2}$

B. $\frac{1}{\sqrt{6}}$

C. $\sqrt{\frac{2}{3}}$

D. $\frac{1}{\sqrt{3}}$

Official Answer by NTA (2)}

Correct answer: B

Source solution

Sol. $(3,5,7)$ satisfy the line L_1

$\frac{3-a}{\ell} = \frac{5-2}{3} = \frac{7-b}{4}$

$\frac{3-a}{\ell} = 1 \quad \& \quad \frac{7-b}{4} = 1$

$$a + \ell = 3 \quad \ldots(1) \quad \& \quad b = 3 \tag{2}$$

$\vec{v_1} = \langle 4,3,8 \rangle - \langle 3,5,7 \rangle$

$\vec{v_1} = \langle 1,-2,1 \rangle$

$\vec{v_2} = \langle \ell, 3, 4 \rangle$

$\vec{v_1} \cdot \vec{v_2} = 0 \Rightarrow \ell - 6 + 4 = 0 \Rightarrow \ell = 2$

$a + \ell = 3 \Rightarrow a = 1$

$L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$

$L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$

$A =< 1,2,3 >$

$B =< 2,4,5 >$

$\overrightarrow{AB} = \langle 1,2,2 \rangle$

$\vec{p} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{q} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$\vec{p} \times \vec{q} = -\hat{i} + 2\hat{j} - \hat{k}$

Shortest distance $= \left| \frac{\overrightarrow{AB} \cdot (\vec{p} \times \vec{q})}{|\vec{p} \times \vec{q}|} \right| = \frac{1}{\sqrt{6}}$

Question 230

The shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$ and $\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}$

A. $5\sqrt{3}$

B. $2\sqrt{3}$

C. $3\sqrt{3}$

D. $4\sqrt{3}$

Correct answer: B

Source solution

Sol. 4

$\mathbf{L_1}=\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}=\lambda$

$\mathbf{L_2}\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}=\mu$

S.D. $=\left|\frac{(\vec{b}-\vec{a})\cdot(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$ $\quad \vec{a}=i-8j+4k \qquad \vec{b}=i+2j+6k$

$\vec{b}_1\times\vec{b}_2=|\mathrm{i}\quad \mathrm{j}\quad \mathrm{k}2\quad -7\quad 52\quad 1\quad -3|$

$=\mathrm{i}(21-5)-\mathrm{j}(-6-10)+\mathrm{k}(2+14)$

$=16\mathrm{i}+16\mathrm{j}+16\mathrm{k}$

$\left|\vec{b}_1\times\vec{b}_2\right|=|16(i+j+k)|$

$=16\times\sqrt{3}$

$\vec{b}-\vec{a}=(10\mathrm{j}+2\mathrm{k})$

S.D. $=\left|\frac{(10j+2k)\cdot 16(i+j+k)}{16\sqrt{3}}\right|$

$=\left|\frac{16(10+2)}{16\sqrt{3}}\right|=\frac{12}{\sqrt{3}}\Rightarrow\frac{12}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$

$=\frac{12\sqrt{3}}{3}=4\sqrt{3}$

Question 231

The distance of line $3\mathrm{y}-2\mathrm{z}-1=0=3\mathrm{x}-\mathrm{z}+4$ from the point (2, -1, 6) is :

A. $\sqrt{26}$

B. $2\sqrt{5}$

C. $2\sqrt{6}$

D. $4\sqrt{2}$

Correct answer: C

Source solution

$$3y-2z-1=0=3x-z+4$$
$$3y-2z-1=0 \qquad \text{D.R's} \Rightarrow (0,3,2)$$
$$3x-z+4=0$$

Let DR's of given line are a, b, c Now $3\,\mathrm{b}-2\mathrm{c}=0$ & $3\mathrm{a}-\mathrm{c}=0$

$$\therefore 6a=3b=2c$$
$$a:b:c=3:6:9$$

Any pt on line

$$3\,\mathrm{K}-1, 6\,\mathrm{K}+1, 9\,\mathrm{K}+1$$

Now $3(3\,\mathrm{K}-1)+6(6\,\mathrm{K}+1)1+9(9\,\mathrm{K}+1)=0$

$$\Rightarrow \mathrm{K}=\frac{1}{3}$$

Point on line $\Rightarrow$ (0, 3, 4) Given point (2, -1, 6) $\Rightarrow$ Distance $=\sqrt{4+16+4}=2\sqrt{6}$

Question 232

The shortest distance between lines $\mathbf{L_1}$ and $\mathbf{L_2}$, where $L_1 : \frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+4}{2}$ and L_2 is the line passing through the points $\mathbf{A(-4, 4, 3) \cdot B(-1, 6, 3)}$ and perpendicular to the line $\frac{x-3}{-2} = \frac{y}{3} = \frac{z-1}{1}$, is

A. $\frac{121}{\sqrt{221}}$

B. $\frac{24}{\sqrt{117}}$

C. $\frac{141}{\sqrt{221}}$

D. $\frac{42}{\sqrt{117}}$

Correct answer: C

Source solution

Sol.-

$\mathbf{L_2} = \frac{x+4}{3} = \frac{y-4}{2} = \frac{z-3}{0}$

$\therefore \text{S.D} = \frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ 2 & -3 & 23 \\ 2 & 0 & \end{vmatrix}}{\left|\vec{n_1} \times \vec{n_2}\right|}$

$= \frac{\begin{vmatrix} 5 & -5 & -7 \\ 2 & -3 & 23 \\ 2 & 0 & \end{vmatrix}}{\left|\vec{n_1} \times \vec{n_2}\right|}$

$= \frac{141}{|-46\hat{i}+6\hat{j}+13\hat{k}|}$

$= \frac{141}{\sqrt{16+36+169}}$

$= \frac{141}{\sqrt{221}}$

Question 233

Let P be the foot of the perpendicular from the point $Q(10, -3, -1)$ on the line

$$\frac{x-3}{7} = \frac{y-2}{-1} = \frac{z+1}{-2}.$$

Then the area of the right angled triangle PQR, where R is the point $(3, -2, 1)$, is

A. $9\sqrt{15}$

B. $\sqrt{30}$

C. $8\sqrt{15}$

D. $3\sqrt{30}$

Correct answer: D

Source solution

$$\text{Let } \frac{x-3}{7} = \frac{y-2}{-1} = \frac{z+1}{-2} = \lambda$$

From this, we can express x, y, and z in terms of λ:

$$x = 7\lambda + 3, \quad y = -\lambda + 2, \quad z = -2\lambda - 1$$

The direction ratios (dr's) of line QP are:

$$7\lambda - 7, \quad -\lambda + 5, \quad -2\lambda$$

Now, we set up the equation for the area of the triangle formed by points P, Q, and R:

$$(7\lambda - 7) \cdot 7 - (-\lambda + 5) + (2\lambda) \cdot 2 = 0$$

Simplifying this gives:

$$54\lambda - 54 = 0 \quad \Rightarrow \quad \lambda = 1$$

Substituting λ back, we find the coordinates of point P:

$$P = (10, 1, -3)$$

Next, we calculate the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$:

$$\overrightarrow{PQ} = -4\hat{j} + 2\hat{k}$$

$$\overrightarrow{PR} = -7\hat{i} - 3\hat{j} + 4\hat{k}$$

The area of the triangle is given by:

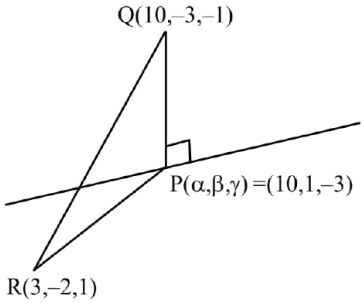
$$\text{Area} = \left|\frac{1}{2}\right| \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -4 & 2 \\ -7 & -3 & 4 \end{matrix}|$$

Calculating the determinant:

$$= 3\sqrt{30}$$

Thus, the final area is:

$$\text{Area} = 3\sqrt{30}$$



Question 234

Let d be the distance of the point of intersection of the lines

$$\frac{x+6}{3}=\frac{y}{2}=\frac{z+1}{1}$$

and

$$\frac{x-7}{4}=\frac{y-9}{3}=\frac{z-4}{2}$$

from the point $(7,8,9)$. Then d^2+6 is equal to :

A. (4) 78

B. (3) 75

C. (2) 69

D. (1) 72

Correct answer: A

Source solution

Sol.

We start with the equations:

$$\frac{x+6}{3}=\frac{y}{2}=\frac{z+1}{1}=\lambda$$

From this, we can express x, y, and z in terms of λ:

$$x=3\lambda-6,\quad y=2\lambda,\quad z=\lambda-1$$

Next, we have another set of equations:

$$\frac{x-7}{4}=\frac{y-9}{3}=\frac{z-4}{2}=\mu$$

This gives us:

$$x=4\mu+7,\quad y=3\mu+9,\quad z=2\mu+4$$

Now, we equate the expressions for x:

$$3\lambda-6=4\mu+7\quad\Rightarrow\quad 3\lambda-4\mu=13 \tag{1}$$

Next, we equate the expressions for y:

$$2\lambda=3\mu+9\quad\Rightarrow\quad 2\lambda-3\mu=9 \tag{2}$$

We can multiply the second equation by 3:

$$6\lambda - 9\mu = 27 \tag{3}$$

Now, we multiply the first equation by 2:

$$6\lambda - 8\mu = 26 \tag{4}$$

Next, we subtract equation (4) from equation (3):

$$(6\lambda - 9\mu) - (6\lambda - 8\mu) = 27 - 26$$

This simplifies to:

$$-\mu = -1 \quad \Rightarrow \quad \mu = -1$$

Substituting $\mu = -1$ back into equation (1):

$$3\lambda - 4(-1) = 13$$

This simplifies to:

$$3\lambda + 4 = 13 \quad \Rightarrow \quad 3\lambda = 9 \quad \Rightarrow \quad \lambda = 3$$

Now we can find the coordinates:

$$x = 3\lambda - 6 = 3(3) - 6 = 3$$

$$y = 2\lambda = 2(3) = 6$$

$$z = \lambda - 1 = 3 - 1 = 2$$

Thus, the intersection point is $(3, 6, 2)$.

Next, we find the distance d^2 from the point $(3, 6, 2)$ to the point $(7, 8, 9)$:

$$d^2 = (7-3)^2 + (8-6)^2 + (9-2)^2 = 4^2 + 2^2 + 7^2 = 16 + 4 + 49 = 69$$

Question 235

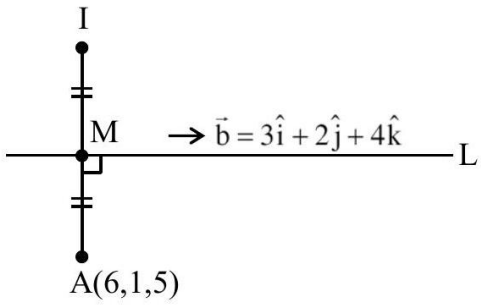
The square of the distance of the image of the point $(6, 1, 5)$ in the line $\frac{x-1}{3} = \frac{y}{2} = \frac{z-2}{4}$, from the origin is .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 62

Source solution



Sol.

Let $\mathbf{M(3\lambda + 1, 2\lambda, 4\lambda + 2)}$

$\overrightarrow{\mathbf{AM}} \cdot \vec{\mathbf{b}} = \mathbf{0}$

$\Rightarrow \quad \mathbf{9\lambda - 15 + 4\lambda - 2 + 16\lambda - 12 = 0}$

$\Rightarrow \quad \mathbf{29\lambda = 29}$

$\Rightarrow \quad \boldsymbol{\lambda = 1}$

$\mathbf{M(4, 2, 6), I = (2, 3, 7)}$

Required Distance $= \sqrt{4 + 9 + 49} = \sqrt{62}$

Ans. 62

Question 236

The distance of the point having position vector $-\hat{\mathbf{i}}+2\hat{\mathbf{j}}+6\hat{\mathbf{k}}$ from the straight line passing through the point $(2,3,-4)$ and parallel to the vector, $6\hat{i}+3\hat{j}-4\hat{k}$ is :

A. 7

B. $4\sqrt{3}$

C. 6

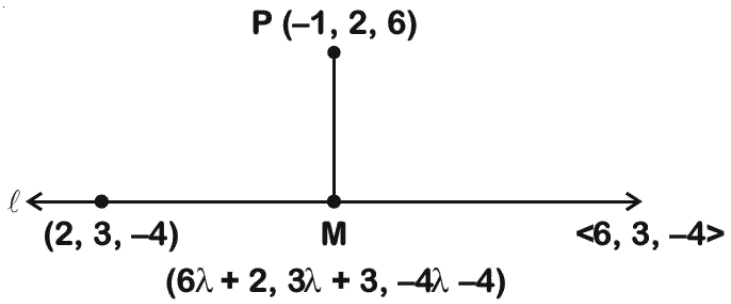
D. $2\sqrt{13}$

Correct answer: A

Source solution

Sol. Equation of l is $\frac{x-2}{6}=\frac{y-3}{3}=\frac{z+4}{-4}$

Let $M(6\lambda+2,3\lambda+3,-4\lambda-4)$
DR's of PM is $\langle 6\lambda+3,3\lambda+1,-4\lambda-10\rangle$
$\Rightarrow (6\lambda+3)(6)+(3\lambda+1)(3)+(-4\lambda-10)(-4)=0$
$\Rightarrow \lambda=-1$
i.e. $M\equiv(-4,0,0)$
$\therefore \quad \mathrm{PM}=\sqrt{9+4+36}=7$



Question 237

Let the line $\frac{x}{1}=\frac{6-y}{2}=\frac{z+8}{5}$ intersect the lines $\frac{x-5}{4}=\frac{y-7}{3}=\frac{z+2}{1}$ and $\frac{x+3}{6}=\frac{3-y}{3}=\frac{z-6}{1}$ at the points A and B respectively. Then the distance of the mid-point of the line segment AB from the plane $2x-2y+z=14$ is

A. 4

B. $\frac{10}{3}$

C. 3

D. $\frac{11}{3}$

Correct answer: A

Source solution

Sol.

We start with the equations:

$$\frac{x}{1}=\frac{y-6}{-2}=\frac{z+8}{5}=\lambda$$

$$\frac{x-5}{4}=\frac{y-7}{3}=\frac{z+2}{1}=\mu \tag{1}$$

$$\frac{x+3}{4}=\frac{y-3}{-3}=\frac{z-6}{1}=\gamma \tag{2}$$

Finding Intersection of (1) & (2) "A":

From (1):

$$x=\lambda, \quad y=-2\lambda+6, \quad z=5\lambda-8$$

From (2):

$$x=4\mu+5, \quad y=3\mu+7, \quad z=\mu-2$$

Setting the equations equal:

$$\lambda=4\mu+5$$

$$-2\lambda+6=3\mu+7$$

$$5\lambda-8=\mu-2$$

Solving these equations, we find:
$\lambda=1$ and $\mu=-1$.

Thus, the intersection point A is:

$$\mathrm{A}(1,4,-3)$$

Finding Intersection of (1) & (3) "B":

From (1):

$$x = \lambda, \quad y = -2\lambda + 6, \quad z = 5\lambda - 8$$

From (3):

$$x = 6\gamma - 3, \quad y = -3\gamma + 3, \quad z = \gamma + 6$$

Setting the equations equal:

$$\lambda = 6\gamma - 3$$

$$-2\lambda + 6 = -3\gamma + 3$$

$$5\lambda - 8 = \gamma + 6$$

Solving these equations, we find:
$\lambda = 3$ and $\gamma = 1$.

Thus, the intersection point B is:

$$\mathbf{B(3, 0, 7)}$$

Finding the Midpoint of A & B:

$$\text{Midpoint} = \left(\frac{1+3}{2}, \frac{4+0}{2}, \frac{-3+7}{2} \right) = (2, 2, 2)$$

Calculating the Perpendicular Distance from the Plane:
The equation of the plane is:

$$2x - 2y + z = 14$$

The distance d from the point $(2, 2, 2)$ to the plane is given by:

$$d = \left| \frac{2(2) - 2(2) + 2 - 14}{\sqrt{4+4+1}} \right| = 4$$

Question 238

Let $\mathbf{L_1}: \frac{x-1}{1}=\frac{y-2}{-1}=\frac{z-1}{2}$ and $\mathbf{L_2}: \frac{x+1}{-1}=\frac{y-2}{2}=\frac{z}{1}$ be two lines.

Let $\mathbf{L_3}$ be a line passing through the point $(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma})$ and be perpendicular to both $\mathbf{L_1}$ and $\mathbf{L_2}$. If $\mathbf{L_3}$ intersects $\mathbf{L_1}$, then $|5\alpha-11\beta-8\gamma|$ equals :

A. 18

B. 16

C. 25

D. 20

Correct answer: C

Source solution

Sol. DR's of $L_3=\vec{m}\times\vec{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}1 \\ -1 & 2-1 & 2 \\ 1 & 1\end{vmatrix}=-5\hat{\mathrm{i}}-3\hat{\mathrm{j}}+\hat{\mathrm{k}}$

$L_3: \frac{x-\alpha}{-5}=\frac{y-\beta}{-3}=\frac{z-\gamma}{1}=\lambda$

$$\mathbf{A}(\alpha-5\lambda, \beta-3\lambda, \gamma+\lambda)$$

$\mathbf{L_1}: \frac{x-1}{1}=\frac{y-2}{-1}=\frac{z-1}{2}=\mathbf{k}$

$$\mathbf{B}(\mathbf{k}+1, -\mathbf{k}+2, 2\mathbf{k}+1)$$

Now

$\alpha-5\lambda=\mathbf{k}+1 \Rightarrow \alpha=5\lambda+\mathbf{k}+1$

$\beta-3\lambda=-\mathbf{k}+2 \Rightarrow \beta=3\lambda-\mathbf{k}+2$

$\gamma+\lambda=2\mathbf{k}+1 \Rightarrow \gamma=-\lambda+2\mathbf{k}+1$

$|5\alpha-11\beta-8\gamma|=|-25|$

$=25$

Question 239

The line l_1 passes through the point $(2, 6, 2)$ and is perpendicular to the plane $2x + y - 2z = 10$. Then the shortest distance between the line l_1 and the line $\frac{x+1}{2} = \frac{y+4}{-3} = \frac{z}{2}$ is :

A. $\frac{13}{3}$

B. $\frac{19}{3}$

C. 7

D. 9

Correct answer: A

Source solution

Sol. 9}
equation of l_1 is $\frac{x-2}{2} = \frac{y-6}{1} = \frac{z-2}{-2}$
Let l_2 is $\frac{x+1}{2} = \frac{y+4}{-3} = \frac{z}{2}$
Point on l_1 is a $= (2, 6, 2)$, direction $\vec{p} = \langle 2, 1, -2 \rangle$
Point on l_2 is $b = (-1, -4, 0)$ direction $\vec{q} = \langle 2, -3, 2 \rangle$
Shortest distance between l_1 and $l_2 = \left| \frac{(\vec{a}-\vec{b}) \cdot (\vec{p} \times \vec{q})}{|\vec{p} \times \vec{q}|} \right|$

$$\because \quad \vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & k2 \\ 1 & -2 & 2 \\ -3 & 2 & \end{vmatrix} = \hat{i}(-4) - \hat{j}(8) + k(-8) \qquad = \left| \frac{\langle 3, 10, 2 \rangle \cdot \langle -4, -8, -8 \rangle}{\sqrt{16 + 64 + 64}} \right| = \left| \frac{-12 - 80 - 16}{\sqrt{144}} \right| \qquad = \frac{108}{12} = 9$$

Shortest distance between the lines is 9 .

Question 240

Let for a triangle ABC,

$$\overrightarrow{AB} = -2\hat{i} + \hat{j} + 3\hat{k}$$

$$\overrightarrow{CB} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$$

$$\overrightarrow{CA} = 4\hat{i} + 3\hat{j} + \delta\hat{k}$$

If $\delta > 0$ and the area of the triangle ABC is $5\sqrt{6}$, then $\overrightarrow{CB} \cdot \overrightarrow{CA}$ is equal to

A. 60

B. 120

C. 108

D. 54

Official Answer. By NTA (1)

Correct answer: A

Source solution

$$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$$

Given:
$\alpha = 2, \beta = 4, \gamma - \delta = 3$

Using the area of the triangle:

$$\frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = 5\sqrt{6}$$

Solving the equation:

$$(\delta - 9)^2 + (2\delta + 12)^2 + 100 = 600$$

This simplifies to:

$$\delta = 5, \quad \gamma = 8$$

Finally, we find:

$$\overrightarrow{CB} \cdot \overrightarrow{CA} = 60$$

Question 241

Let the shortest distance between the lines $L: \frac{x-5}{-2} = \frac{y-\lambda}{0} = \frac{z+\lambda}{1}, \lambda \geq 0$ and $L_1: x+1 = y-1 = 4-z$ be $2\sqrt{6}$. If (α, β, γ) lies on L, then which of the following is NOT possible?

A. $\alpha - 2\gamma = 19$

B. $2\alpha + \gamma = 7$

C. $2\alpha - \gamma = 9$

D. $\alpha + 2\gamma = 24$

Correct answer: D

Source solution

Let

$$\vec{b}_1 = \langle -2, 0, 1 \rangle, \qquad \vec{a}_1 = (5, \lambda, -\lambda),$$
$$\vec{b}_2 = \langle 1, 1, -1 \rangle, \qquad \vec{a}_2 = (-1, 1, 4).$$

A vector perpendicular to both lines is $\vec{b}_1 \times \vec{b}_2$.

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$
$$= \hat{i}(0 \cdot (-1) - 1 \cdot 1) - \hat{j}((-2)(-1) - 1 \cdot 1) + \hat{k}((-2)(1) - 0 \cdot 1)$$
$$= -\hat{i} - \hat{j} - 2\hat{k}.$$

Hence,

$$\vec{b}_1 \times \vec{b}_2 = \langle -1, -1, -2 \rangle.$$

Also,

$$\vec{a}_1 - \vec{a}_2 = \langle 6, \lambda - 1, -\lambda - 4 \rangle.$$

Using the formula for the shortest distance between two skew lines,

$$d = \left| \frac{(\vec{a}_1 - \vec{a}_2) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|.$$

Given $d = 2\sqrt{6}$,

$$2\sqrt{6} = \left| \frac{\langle 6, \lambda - 1, -\lambda - 4 \rangle \cdot \langle -1, -1, -2 \rangle}{\sqrt{1+1+4}} \right|.$$

Since

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{6},$$

we get

$$2\sqrt{6} = \left| \frac{-6 - (\lambda - 1) + 2\lambda + 8}{\sqrt{6}} \right|.$$

$$12 = |-6 - \lambda + 1 + 2\lambda + 8|.$$

$$12 = |\lambda + 3|.$$

Therefore,

$$\lambda + 3 = \pm 12.$$

$$\lambda = 9 \quad \text{or} \quad \lambda = -15.$$

Since $\lambda \geq 0$,

$$\boxed{\lambda = 9}.$$

Now, since (α, β, γ) lies on line L,

$$\frac{\alpha - 5}{-2} = \frac{\beta - 9}{0} = \frac{\gamma + 9}{1} = k.$$

Hence,

$$\alpha = 5 - 2k, \qquad \beta = 9, \qquad \gamma = -9 + k.$$

Therefore,

$$\alpha + 2\gamma = (5 - 2k) + 2(-9 + k).$$

$$\alpha + 2\gamma = 5 - 2k - 18 + 2k.$$

$$\boxed{\alpha + 2\gamma = -13}.$$

Hence,

$$\boxed{\alpha + 2\gamma = 24}$$

is not possible.

Question 242

Consider the line L passing through the points $(1,2,3)$ and $(2,3,5)$. The distance of the point $\left(\frac{11}{3}, \frac{11}{3}, \frac{19}{3}\right)$ from the line L along the line $\frac{3x-11}{2} = \frac{3y-11}{1} = \frac{3z-19}{2}$ is equal to :

A. 3

B. 5

C. 4

D. 6

Correct answer: A

Source solution

Sol.

Given the equations:

$$\frac{x-1}{2-1} = \frac{y-2}{3-2} = \frac{z-3}{5-3}$$

This simplifies to:

$$\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{2} = \lambda$$

From this, we can express x, y, and z in terms of λ:

$$x = 1+\lambda, \quad y = 2+\lambda, \quad z = 3+2\lambda$$

The point B can be represented as:

$$B(1+\lambda, 2+\lambda, 3+2\lambda)$$

The direction ratios (D.R.) of line segment AB are given by:

$$\langle \frac{3\lambda-8}{3}, \frac{3\lambda-5}{3}, \frac{6\lambda-10}{3} \rangle$$

Substituting B into the equation:

$$B\left(\frac{5}{3}, \frac{8}{3}, \frac{13}{3}\right)$$

Setting the ratios equal:

$$\frac{3\lambda-8}{3\lambda-5} = \frac{2}{1}$$

Cross-multiplying gives:

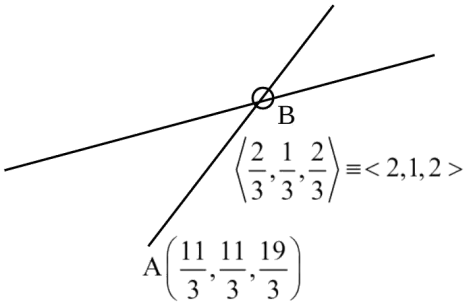
$$3\lambda - 8 = 6\lambda - 10$$

Rearranging the equation:

$$3\lambda = 2 \quad \Rightarrow \quad \lambda = \frac{2}{3}$$

Now, we calculate the length of AB:

$$AB = \frac{\sqrt{36+9+36}}{3} = \frac{9}{3} = 3$$



Question 243

The distance of the point having position vector $-\hat{\mathbf{i}}+2\hat{\mathbf{j}}+6\hat{\mathbf{k}}$ from the straight line passing through the point $(2,3,-4)$ and parallel to the vector $6\hat{\mathbf{i}}+3\hat{\mathbf{j}}-4\hat{\mathbf{k}}$ is :

A. 7

B. $4\sqrt{3}$

C. 6

D. $2\sqrt{13}$

Correct answer: A

Source solution

Sol.

The equation of line I is given by

$$\frac{x-2}{6}=\frac{y-3}{3}=\frac{z+4}{-4}$$

Let the point M be represented as

$$M(6\lambda+2,3\lambda+3,-4\lambda-4)$$

The direction ratios of the vector PM are

$$\langle 6\lambda+3,3\lambda+1,-4\lambda-10\rangle$$

Setting up the equation for the dot product with the direction ratios of line I:

$$(6\lambda+3)(6)+(3\lambda+1)(3)+(-4\lambda-10)(-4)=0$$

Expanding this gives:

$$36\lambda+18+9\lambda+3+16\lambda+40=0$$

Combining like terms results in:

$$61\lambda+61=0$$

Solving for λ:

$$\lambda=-1$$

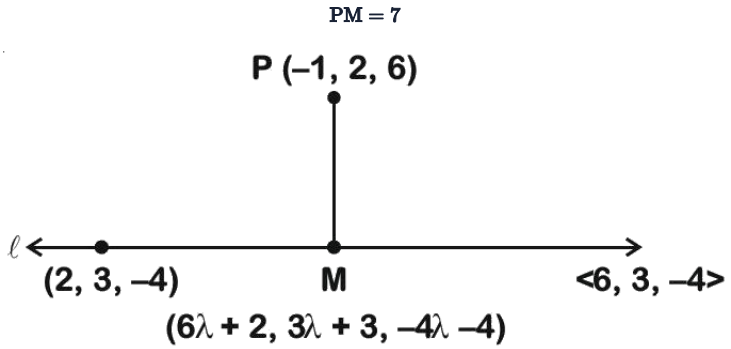
Substituting λ back into the coordinates of M:

$$M\equiv(-4,0,0)$$

Finally, calculating the distance PM:

$$PM = \sqrt{(0-(-4))^2 + (0-0)^2 + (0-0)^2} = \sqrt{9+0+36} = 7$$

Thus, the final answer is



Question 244

The distance of the point $\mathbf{Q(0,2,-2)}$ form the line passing through the point $\mathbf{P(5,-4,3)}$ and perpendicular to the lines $\vec{r}=(-3\hat{i}+2\hat{k}) + \lambda(2\hat{i}+3\hat{j}+5\hat{k}), \quad \lambda \in \mathbb{R}$ and $\vec{r}=(\hat{i}-2\hat{j}+\hat{k}) + \mu(-\hat{i}+3\hat{j}+2\hat{k}), \mu \in \mathbb{R}$ is

A. $\sqrt{86}$

B. $\sqrt{20}$

C. $\sqrt{54}$

D. $\sqrt{74}$

Correct answer: D

Source solution

Sol. A vector in the direction of the required line can be obtained by cross product of

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 5 \\ -1 & 3 & 2 \end{vmatrix}$

$= -9\hat{i}-9\hat{j}+9\hat{k}$

Required line,

$\vec{r}=(5\hat{i}-4\hat{j}+3\hat{k})+\lambda'(-9\hat{i}-9\hat{j}+9\hat{k})$

$\vec{r}=(5\hat{i}-4\hat{j}+3\hat{k})+\lambda(\hat{i}+\hat{j}-\hat{k})$

Now distance of (0, 2, -2)

P.V. of $\mathbf{P} \equiv (5+\lambda)\hat{i}+(\lambda-4)\hat{j}+(3-\lambda)\hat{k}$

$\overrightarrow{AP}=(5+\lambda)\hat{i}+(\lambda-6)\hat{j}+(5-\lambda)\hat{k}$

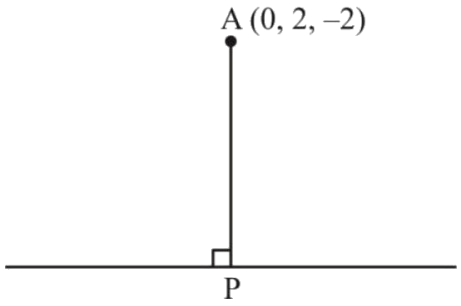
$\overrightarrow{AP}\cdot(\hat{i}+\hat{j}-\hat{k})=0$

$5+\lambda+\lambda-6-5+\lambda=0$

$\lambda=2$

$|\overrightarrow{AP}|=\sqrt{49+16+9}$

$|\overrightarrow{AP}|=\sqrt{74}$



Question 245

If the shortest distance between the lines $\frac{x-\lambda}{2}=\frac{y-4}{3}=\frac{z-3}{4}$ and $\frac{x-2}{4}=\frac{y-4}{6}=\frac{z-7}{8}$ is $\frac{13}{\sqrt{29}}$, then a value of $\boldsymbol{\lambda}$ is :

A. $-\frac{13}{25}$

B. $\frac{13}{25}$

C. 1

D. -1

Correct answer: C

Source solution

Sol. $\bar{r}_1 = (\lambda\hat{i}+4\hat{j}+3\hat{k})+\alpha(2\hat{i}+3\hat{j}+4\hat{k})\bar{r}_2 = (2\hat{i}+4\hat{j}+7\hat{k})+\beta(2\hat{i}+3\hat{j}+4\hat{k})\}\bar{b}=2\hat{i}+3\hat{j}+4\hat{k} \quad \bar{a}_1+\lambda\hat{i}+4\hat{j}+3\hat{k}$

$\bar{r}_2=(2\hat{i}+4\hat{j}+7\hat{k})+\beta(2\hat{i}+3\hat{j}+4\hat{k})\int_{\bar{a}_2}=2\hat{i}+4\hat{j}+7\hat{k}$ Shortest dist. $=\frac{\left|\bar{b}\times(\bar{a}_2-\bar{a}_1)\right|}{|b|}=\frac{13}{\sqrt{29}}$

$\frac{|(2\hat{i}+3\hat{j}+4\hat{k})\times((2-\lambda)\hat{i}+4\hat{k})|}{\sqrt{29}}=\frac{13}{\sqrt{29}}$

$|-8\hat{j}-3(2-\lambda)\hat{k}+12\hat{i}+4(2-\lambda)\hat{j}|=13$

$|12\hat{i}-4\lambda\hat{j}+(3\lambda-6)\hat{k}|=13$

$144+16\lambda^2+(3\lambda-6)^2=169$

$16\lambda^2+(3\lambda-6)^2=25=\lambda \Rightarrow =1$

Question 246

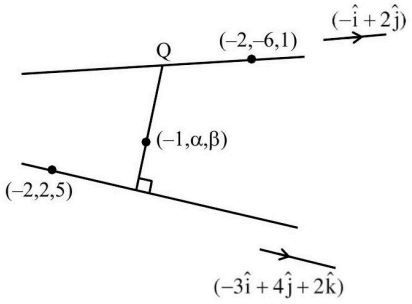
Let the point $(-1, \alpha, \beta)$ lie on the line of the shortest distance between the lines $\frac{x+2}{-3} = \frac{y-2}{4} = \frac{z-5}{2}$ and $\frac{x+2}{-1} = \frac{y+6}{2} = \frac{z-1}{0}$.
Then $(\alpha - \beta)^2$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 25

Source solution



$\mathbf{P}(-3\lambda - 2, 4\lambda + 2, 2\lambda + 5)$
$\mathbf{Q}(-\mu - 2, 2\mu - 6, 1)$
DRS of PQ $= (3\lambda - \mu, 2\mu - 4\lambda - 8, -2\lambda - 4)$
DRS of $\boldsymbol{PQ} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 0 \\ -3 & 4 & 2 \end{vmatrix}$
$= (4\hat{i} + 2\hat{j} + 2\hat{k})$
OR
(2, 1, 1)
$\frac{3\lambda-\mu}{2} = \frac{2\mu-4\lambda-8}{1} = \frac{-2\lambda-4}{1}$
$\Rightarrow \mu = \lambda + 2 \& 7\lambda = \mu - 8$
$\lambda = -1 \quad \mu = 1$
$\mathbf{Q} : (-3, -4, 1)$
$\mathbf{L_{PQ}} = \frac{x+3}{2} = \frac{y+4}{1} = \frac{z-1}{1}$
$(-1, \alpha, \beta) \Rightarrow 1 = \frac{\alpha+4}{1} = \frac{\beta-1}{1}$
$\Rightarrow \alpha = -3, \beta = 2$
$(\alpha - \beta)^2 = 25$

Question 247

Let $L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ be two lines. Then which of the following points lies on the line of the shortest distance between L_1 and L_2?

A. $\left(-\frac{5}{3}, -7, 1\right)$

B. $\left(2, 3, \frac{1}{3}\right)$

C. $\left(\frac{8}{3}, -1, \frac{1}{3}\right)$

D. $\left(\frac{14}{3}, -3, \frac{22}{3}\right)$

Correct answer: D

Source solution

$$\text{Let } P(2\lambda+1, 3\lambda+2, 4\lambda+3) \text{ on } L_1$$

$$Q(3\mu+2, 4\mu+4, 5\mu+5) \text{ on } L_2$$

The direction ratios of PQ are given by:

$$PQ = (3\mu - 2\lambda + 1, 4\mu - 3\lambda + 2, 5\mu - 4\lambda + 2)$$

Since $PQ \perp L_1$:

$$(3\mu - 2\lambda + 1) \cdot 2 + (4\mu - 3\lambda + 2) \cdot 3 + (5\mu - 4\lambda + 2) \cdot 4 = 0$$

This simplifies to:

$$38\mu - 29\lambda + 16 = 0 \tag{1}$$

Since $PQ \perp L_2$:

$$(3\mu - 2\lambda + 1) \cdot 3 + (4\mu - 3\lambda + 2) \cdot 4 + (5\mu - 4\lambda + 2) \cdot 5 = 0$$

This simplifies to:

$$50\mu - 38\lambda + 21 = 0 \tag{2}$$

Now, solving equations (1) and (2):
From (1):

$$\lambda = \frac{1}{3}$$

Substituting λ into (1):

$$38\mu - 29\left(\frac{1}{3}\right) + 16 = 0$$

$$38\mu - \frac{29}{3} + 16 = 0$$

$$38\mu = \frac{29}{3} - 16 = \frac{29 - 48}{3} = \frac{-19}{3}$$

$$\mu = \frac{-1}{6}$$

Thus, we have:

$$P\left(\frac{5}{3}, 3, \frac{13}{3}\right) \quad \text{and} \quad Q\left(\frac{3}{2}, \frac{10}{3}, \frac{25}{6}\right)$$

The line PQ can be expressed as:

$$\frac{x-\frac{5}{3}}{\frac{1}{6}} = \frac{y-3}{-\frac{1}{3}} = \frac{z-\frac{13}{3}}{\frac{1}{6}}$$

This simplifies to:

$$\frac{x-\frac{5}{3}}{1} = \frac{y-3}{-2} = \frac{z-\frac{13}{3}}{1}$$

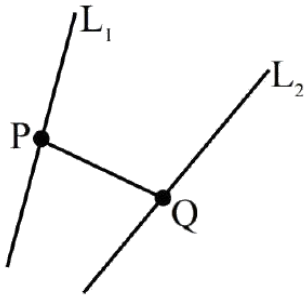
To find a point on line PQ:
Let $t = 1$:

$$x = \frac{5}{3} + 1 = \frac{8}{3}$$

$$y = 3 - 2 = 1$$

$$z = \frac{13}{3} + 1 = \frac{16}{3}$$

Thus, the point $\left(\frac{14}{3}, -3, \frac{22}{3}\right)$ lies on the line PQ.



Question 248

The shortest distance between the line
$\frac{x-3}{4}=\frac{y+7}{-11}=\frac{z-1}{5}$ and $\frac{x-5}{3}=\frac{y-9}{-6}=\frac{z+2}{1}$ is :

A. $\frac{187}{\sqrt{563}}$

B. $\frac{178}{\sqrt{563}}$

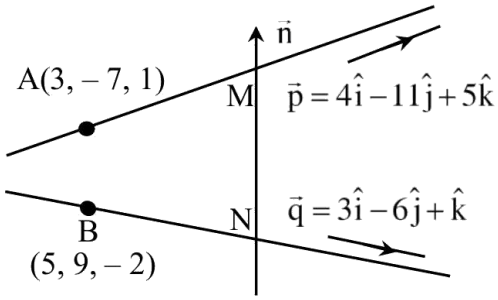
C. $\frac{185}{\sqrt{563}}$

D. $\frac{179}{\sqrt{563}}$

Correct answer: A

Source solution

Sol.
$\vec{n}=\vec{p}\times\vec{q}$
$\vec{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}4 & -11 & 53 & -6 & 1\end{vmatrix}=19\hat{i}+11\hat{j}+9\hat{k}$
S.d. = projection of $\overrightarrow{AB}$ on $\vec{n}$
$=\left|\frac{\overrightarrow{AB}\cdot\vec{n}}{|\vec{n}|}\right|=\left|\frac{(2\hat{i}+16\hat{j}-3\hat{k})\cdot(19\hat{i}+11\hat{j}+9\hat{k})}{\sqrt{361+121+81}}\right|$
$=\frac{38+176-27}{\sqrt{563}}$
S.d. $=\frac{187}{\sqrt{563}}$



Question 249

Let Q be the foot of the perpendicular from the point $\mathbf{P(7,-2,13)}$ on the plane containing the lines $\frac{x+1}{6}=\frac{y-1}{7}=\frac{z-3}{8}$ and $\frac{x-1}{3}=\frac{y-2}{5}=\frac{z-3}{7}$. Then $\mathbf{(PQ)^2}$, is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 96

Source solution

Containing the line $\begin{vmatrix}x+1 & y-1 & z-3\\ 6 & 7 & 8\\ 3 & 5 & 7\end{vmatrix}=0$
$9(x+1)-18(y-1)+9(z-3)=0$
$x-2y+z=0$
$PQ=\left|\frac{7+4+13}{\sqrt{6}}\right|=4\sqrt{6}$
$PQ^2=96$

Question 250

Shortest distance between the lines $\frac{x-3}{1}=\frac{y-8}{4}=\frac{z-3}{22}, \frac{x+3}{1}=\frac{y+7}{1}=\frac{z-6}{7}$ is

A. $3\sqrt{30}$

B. $\sqrt{30}$

C. $2\sqrt{30}$

D. $4\sqrt{30}$

Correct answer: A

Source solution

Sol.

$\overrightarrow{AB}=-3\hat{i}-7\hat{j}+6\hat{k}-(3\hat{i}+8\hat{j}+3\hat{k})=-6\hat{i}-15\hat{j}+3\hat{k}$

$\vec{p}=\hat{i}+4\hat{j}+22\hat{k}$

$\vec{q}=\hat{i}+\hat{j}+7\hat{k}$

$\vec{p}\times\vec{q}=\left|\hat{i} \quad \hat{j} \quad \hat{k}1 \quad 4 \quad 221 \quad 1 \quad 7\right|=6\hat{i}+15\hat{j}-3\hat{k}$

Shortest distance $=\frac{|\overrightarrow{AB}\cdot(\vec{p}\times\vec{q})|}{|\vec{p}\times\vec{q}|}=\frac{|36+225+9|}{\sqrt{36+225+9}}=3\sqrt{30}$.

Question 251

Let P be the foot of the perpendicular from the point $\mathbf{Q(10,-3,-1)}$ on the line $\frac{x-3}{7}=\frac{y-2}{-1}=\frac{z+1}{-2}$. Then the area of the right angled triangle PQR , where R is the point $\mathbf{(3,-2,1)}$, is

A. $9\sqrt{15}$

B. $\sqrt{30}$

C. $8\sqrt{15}$

D. $3\sqrt{30}$

Correct answer: D

Source solution

Sol.

$\frac{x-3}{7}=\frac{y-2}{-1}=\frac{z+1}{-2}=\lambda$

$\Rightarrow 7\lambda+3,-\lambda+2,-2\lambda-1$

dr's of QP $\Rightarrow$

$7\lambda-7,-\lambda+5,-2\lambda$

Now

$(7\lambda-7)\cdot 7-(-\lambda+5)+(2\lambda)\cdot 2=0$

$54\lambda-54=0\Rightarrow\lambda=1$

$\therefore \mathrm{P}=(10,1,-3)$

$\overrightarrow{\mathrm{PQ}}=-4\hat{\mathrm{j}}+2\hat{\mathrm{k}}$

$\overrightarrow{\mathrm{PR}}=-7\hat{\mathrm{i}}-3\hat{\mathrm{j}}+4\hat{\mathrm{k}}$

Area $=\left|\frac{1}{2}\right|\mathrm{i} \quad \mathrm{j} \quad \mathrm{k}0 \quad -4 \quad 2-7 \quad -3 \quad 4||=3\sqrt{30}$

Question 252

The distance of the line

$$\frac{x-2}{2}=\frac{y-6}{3}=\frac{z-3}{4}$$

from the point $(1,4,0)$ along the line

$$\frac{x}{1}=\frac{y-2}{2}=\frac{z+3}{3}$$

is :

A. $\sqrt{17}$

B. $\sqrt{14}$

C. $\sqrt{15}$

D. $\sqrt{13}$

Correct answer: B

Source solution

Let the equation of the parallel line be

$$\frac{x-1}{1}=\frac{y-4}{2}=\frac{z-0}{3}$$

The point of intersection can be expressed as

$$(\lambda+1,2\lambda+4,3\lambda)$$

We also have the parametric representation of another line:

$$(2t+2,3t+6,4t+3)$$

Setting the two expressions equal gives:

$$\lambda+1=2t+2$$

$$2\lambda+4=3t+6$$

$$3\lambda=4t+3$$

From the first equation, we can express λ in terms of t:

$$\lambda=2t+1$$

Substituting λ into the second equation:

$$2(2t+1)+4=3t+6$$

This simplifies to:

$$4t+2+4=3t+6$$

Thus,

$$4t+6=3t+6$$

Solving for t:

$$t=0$$

Now substituting $t=0$ back into the parametric equations gives the point of intersection:

$$(2(0)+2,3(0)+6,4(0)+3)=(2,6,3)$$

Next, we calculate the distance from the point $(1,4,0)$ to the point of intersection $(2,6,3)$:

$$\text{Distance}=\sqrt{(2-1)^2+(6-4)^2+(3-0)^2}$$

Calculating each term:

$$=\sqrt{1^2+2^2+3^2}=\sqrt{1+4+9}=\sqrt{14}$$

Thus, the final answer is

$$\sqrt{14}$$

Question 253

If the shortest distance between the lines $\frac{x+\sqrt{6}}{2}=\frac{y-\sqrt{6}}{3}=\frac{z-\sqrt{6}}{4}$ and $\frac{x-\lambda}{3}=\frac{y-2\sqrt{6}}{4}=\frac{z+2\sqrt{6}}{5}$ is 6 , then the square of sum of all possible values of λ is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 384

Source solution

384

$P(-\sqrt{6},\sqrt{6},\sqrt{6})\quad Q(\lambda,2\sqrt{6},-2\sqrt{6})$

$\bar{n}_1=(2,3,4)\quad \bar{n}_2=(3,4,5)$

$\bar{n}_1\times\bar{n}_2\Rightarrow\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&3&4\\3&4&5\end{vmatrix}=\hat{i}(-1)-\hat{j}(-2)+\hat{k}(-1)$

$=(-1,2,-1)$

$\therefore S_d\left|\frac{\overline{PQ}\cdot(-1,2,-1)}{\sqrt{6}}\right|=\frac{(\lambda+\sqrt{6},\sqrt{6},-3\sqrt{6})\cdot(-1,2,-1)}{\sqrt{6}}$

$$=\left|\frac{-\lambda-\sqrt{6}+2\sqrt{6}+3\sqrt{6}}{\sqrt{6}}\right|=6$$

$\Rightarrow|-\lambda+4\sqrt{6}|=6\sqrt{6}$

(+) $\quad-\lambda+4\sqrt{6}=6\sqrt{6}$

(-) $\lambda-4\sqrt{6}=6\sqrt{6}$

$\lambda=-2\sqrt{6}$

$\lambda=10\sqrt{6}$

$\therefore(8\sqrt{6})^2=384$

Question 254

The distance of the line $\frac{x-2}{2}=\frac{y-6}{3}=\frac{z-3}{4}$ from the point $(1,4,0)$ along the line $\frac{x}{1}=\frac{y-2}{2}=\frac{z+3}{3}$ is :

A. $\sqrt{17}$

B. $\sqrt{14}$

C. $\sqrt{15}$

D. $\sqrt{13}$

Correct answer: B

Source solution

Sol. Let the parallel line is

$\frac{x-1}{1}=\frac{y-4}{2}=\frac{z-0}{3}$

so their point of intersection is

$(\lambda+1,2\lambda+43\lambda)=(2t+2,3t+6,4t+3)$

$\lambda=2t+1$

$2\lambda+4=3t+6\Rightarrow t=0$

so POI is $(2,6,3)$

so distance $=\sqrt{(2-1)^2+(6-4)^2+(3-0)^2}=\sqrt{14}$

Question 255

If $\mathbf{d_1}$ is the shortest distance between the lines $x+1=2y=-12z, x=y+2=6z-6$ and d_2 is the shortest distance between the lines $\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}, \frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$, then the value of $\frac{32\sqrt{3}\,\mathrm{d_1}}{\mathrm{d_2}}$ is :

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 16

Source solution

$\mathbf{L_1}: \frac{x+1}{1}=\frac{y}{1/2}=\frac{z}{-1/12}$, $\mathbf{L_2}: \frac{x}{1}=\frac{y+2}{1}=\frac{z-1}{\frac{1}{6}}$

$\mathbf{d_1}=$ shortest distance between $\mathbf{L_1}$& $\mathbf{L_2}$

$=\left|\frac{(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)}{\left|(\vec{b}_1\times\vec{b}_2)\right|}\right|$

$\mathbf{d_1}=2$

$\mathbf{L_3}: \frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}$, $\mathbf{L_4}: \frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$

$\mathbf{d_2}=$ shortest distance between $\mathbf{L_3}$& $\mathbf{L_4}$

$\mathbf{d_2}=\frac{12}{\sqrt{3}}$ Hence

$=\frac{32\sqrt{3}\,\mathrm{d_1}}{\mathrm{d_2}}=\frac{32\sqrt{3}\times 2}{\frac{12}{\sqrt{3}}}=16$

Question 256

The shortest distance between the lines $x+1=2y=-12z$ and $x=y+2=6z-6$ is

A. $\frac{3}{2}$

B. 2

C. $\frac{5}{2}$

D. 3

Correct answer: B

Source solution

Sol.

We start with the equations:

$$\frac{x+1}{1}=\frac{y}{\frac{1}{2}}=\frac{z}{\frac{-1}{12}}, \quad \frac{x}{1}=\frac{y+2}{1}=\frac{z-1}{\frac{1}{6}}$$

Let $\vec{a}=(-1,0,0)$ and $\vec{b}=(0,-2,1)$.

Define the vectors:

$$\vec{p}=\left(1,\frac{1}{2},\frac{-1}{12}\right), \quad \vec{q}=\left(1,1,\frac{1}{6}\right)$$

Calculate $\vec{b}-\vec{a}$:

$$\vec{b}-\vec{a}=\hat{i}-2\hat{j}+\hat{k}$$

Next, compute the cross product $\vec{p}\times\vec{q}$:

$$\vec{p}\times\vec{q}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & \frac{1}{2} & -\frac{1}{12}\\ 1 & 1 & \frac{1}{6}\end{vmatrix}$$

Calculating the determinant:

$$=\hat{i}\left(\frac{1}{12}+\frac{1}{12}\right)-\hat{j}\left(\frac{1}{6}+\frac{1}{12}\right)+\hat{k}\left(1-\frac{1}{2}\right)$$

This simplifies to:

$$=\frac{\hat{i}}{6}-\frac{\hat{j}}{4}+\frac{\hat{k}}{2}$$

Now, find the magnitude of $\vec{p}\times\vec{q}$:

$$|\vec{p}\times\vec{q}|=\sqrt{\frac{1}{36}+\frac{1}{16}+\frac{1}{4}}=\frac{7}{12}$$

Now, calculate the distance d:

$$d = \left| \frac{(\vec{b} - \vec{a}) \cdot (\vec{p} \times \vec{q})}{|\vec{p} \times \vec{q}|} \right|$$

Substituting the values:

$$d = \left| \frac{\left(\hat{i} - 2\hat{j} + \hat{k}\right) \cdot \left(\frac{\hat{i}}{6} - \frac{\hat{j}}{4} + \frac{\hat{k}}{2}\right)}{\frac{7}{12}} \right|$$

Calculating the dot product:

$$= \left| \frac{\frac{1}{6} + 1 + \frac{1}{2}}{\frac{7}{12}} \right| = \left| \frac{\frac{7}{6}}{\frac{7}{12}} \right| = 2$$

Thus, the final answer is:

$$\boxed{2}$$

Question 257

The shortest distance between the lines $x+1=2y=-12z$ and $x=y+2=6z-6$ is

A. $\frac{3}{2}$

B. 2

C. $\frac{5}{2}$

D. 3

Correct answer: B

Source solution

Given lines

$$\frac{x+1}{1}=\frac{y}{\frac{1}{2}}=\frac{z}{-\frac{1}{12}}, \qquad \frac{x}{1}=\frac{y+2}{1}=\frac{z-1}{\frac{1}{6}}.$$

Let the lines be

$$L_1:\ \vec{r}=\vec{a}+\lambda\vec{p}, \qquad L_2:\ \vec{r}=\vec{b}+\mu\vec{q}.$$

From the given equations,

$$\vec{a}=(-1,0,0), \qquad \vec{p}=\left(1,\frac{1}{2},-\frac{1}{12}\right),$$

and

$$\vec{b}=(0,-2,1), \qquad \vec{q}=\left(1,1,\frac{1}{6}\right).$$

The shortest distance between two skew lines is

$$d=\left|\frac{(\vec{b}-\vec{a})\cdot(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right|.$$

First,

$$\vec{b}-\vec{a}=(0,-2,1)-(-1,0,0)=(1,-2,1).$$

Hence,

$$\vec{b}-\vec{a}=\hat{i}-2\hat{j}+\hat{k}.$$

Now compute $\vec{p}\times\vec{q}$:

$$\vec{p}\times\vec{q}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & \frac{1}{2} & -\frac{1}{12}\\ 1 & 1 & \frac{1}{6}\end{vmatrix}.$$

$$\vec{p}\times\vec{q}=\hat{i}\left(\frac{1}{2}\cdot\frac{1}{6}-\left(-\frac{1}{12}\right)\cdot 1\right)-\hat{j}\left(1\cdot\frac{1}{6}-\left(-\frac{1}{12}\right)\cdot 1\right)$$
$$+\hat{k}\left(1\cdot 1-\frac{1}{2}\cdot 1\right).$$

$$=\hat{i}\left(\frac{1}{12}+\frac{1}{12}\right)-\hat{j}\left(\frac{1}{6}+\frac{1}{12}\right)+\hat{k}\left(1-\frac{1}{2}\right).$$

$$=\frac{\hat{i}}{6}-\frac{\hat{j}}{4}+\frac{\hat{k}}{2}.$$

Its magnitude is

$$|\vec{p}\times\vec{q}|=\sqrt{\left(\frac{1}{6}\right)^2+\left(-\frac{1}{4}\right)^2+\left(\frac{1}{2}\right)^2}.$$

$$=\sqrt{\frac{1}{36}+\frac{1}{16}+\frac{1}{4}}.$$

$$=\sqrt{\frac{4+9+36}{144}}=\sqrt{\frac{49}{144}}=\frac{7}{12}.$$

Next,

$$(\vec{b}-\vec{a})\cdot(\vec{p}\times\vec{q})=(\hat{i}-2\hat{j}+\hat{k})\cdot\left(\frac{\hat{i}}{6}-\frac{\hat{j}}{4}+\frac{\hat{k}}{2}\right).$$

$$=\frac{1}{6}+\frac{1}{2}+\frac{1}{2}.$$

$$=\frac{7}{6}.$$

Therefore,

$$d=\left|\frac{\frac{7}{6}}{\frac{7}{12}}\right|.$$

$$= \frac{7}{6} \cdot \frac{12}{7}.$$

$$= 2.$$

Hence, the shortest distance between the two lines is

$$\boxed{2}.$$

Question 258

If the shortest distance between the lines

$$\vec{r_1} = \alpha\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k}), \lambda \in \mathbf{R}, \alpha > 0$$

and $\vec{r_2} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k}), \mu \in \mathbf{R}$ is 9 , then α is equal to ___.

Numerical answer: _____

Enter the requested integer (0 decimal places).

Correct answer: 6

Source solution

If $\vec{r} = \vec{a} + \lambda\vec{b}$ and $\vec{r} = \vec{c} + \lambda\vec{d}$ then shortest distance between two lines is

$$L = \frac{(\vec{a} - \vec{c}) \cdot (\vec{b} \times \vec{d})}{|\mathrm{b} \times \mathrm{d}|}$$

$$\therefore \vec{a} - \vec{c} = ((\alpha + 4)\hat{i} + 2\hat{j} + 3\hat{k})$$

$$\frac{\vec{b} \times \vec{d}}{|\mathrm{b} \times \mathrm{d}|} = \frac{(2\hat{i} + 2\hat{j} + \hat{k})}{3}$$

$$\therefore ((\alpha + 4)\hat{i} + 2\hat{j} + 3\hat{k}) \cdot \frac{(2\hat{i} + 2\hat{j} + \hat{k})}{3} = 9$$

or $\alpha = 6$

Question 259

Let the line passing through the points $(-1, 2, 1)$ and parallel to the line

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{4}$$

intersect the line

$$\frac{x+2}{3} = \frac{y-3}{2} = \frac{z-4}{1}$$

at the point P. Then the distance of P from the point $Q(4, -5, 1)$ is :

A. 5

B. 10

C. $5\sqrt{6}$

D. $5\sqrt{5}$

Correct answer: D

Source solution

$$\text{Let the line through } (-1, 2, 1) \text{ parallel to } \frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{4} :$$

$$\frac{x+1}{2} = \frac{y-2}{3} = \frac{z-1}{4} = \lambda$$

$$\Rightarrow x = 2\lambda - 1, y = 3\lambda + 2, z = 4\lambda + 1$$

$$\text{Let } \frac{x+2}{3} = \frac{y-3}{2} = \frac{z-4}{1} = \mu$$

$$\Rightarrow x = 3\mu - 2, y = 2\mu + 3, z = \mu + 4$$

$$\text{For intersection: } 2\lambda - 1 = 3\mu - 2,\ 3\lambda + 2 = 2\mu + 3,\ 4\lambda + 1 = \mu + 4$$

$$\lambda = 1, \mu = 1$$

$$P(1, 5, 5), Q(4, -5, 1)$$

$$PQ = \sqrt{9 + 100 + 16} = \sqrt{125} = 5\sqrt{5}$$

Question 260

If the shortest distance between the lines

$$\frac{x-\lambda}{2}=\frac{y-4}{3}=\frac{z-3}{4}$$

and

$$\frac{x-2}{4}=\frac{y-4}{6}=\frac{z-7}{8}$$

is

$$\frac{13}{\sqrt{29}},$$

then a value of λ is :

A. $-\frac{13}{25}$

B. $\frac{13}{25}$

C. 1

D. -1

Correct answer: C

Source solution

$$\text{Let } \bar{r}_1=(\lambda\hat{i}+4\hat{j}+3\hat{k})+\alpha(2\hat{i}+3\hat{j}+4\hat{k})$$

$$\bar{r}_2=(2\hat{i}+4\hat{j}+7\hat{k})+\beta(2\hat{i}+3\hat{j}+4\hat{k})$$

$$\bar{b}=2\hat{i}+3\hat{j}+4\hat{k}$$

$$\bar{a}_1=\lambda\hat{i}+4\hat{j}+3\hat{k}$$

$$\bar{a}_2=2\hat{i}+4\hat{j}+7\hat{k}$$

The shortest distance is given by:

$$\text{Shortest distance}=\frac{\left|\bar{b}\times(\bar{a}_2-\bar{a}_1)\right|}{|\bar{b}|}$$

Calculating $\bar{a}_2-\bar{a}_1$:

$$\bar{a}_2-\bar{a}_1=(2-\lambda)\hat{i}+(4-4)\hat{j}+(7-3)\hat{k}=(2-\lambda)\hat{i}+0\hat{j}+4\hat{k}$$

Now, compute the cross product:

$$\bar{b}\times(\bar{a}_2-\bar{a}_1)=(2\hat{i}+3\hat{j}+4\hat{k})\times((2-\lambda)\hat{i}+4\hat{k})$$

Calculating the cross product:

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 2-\lambda & 0 & 4 \end{vmatrix}$$

$$= \hat{i}(3 \cdot 4 - 0 \cdot 4) - \hat{j}(2 \cdot 4 - (2-\lambda) \cdot 4) + \hat{k}(2 \cdot 0 - 3(2-\lambda))$$

$$= 12\hat{i} - (8 - 4(2-\lambda))\hat{j} - 3(2-\lambda)\hat{k}$$

$$= 12\hat{i} - (8 - 8 + 4\lambda)\hat{j} - (6 - 3\lambda)\hat{k}$$

$$= 12\hat{i} - 4\lambda\hat{j} + (3\lambda - 6)\hat{k}$$

Now, find the magnitude:

$$\left|12\hat{i} - 4\lambda\hat{j} + (3\lambda - 6)\hat{k}\right| = 13$$

Calculating the magnitude:

$$\sqrt{12^2 + (-4\lambda)^2 + (3\lambda - 6)^2} = 13$$

Squaring both sides:

$$144 + 16\lambda^2 + (3\lambda - 6)^2 = 169$$

Expanding:

$$(3\lambda - 6)^2 = 9\lambda^2 - 36\lambda + 36$$

Thus:

$$144 + 16\lambda^2 + 9\lambda^2 - 36\lambda + 36 = 169$$

Combining terms:

$$25\lambda^2 - 36\lambda + 180 = 169$$

Simplifying:

$$25\lambda^2 - 36\lambda + 11 = 0$$

Using the quadratic formula:

$$\lambda = \frac{-(-36) \pm \sqrt{(-36)^2 - 4 \cdot 25 \cdot 11}}{2 \cdot 25}$$

Calculating the discriminant:

$$= \frac{36 \pm \sqrt{1296 - 1100}}{50} = \frac{36 \pm \sqrt{196}}{50} = \frac{36 \pm 14}{50}$$

Thus:

$$\lambda = \frac{50}{50} = 1 \quad \text{or} \quad \lambda = \frac{22}{50} = \frac{11}{25}$$

Final answer:

$$\lambda = 1$$

Question 261

If the square of the shortest distance between the lines $\frac{x-2}{1}=\frac{y-1}{2}=\frac{z+3}{-3}$ and $\frac{x+1}{2}=\frac{y+3}{4}=\frac{z+5}{-5}$ is $\frac{m}{n}$, where m, n are coprime numbers, then $m+n$ is equal to:

A. 6

B. 9

C. 21

D. 14

Correct answer: B

Source solution

Sol. $\vec{a}=(2,1,-3)$

$\vec{b}=(-1,-3,-5)$

$\vec{p}\times\vec{q}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & -3\\ 2 & 4 & -5\end{vmatrix}$

$=2\hat{i}-\hat{j}$

$\vec{b}-\vec{a}=-3\hat{i}-4\hat{j}-2\hat{k}$

$S_d=\frac{|(\vec{b}-\vec{a})\cdot(\vec{p}\times\vec{q})|}{|\vec{p}\times\vec{q}|}=$

$=\frac{2}{\sqrt{5}}$

$(S_d)^2=\frac{4}{5}$

$m=4, n=5 \Rightarrow m+n=9$

Question 262

Let PQR be a triangle with $R(-1,4,2)$. Suppose $M(2,1,2)$ is the mid point of PQ . The distance of the centroid of $\triangle PQR$ from the point of intersection of the line $\frac{x-2}{0}=\frac{y}{2}=\frac{z+3}{-1}$ and $\frac{x-1}{1}=\frac{y+3}{-3}=\frac{z+1}{1}$ is

A. 69

B. 9

C. $\sqrt{69}$

D. $\sqrt{99}$

Correct answer: C

Source solution

Sol. Centroid G divides MR in $1:2$

$G(1,2,2)$

Point of intersection A of given lines is ($2,-6,0$)

$AG=\sqrt{69}$

Question 263

Let a line pass through two distinct points $\mathbf{P(-2,-1,3)}$ and Q , and be parallel to the vector $\mathbf{3\hat{i}+2\hat{j}+2k}$. If the distance of the point Q from the point $\mathbf{R(1,3,3)}$ is 5 , then the square of the area of $\mathbf{\triangle PQR}$ is equal to:

A. 136

B. 140

C. 144

D. 148

Correct answer: A

Source solution

Sol. $\overrightarrow{\mathbf{PQ}}$ parallel to $3\hat{i}+2\hat{j}+2\hat{k}, R(1,3,3)$

$\Rightarrow \mathbf{Q(3\lambda-2,2\lambda-1,2\lambda+3),\lambda\in R-\{0\}}$

$|\overrightarrow{QR}|=5=\sqrt{(3\lambda-3)^2+(2\lambda-4)^2+(2\lambda)^2}$

$\therefore 17\lambda^2-34\lambda+25=25\Rightarrow\lambda=2(\because\lambda\neq 0)$

$\therefore \mathbf{Q(4,3,7),P(-2,-1,3),R(1,3,3)}$

Area of $\triangle PQR=[PQR]=\frac{1}{2}|\overrightarrow{PQ}\times\overrightarrow{PR}|$

$[PQR]=\frac{1}{2}\left\|\begin{matrix}\hat{i} & \hat{j} & \hat{k}\\ 6 & 4 & 4\\ 3 & 2 & 3\end{matrix}\right.\left.\begin{matrix}\\ 4\\ 4\end{matrix}\right.\left.\begin{matrix}\\ 0\\ 0\end{matrix}\right\|=\left\|\begin{matrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 2 & 3\\ 4 & 4 & 0\end{matrix}\right\|$

$[PQR]=|-8\hat{i}+6\hat{j}+6\hat{k}|=\sqrt{136}$

$\therefore [PQR]^2=136$

Question 264

Let P be the plane passing through the line $\frac{x-1}{1}=\frac{y-2}{-3}=\frac{z+5}{7}$ and the point $(2,4,-3)$. If the image of the point $(-1,3,4)$ in the plane P is (α, β, γ), then $\alpha+\beta+\gamma$ is equal to

A. 12

B. 11

C. 9

D. 10

Correct answer: D

Source solution

The plane P passes through the line

$$\frac{x-1}{1}=\frac{y-2}{-3}=\frac{z+5}{7}$$

and the point

$$A=(2,4,-3).$$

A point on the line is

$$B=(1,2,-5),$$

and its direction vector is

$$\vec{d}=(1,-3,7).$$

Also,

$$\overrightarrow{BA}=(2-1,\ 4-2,\ -3+5)=(1,2,2).$$

Hence a normal vector to the plane is

$$\vec{n}=\vec{d}\times\overrightarrow{BA}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -3 & 7\\ 1 & 2 & 2\end{vmatrix}.$$

$$=\hat{i}(-6-14)-\hat{j}(2-7)+\hat{k}(2+3)$$

$$=(-20,5,5)=(-4,1,1).$$

Therefore, the equation of the plane is

$$-4(x-1)+(y-2)+(z+5)=0,$$

i.e.,

$$4x-y-z-7=0.$$

Let

$$P_0=(-1,3,4)$$

and let its image in the plane be

$$P_0'=(\alpha,\beta,\gamma).$$

For the plane

$$ax+by+cz+d=0,$$

the reflection of (x_1,y_1,z_1) is

$$\left(x_1-\frac{2aD}{a^2+b^2+c^2},\ y_1-\frac{2bD}{a^2+b^2+c^2},\ z_1-\frac{2cD}{a^2+b^2+c^2}\right),$$

where

$$D=ax_1+by_1+cz_1+d.$$

Here,

$$a=4,\quad b=-1,\quad c=-1,\quad d=-7.$$

Thus,

$$D=4(-1)-3-4-7=-18.$$

Also,

$$a^2+b^2+c^2=16+1+1=18.$$

Hence,

$$\alpha=-1-\frac{2(4)(-18)}{18}=-1+8=7,$$

$$\beta = 3 - \frac{2(-1)(-18)}{18} = 3 - 2 = 1,$$

$$\gamma = 4 - \frac{2(-1)(-18)}{18} = 4 - 2 = 2.$$

Therefore,

$$(\alpha, \beta, \gamma) = (7, 1, 2).$$

So,

$$\alpha + \beta + \gamma = 7 + 1 + 2 = 10.$$

$$\boxed{10}$$

Hence, the correct option is

$$\boxed{\text{(4) } 10}.$$

Question 265

The perpendicular distance, of the line

$$\frac{x-1}{2}=\frac{y+2}{-1}=\frac{z+3}{2}$$

from the point $P(2,-10,1)$, is:

A. 6

B. $5\sqrt{2}$

C. $3\sqrt{5}$

D. $4\sqrt{3}$

Correct answer: C

Source solution

$$\text{Let } \frac{x-1}{2}=\frac{y+2}{-1}=\frac{z+3}{2}=\lambda$$

$$\Rightarrow (x,y,z)=(2\lambda+1,-\lambda-2,2\lambda-3)$$

$$\text{Since } \overrightarrow{PA}\cdot\vec{n}=0$$

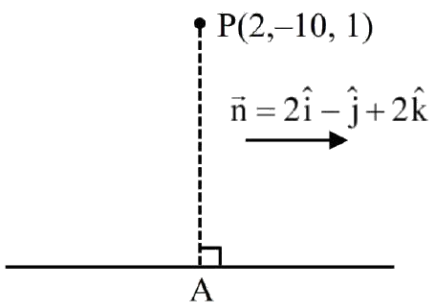
$$\Rightarrow (2\lambda-1)\cdot 2+(-\lambda+8)(-1)+(2\lambda-4)\cdot 2=0$$

$$\Rightarrow 4\lambda-2+\lambda-8+4\lambda-8=0$$

$$\Rightarrow 9\lambda-18=0\Rightarrow \lambda=2$$

$$\therefore A(5,-4,1)$$

$$\therefore AP=\sqrt{3^2+6^2+0^2}=\sqrt{45}=3\sqrt{5}$$



Question 266

Let $L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ be two lines. Then which of the following points lies on the line of the shortest distance between L_1 and L_2 ?

A. $\left(-\frac{5}{3}, -7, 1\right)$

B. $\left(2, 3, \frac{1}{3}\right)$

C. $\left(\frac{8}{3}, -1, \frac{1}{3}\right)$

D. $\left(\frac{14}{3}, -3, \frac{22}{3}\right)$

Correct answer: D

Source solution

Sol.

$P(2\lambda+1, 3\lambda+2, 4\lambda+3)$ on L_1

$Q(3\mu+2, 4\mu+4, 5\mu+5)$ on L_2

Dr's of $PQ = 3\mu - 2\lambda + 1, 4\mu - 3\lambda + 2, 5\mu - 4\lambda + 2$

$PQ \perp L_1$

$\Rightarrow (3\mu - 2\lambda + 1)2 + (4\mu - 3\lambda + 2)3 + (5\mu - 4\lambda + 2)\,4 = 0$

$$38\mu - 29\lambda + 16 = 0 \quad (1)$$

$PQ \perp L_2$

$\Rightarrow (3\mu - 2\lambda + 1)3 + (4\mu - 3\lambda + 2)4 + (5\mu - 4\lambda + 2)\,5 = 0$

$$50\mu - 38\lambda + 21 = 0 \quad (2)$$

By (1) & (2)

$\lambda = \frac{1}{3}; \mu = \frac{-1}{6}$

$\therefore P\left(\frac{5}{3}, 3, \frac{13}{3}\right) \& Q\left(\frac{3}{2}, \frac{10}{3}, \frac{25}{6}\right)$

Line PQ

$\frac{x-\frac{5}{3}}{\frac{1}{6}} \quad \frac{y-3}{\frac{-1}{3}} \quad \frac{z-\frac{13}{3}}{\frac{1}{6}}$

$\frac{x-\frac{5}{3}}{1} = \frac{y-3}{-2} = \frac{z-\frac{13}{3}}{1}$

Point $\left(\frac{14}{3}, -3, \frac{22}{3}\right)$

lies on the line PQ

Question 267

Let a line having direction ratio $1, -4, 2$ intersect the lines $\frac{x-7}{3} = \frac{y-1}{-1} = \frac{z+2}{1}$ and $\frac{x}{2} = \frac{y-7}{3} = \frac{z}{1}$ at the point A and B . Then $(AB)^2$ is equal to .

Numerical answer: _____

Enter the requested value or coefficient rounded to the nearest integer (0 decimal places).

Correct answer: 84

Source solution

Write A=(7+3t,1-t,-2+t), B=(2s,7+3s,s). Requiring B-A to have direction ratios (1,-4,2) gives t=-5,s=-3. Thus A=(-8,6,-7), B=(-6,-2,-3), and $AB^2 = 2^2 + (-8)^2 + 4^2 = 84$. The printed source coordinate +7 for the last component of A is a sign typo.

Question 268

Consider the line L passing through the points $(1,2,3)$ and $(2,3,5)$. The distance of the point $\left(\frac{11}{3}, \frac{11}{3}, \frac{19}{3}\right)$ from the line L along the line $\frac{3x-11}{2} = \frac{3y-11}{1} = \frac{3z-19}{2}$ is equal to :

A. 3

B. 5

C. 4

D. 6

Correct answer: A

Source solution

Sol. $\frac{x-1}{2-1} = \frac{y-2}{3-2} = \frac{z-3}{5-3}$

$\Rightarrow \frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{2} = \lambda$

$\mathbf{B(1+\lambda, 2+\lambda, 3+2\lambda)}$

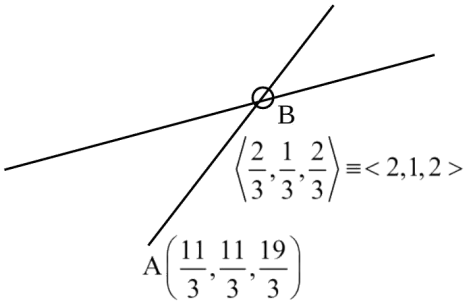
D.R. of $\mathbf{AB} =< \frac{3\lambda-8}{3}, \frac{3\lambda-5}{3}, \frac{6\lambda-10}{3} >$

B $\left(\frac{5}{3}, \frac{8}{3}, \frac{13}{3}\right) \frac{3\lambda-8}{3\lambda-5} = \frac{2}{1} \Rightarrow 3\lambda - 8 = 6\lambda - 10$

$3\lambda = 2$

$\lambda = \frac{2}{3}$

$\mathbf{AB} = \frac{\sqrt{36+9+36}}{3} = \frac{9}{3} = 3$



Question 269

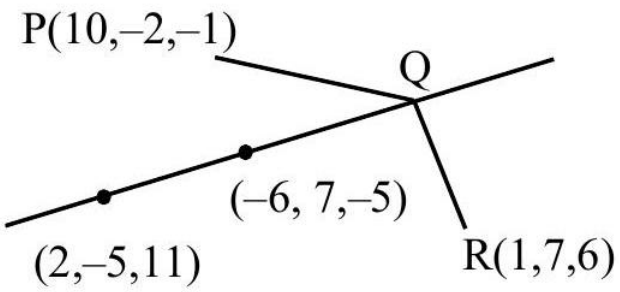
Let P be the point $(10, -2, -1)$ and Q be the foot of the perpendicular drawn from the point $R(1, 7, 6)$ on the line passing through the points $(2, -5, 11)$ and $(-6, 7, -5)$. Then the length of the line segment PQ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 13

Source solution



Line : $\frac{x+6}{-8} = \frac{y-7}{12} = \frac{z+5}{-16}$

$\frac{x+6}{2} = \frac{y-7}{-3} = \frac{z+5}{4} = \lambda$

$Q(2\lambda - 6, 7 - 3\lambda, 4\lambda - 5)$

$\overrightarrow{QR}(2\lambda - 7, -3\lambda, 4\lambda - 11)$

$\overrightarrow{QR} \cdot \mathbf{dr}$'s of line $= 0$

$4\lambda - 14 + 9\lambda + 16\lambda - 44 = 0$

$29\lambda = 58 \Rightarrow \lambda = 2$

Q(-2, 1, 3)

$PQ = \sqrt{144 + 9 + 16} = \sqrt{169} = 13$

Question 270

Let in a $\triangle ABC$, the length of the side AC be 6 , the vertex B be $(1,2,3)$ and the vertices A, C lie on the line $\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$. Then the area (in sq. units) of $\triangle ABC$ is

A. 42

B. 21

C. 56

D. 17

Correct answer: B

Source solution

Sol.

Let $M(3\lambda+6, 2\lambda+7, -2\lambda+7)$

$$\overrightarrow{BM}=(3\lambda+5)\hat{i}+(2\lambda+5)\hat{j}+(-2\lambda+4)\hat{k}$$

$\overrightarrow{AC} \cdot \overrightarrow{BM}=0=3(3\lambda+5)+2(2\lambda+5)-2(-2\lambda+4)$
$\overrightarrow{BM}=2\hat{i}+3\hat{j}+6\hat{k}$
$|\overrightarrow{BM}|=7$
Area $=\frac{1}{2}\times 6\times 7=21$
Option (2) is the correct answer.

Question 271

If the shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{\lambda}$ and $\frac{x-2}{1}=\frac{y-4}{4}=\frac{z-5}{5}$ is $\frac{1}{\sqrt{3}}$, then the sum of all possible values of λ is :

A. 16

B. (1) is correct option.

C. (2) 6

D. (3) 12

Correct answer: A

Source solution

$$L_1: \quad \frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{\lambda}$$

and

$$L_2: \quad \frac{x-2}{1}=\frac{y-4}{4}=\frac{z-5}{5}.$$

For L_1,

$$A=(1,2,3), \qquad \vec{d}_1=(2,3,\lambda).$$

For L_2,

$$B=(2,4,5), \qquad \vec{d}_2=(1,4,5).$$

Also,

$$\overrightarrow{AB}=(1,2,2).$$

The shortest distance between two lines is

$$D=\frac{\left|\overrightarrow{AB}\cdot(\vec{d}_1\times\vec{d}_2)\right|}{\left|\vec{d}_1\times\vec{d}_2\right|}.$$

Given

$$D=\frac{1}{\sqrt{3}}.$$

First,

$$\vec{d}_1\times\vec{d}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & \lambda\\ 1 & 4 & 5\end{vmatrix}=(15-4\lambda,\ \lambda-10,\ 5).$$

Therefore,

$$\overrightarrow{AB} \cdot (\vec{d}_1 \times \vec{d}_2) = (1,2,2) \cdot (15 - 4\lambda, \lambda - 10, 5).$$

$$= 15 - 4\lambda + 2\lambda - 20 + 10 = 5 - 2\lambda.$$

Also,

$$\left|\vec{d}_1 \times \vec{d}_2\right|^2 = (15 - 4\lambda)^2 + (\lambda - 10)^2 + 25.$$

$$= 17\lambda^2 - 140\lambda + 350.$$

Hence,

$$\frac{|5 - 2\lambda|}{\sqrt{17\lambda^2 - 140\lambda + 350}} = \frac{1}{\sqrt{3}}.$$

Squaring,

$$3(5 - 2\lambda)^2 = 17\lambda^2 - 140\lambda + 350.$$

$$75 - 60\lambda + 12\lambda^2 = 17\lambda^2 - 140\lambda + 350.$$

$$5\lambda^2 - 80\lambda + 275 = 0.$$

$$\lambda^2 - 16\lambda + 55 = 0.$$

$$(\lambda - 5)(\lambda - 11) = 0.$$

Thus,

$$\lambda = 5 \quad \text{or} \quad \lambda = 11.$$

Therefore, the sum of all possible values of λ is

$$5 + 11 = 16.$$

$$\boxed{16}$$

Hence, the correct option is

(1) 16.

Question 272

The shortest distance between the line

$$\frac{x-3}{4}=\frac{y+7}{-11}=\frac{z-1}{5}$$

and

$$\frac{x-5}{3}=\frac{y-9}{-6}=\frac{z+2}{1}$$

is :

A. $\frac{187}{\sqrt{563}}$

B. $\frac{178}{\sqrt{563}}$

C. $\frac{185}{\sqrt{563}}$

D. $\frac{179}{\sqrt{563}}$

Correct answer: A

Source solution

$$\vec{n}=\vec{p}\times\vec{q}$$

Calculating the cross product:

$$\vec{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & -11 & 5\\ 3 & -6 & 1\end{vmatrix}=19\hat{i}+11\hat{j}+9\hat{k}$$

Next, we find the shortest distance (S.d.) as the projection of $\overrightarrow{AB}$ on $\vec{n}$:

$$\text{S.d.}=\left|\frac{\overrightarrow{AB}\cdot\vec{n}}{|\vec{n}|}\right|$$

Calculating the dot product:

$$\overrightarrow{AB}=2\hat{i}+16\hat{j}-3\hat{k}$$

$$\overrightarrow{AB}\cdot\vec{n}=(2\hat{i}+16\hat{j}-3\hat{k})\cdot(19\hat{i}+11\hat{j}+9\hat{k})=38+176-27=187$$

Calculating the magnitude of $\vec{n}$:

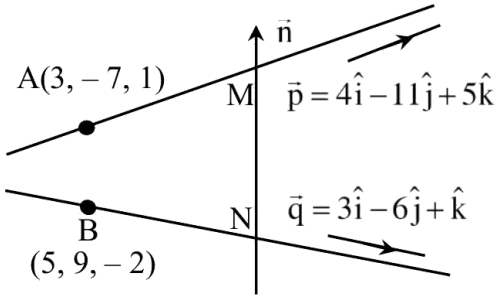
$$|\vec{n}|=\sqrt{361+121+81}=\sqrt{563}$$

Thus, the shortest distance is:

$$\text{S.d.} = \left|\frac{187}{\sqrt{563}}\right|$$

Final answer:

$$\text{S.d.} = \frac{187}{\sqrt{563}}$$



Question 273

Let λ be an integer. If the shortest distance between the lines $x - \lambda = 2y - 1 = -2z$ and $x = y + 2\lambda = z - \lambda$ is $\frac{\sqrt{7}}{2\sqrt{2}}$, then the value of $|\lambda|$ is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 1

Source solution

$\frac{x-\lambda}{1} = \frac{y-\frac{1}{2}}{\frac{1}{2}} = \frac{z}{-\frac{1}{2}}$

$$\frac{x-\lambda}{2} = \frac{y-\frac{1}{2}}{1} = \frac{2}{-1} \quad (1)$$

Point on line $= \left(\lambda, \frac{1}{2}, 0\right)$

$$\frac{x}{1} = \frac{y+2\lambda}{1} = \frac{z-\lambda}{1} \quad (2)$$

Point on line $= (0, -2\lambda, \lambda)$

Distance between skew lines $= \frac{\left[\vec{a}_2 - \vec{a}_1\, \vec{b}_1\, \vec{b}_2\right]}{\left|\vec{b}_1 \times \vec{b}_2\right|}$

$$\frac{\begin{vmatrix} \lambda & \frac{1}{2}+2\lambda & -\lambda \\ 2 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix}}$$

$= \frac{\left|-5\lambda - \frac{3}{2}\right|}{\sqrt{14}} = \frac{\sqrt{7}}{2\sqrt{2}}$ (given)

$= |10\lambda + 3| = 7 \Rightarrow \lambda = -1$

$\Rightarrow |\lambda| = 1$

Question 274

Let $\boldsymbol{P}$ the point of intersection of the lines $\frac{x-2}{1}=\frac{y-4}{5}=\frac{z-2}{1}$ and $\frac{x-3}{2}=\frac{y-2}{3}=\frac{z-3}{2}$.

Then, the shortest distance of P from the line $\mathbf{4x=2y=z}$ is

A. $\frac{5\sqrt{14}}{7}$

B. $\frac{\sqrt{14}}{7}$

C. $\frac{3\sqrt{14}}{7}$

D. $\frac{6\sqrt{14}}{7}$

Correct answer: C

Source solution

$$\text{Line } L_1: \quad \frac{x-2}{1}=\frac{y-4}{5}=\frac{z-2}{1}=\lambda$$

Hence,

$$x=2+\lambda, \qquad y=4+5\lambda, \qquad z=2+\lambda.$$

Similarly,

$$\text{Line } L_2: \quad \frac{x-3}{2}=\frac{y-2}{3}=\frac{z-3}{2}=\mu$$

gives

$$x=3+2\mu, \qquad y=2+3\mu, \qquad z=3+2\mu.$$

Since the two lines intersect at $\boldsymbol{P}$,

$$2+\lambda=3+2\mu,$$

$$4+5\lambda=2+3\mu.$$

From the first equation,

$$\lambda-2\mu=1.$$

From the second,

$$5\lambda-3\mu=-2.$$

Substituting $\boldsymbol{\lambda=1+2\mu}$,

$$5(1+2\mu)-3\mu=-2$$

$$5+7\mu=-2$$

$$\mu=-1.$$

Therefore,

$$\lambda=1+2(-1)=-1.$$

Hence,

$$P=(2-1,\ 4-5,\ 2-1)=(1,-1,1).$$

The line

$$4x=2y=z$$

can be written as

$$\frac{x}{1}=\frac{y}{2}=\frac{z}{4}=t.$$

Thus it passes through the origin and has direction vector

$$\vec{d}=(1,2,4).$$

Let

$$\vec{r}=(1,-1,1)$$

be the position vector of P.

The shortest distance from a point to a line through the origin is

$$D=\frac{\left|\vec{r}\times\vec{d}\right|}{|\vec{d}|}.$$

Now,

$$\vec{r}\times\vec{d}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&-1&1\\1&2&4\end{vmatrix}=(-6,-3,3).$$

Therefore,

$$|\vec{r} \times \vec{d}| = \sqrt{(-6)^2 + (-3)^2 + 3^2} = \sqrt{54} = 3\sqrt{6}.$$

Also,

$$|\vec{d}| = \sqrt{1^2 + 2^2 + 4^2} = \sqrt{21}.$$

Hence,

$$D = \frac{3\sqrt{6}}{\sqrt{21}} = 3\sqrt{\frac{2}{7}} = \frac{3\sqrt{14}}{7}.$$

$$\boxed{\frac{3\sqrt{14}}{7}}$$

Hence, the correct option is

$$\boxed{(3)\ \frac{3\sqrt{14}}{7}}.$$

Question 275

Let a line pass through two distinct points $\mathbf{P(-2,-1,3)}$ and $\mathbf{Q}$, and be parallel to the vector $\mathbf{3\hat{i}+2\hat{j}+2\hat{k}}$. If the distance of the point $\mathbf{Q}$ from the point $\mathbf{R(1,3,3)}$ is 5, then the square of the area of $\mathbf{\triangle PQR}$ is equal to:

A. 136

B. 140

C. 144

D. 148

Correct answer: A

Source solution

Sol.

1. Let $\overrightarrow{\mathbf{PQ}}$ be parallel to $3\hat{i}+2\hat{j}+2\hat{k}$ and point $R(1,3,3)$.
2. Define point Q as $Q(3\lambda-2, 2\lambda-1, 2\lambda+3)$, where $\lambda \in \mathbb{R}-\{0\}$.
3. The distance $|\overrightarrow{QR}|$ is given by:

$$|\overrightarrow{QR}| = 5 = \sqrt{(3\lambda-3)^2+(2\lambda-4)^2+(2\lambda)^2}$$

4. Squaring both sides, we have:

$$25 = (3\lambda-3)^2+(2\lambda-4)^2+(2\lambda)^2$$

5. Expanding and simplifying:

$$17\lambda^2-34\lambda+25 = 25 \Rightarrow 17\lambda^2-34\lambda = 0$$

6. Factoring out λ:

$$\lambda(17\lambda-34)=0 \Rightarrow \lambda = 2 \quad (\text{since } \lambda \neq 0)$$

7. Substitute λ back to find Q:

$$Q(4,3,7), \quad P(-2,-1,3), \quad R(1,3,3)$$

8. The area of triangle $\triangle PQR$ is given by:

$$[\mathrm{PQR}] = \frac{1}{2}|\overrightarrow{\mathrm{PQ}} \times \overrightarrow{\mathrm{PR}}|$$

9. Calculate the cross product:

$$\overrightarrow{\mathrm{PQ}} = Q - P = (4+2, 3+1, 7-3) = (6,4,4)$$

$$\overrightarrow{\mathrm{PR}} = R - P = (1+2, 3+1, 3-3) = (3,4,0)$$

10. The cross product is:

$$\overrightarrow{\mathrm{PQ}} \times \overrightarrow{\mathrm{PR}} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 4 & 4 \\ 3 & 4 & 0 \end{vmatrix}$$

11. Evaluating the determinant:

$$= \hat{i}(4 \cdot 0 - 4 \cdot 4) - \hat{j}(6 \cdot 0 - 4 \cdot 3) + \hat{k}(6 \cdot 4 - 4 \cdot 3)$$

$$= -16\hat{i} + 12\hat{j} + 6\hat{k}$$

12. The magnitude is:

$$\left\| -16\hat{i} + 12\hat{j} + 6\hat{k} \right\| = \sqrt{(-16)^2 + 12^2 + 6^2} = \sqrt{256 + 144 + 36} = \sqrt{436}$$

13. Therefore, the area is:

$$[\mathrm{PQR}] = \frac{1}{2}\sqrt{436} = \sqrt{109}$$

14. Finally, the squared area is:

$$[\mathrm{PQR}]^2 = 136$$

Question 276

The shortest distance between the lines $\frac{x-4}{4}=\frac{y+2}{5}=\frac{z+3}{3}$ and $\frac{x-1}{3}=\frac{y-3}{4}=\frac{z-4}{2}$ is

A. $3\sqrt{6}$

B. $6\sqrt{3}$

C. $6\sqrt{2}$

D. $2\sqrt{6}$

Correct answer: A

Source solution

$$\text{Line } L_1: \quad \frac{x-4}{4}=\frac{y+2}{5}=\frac{z+3}{3}=t$$

Hence,

$$(x,y,z)=(4,-2,-3)+t(4,5,3).$$

So,

$$A=(4,-2,-3), \qquad \vec{d}_1=(4,5,3).$$

Similarly,

$$\text{Line } L_2: \quad \frac{x-1}{3}=\frac{y-3}{4}=\frac{z-4}{2}=s$$

gives

$$(x,y,z)=(1,3,4)+s(3,4,2).$$

Thus,

$$B=(1,3,4), \qquad \vec{d}_2=(3,4,2).$$

The shortest distance between two skew lines is

$$D=\frac{\left|(\overrightarrow{AB})\cdot(\vec{d}_1\times\vec{d}_2)\right|}{\left|\vec{d}_1\times\vec{d}_2\right|}.$$

First,

$$\overrightarrow{AB}=(1-4,\ 3-(-2),\ 4-(-3))=(-3,5,7).$$

Now,

$$\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 5 & 3 \\ 3 & 4 & 2 \end{vmatrix}.$$

$$= \hat{i}(10-12) - \hat{j}(8-9) + \hat{k}(16-15)$$

$$= (-2, 1, 1).$$

Therefore,

$$\left|\vec{d}_1 \times \vec{d}_2\right| = \sqrt{(-2)^2 + 1^2 + 1^2} = \sqrt{6}.$$

Also,

$$(\overrightarrow{AB}) \cdot (\vec{d}_1 \times \vec{d}_2) = (-3, 5, 7) \cdot (-2, 1, 1)$$

$$= 6 + 5 + 7 = 18.$$

Hence,

$$D = \frac{18}{\sqrt{6}} = 3\sqrt{6}.$$

$$\boxed{3\sqrt{6}}$$

Hence, the correct option is

$$\boxed{(1)\ 3\sqrt{6}}.$$

Question 277

The shortest distance between the lines $\frac{x+7}{-6} = \frac{y-6}{7} = z$ and $\frac{7-x}{2} = y-2 = z-6$ is

A. $2\sqrt{29}$

B. 1

C. $\sqrt{\frac{37}{29}}$

D. $\frac{\sqrt{29}}{2}$

Correct answer: A

Source solution

$$L_1: \quad \frac{x+7}{-6} = \frac{y-6}{7} = z = t.$$

Hence,

$$x = -6t - 7, \qquad y = 7t + 6, \qquad z = t.$$

So a point on L_1 is

$$A = (-7, 6, 0),$$

and its direction vector is

$$\vec{d}_1 = (-6, 7, 1).$$

The second line is

$$\frac{7-x}{2} = y - 2 = z - 6 = s.$$

Thus,

$$x = 7 - 2s, \qquad y = s + 2, \qquad z = s + 6.$$

Hence a point on L_2 is

$$B = (7, 2, 6),$$

and its direction vector is

$$\vec{d}_2 = (-2, 1, 1).$$

The shortest distance between two skew lines is

$$D = \frac{\left|\overrightarrow{AB} \cdot (\vec{d}_1 \times \vec{d}_2)\right|}{\left|\vec{d}_1 \times \vec{d}_2\right|}.$$

First,

$$\overrightarrow{AB} = B - A = (14, -4, 6).$$

Now,

$$\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -6 & 7 & 1 \\ -2 & 1 & 1 \end{vmatrix}.$$

$$= \hat{i}(7 \cdot 1 - 1 \cdot 1) - \hat{j}(-6 \cdot 1 - 1(-2)) + \hat{k}(-6 \cdot 1 - 7(-2)).$$

$$= 6\hat{i} + 4\hat{j} + 8\hat{k}.$$

$$= 2(3, 2, 4).$$

Therefore,

$$\left|\vec{d}_1 \times \vec{d}_2\right| = \sqrt{6^2 + 4^2 + 8^2} = \sqrt{116} = 2\sqrt{29}.$$

Now,

$$\overrightarrow{AB} \cdot (\vec{d}_1 \times \vec{d}_2) = (14, -4, 6) \cdot (6, 4, 8).$$

$$= 84 - 16 + 48 = 116.$$

Hence,

$$D = \frac{116}{2\sqrt{29}} = \frac{58}{\sqrt{29}} = 2\sqrt{29}.$$

$$\boxed{D = 2\sqrt{29}}$$

Therefore, the correct option is

$\boxed{(1)\ 2\sqrt{29}}$.

Question 278

The shortest distance between the lines
$\frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5}$ and $\frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3}$ is

A. $6\sqrt{3}$

B. $4\sqrt{3}$

C. $5\sqrt{3}$

D. $8\sqrt{3}$

Correct answer: B

Source solution

Sol. $\frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5}\&\frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3}$

S.D $=\frac{\left|\left(\bar{a}_2-\bar{a}_1\right)\cdot\left(\bar{b}_1\times\bar{b}_2\right)\right|}{\left|\bar{b}_1\times\bar{b}_2\right|}$

$a_1=3,-15,9$

$\mathrm{b}_1=2,-7,5$

$a_2=-1,1,9$

$b_2=2,1,-3$

$a_2-a_1=-4,16,0$

$\overline{\mathrm{b}}_1\times\overline{\mathrm{b}}_2=\left|\hat{\mathrm{i}}\quad\hat{\mathrm{j}}\quad\hat{\mathrm{k}}2\quad-7\quad52\quad1\quad-3\right|=\hat{\mathrm{i}}(16)-\hat{\mathrm{j}}(-16)+\hat{\mathrm{k}}(16)$

$16(\hat{i}+\hat{j}+\hat{k})$

$\left|\overline{\mathrm{b}}_1\times\overline{\mathrm{b}}_2\right|=16\sqrt{3}$

$\therefore\left(\bar{a}_2-\bar{a}_1\right)\cdot\left(\overline{\mathrm{b}}_1-\overline{\mathrm{b}}_2\right)=16[-4+16]=(16)(12)$

S.D. $=\frac{(16)(12)}{16\sqrt{3}}=4\sqrt{3}$

Question 279

Let the image of the point $\mathbf{P(1,2,6)}$ in the plane passing through the points $\mathbf{A(1,2,0), B(1,4,1)}$ and $\mathbf{C(0,5,1)}$ be $\mathbf{Q(\alpha,\beta,\gamma)}$. Then $\left(\alpha^2+\beta^2+\gamma^2\right)$ is equal to :

A. 65

B. 70

C. 76

D. 62

Correct answer: A

Source solution

Sol.
Equation of plane $\mathbf{A(x-1)+B(y-2)+C(z-0)=0}$

Put $\mathbf{(1,4,1) \Rightarrow 2\,B+C=0}$

Put $\mathbf{(0,5,1) \Rightarrow -A+3\,B+C=0}$

Sub : $\mathbf{B-A=0 \Rightarrow\ A=B, C=-2\,B}$
$\mathbf{1(x-1)+1(y-2)-2(z-0)=0}$
$\mathbf{x+y-2z-3=0}$

Image is $\mathbf{(\alpha,\beta,\gamma)pt \equiv (1,2,6)}$
$\frac{\alpha-1}{1}=\frac{\beta-2}{1}=\frac{\gamma-6}{-2}=\frac{-2(1+2-12-3)}{6}$
$\frac{\alpha-1}{1}=\frac{\beta-2}{1}=\frac{\gamma-6}{-2}=4$
$\alpha=5, \beta=6, \gamma=-2 \Rightarrow \alpha^2+\beta^2+\gamma^2$
$=25+36+4=65$

Question 280

Let the line $\frac{x}{1}=\frac{6-y}{2}=\frac{z+8}{5}$ intersect the lines $\frac{x-5}{4}=\frac{y-7}{3}=\frac{z+2}{1}$ and $\frac{x+3}{6}=\frac{3-y}{3}=\frac{z-6}{1}$ at the points A and B respectively. Then the distance of the mid-point of the line segment AB from the plane $2x-2y+z=14$ is

A. 4

B. $\frac{10}{3}$

C. 3

D. $\frac{11}{3}$

Correct answer: A

Source solution

Sol. $\frac{x}{1}=\frac{y-6}{-2}=\frac{z+8}{5}=\lambda$

$$\frac{x-5}{4}=\frac{y-7}{3}=\frac{z+2}{1}=\mu \quad (1)$$

$$\frac{x+3}{4}=\frac{y-3}{-3}=\frac{z-6}{1}=\gamma \quad (2)$$

Intersection of (1) & (2) "A"

$(\lambda,-2\lambda+6,5\lambda-8)\&(4\mu+5,3\mu+7,\mu-2)$

$\lambda=1,\mu=-1$

$A(1,4,-3)$

Intersection of (1) & (3) "B"

$(\lambda,-2\lambda+6,5\lambda-8)\&(6\gamma-3,-3\gamma+3,\gamma+6)$

$\lambda=3$

$\gamma=1$

B(3, 0, 7)

Mid point of $A\&B \Rightarrow (2,2,2)$

Perpendicular distance from the plane

$2x-2y+z=14$

$\Rightarrow \left|\frac{2(2)-2(2)+2-14}{\sqrt{4+4+1}}\right|=4$

Question 281

If the foot of the perpendicular drawn from $(1, 9, 7)$ to the line passing through the point $(3, 2, 1)$ and parallel to the planes $x + 2y + z = 0$ and $3y - z = 3$ is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to

A. 3

B. 1

C. -1

D. 5

Correct answer: D

Source solution

Sol.

Given vectors:

$$\overline{n}_1 = (1, 2, 1), \quad \overline{n}_2 = (0, 3, -1)$$

Calculate the cross product $\overline{P} = \overline{n}_1 \times \overline{n}_2$:

$$\overline{P} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{vmatrix} = \hat{i}(-5) - \hat{j}(-1) + \hat{k}(3) = (-5, 1, 3)$$

The equation of the line is:

$$\frac{x-3}{-5} = \frac{y-2}{1} = \frac{z-1}{3} = \lambda$$

Substituting into the equation:

$$(2 - 5\lambda)(-5) + (\lambda - 7)(1) + (3\lambda - 6)(3) = 0$$

Expanding and simplifying:

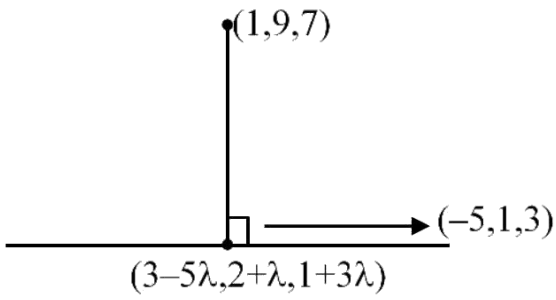
$$25\lambda - 10 + \lambda - 7 + 9\lambda - 18 = 0$$

$$35\lambda - 35 = 0$$

$$\lambda = 1$$

Substituting λ back to find the point:

$$(-2, 3, 4) \quad \text{where} \quad \alpha + \beta + \gamma = 5$$



Question 282

If the shortest distance between the lines $\frac{x-\lambda}{3}=\frac{y-2}{-1}=\frac{z-1}{1}$ and $\frac{x+2}{-3}=\frac{y+5}{2}=\frac{z-4}{4}$ is $\frac{44}{\sqrt{30}}$, then the largest possible value of $|\lambda|$ is equal to .

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 43

Source solution

$\bar{a}_1=\lambda\hat{i}+2\hat{j}+\hat{k}$

$\bar{a}_2=-2\hat{i}-5\hat{j}+4\hat{k}$

$\vec{p}=3\hat{i}-\hat{j}+\hat{k}$

$\vec{q}=-3\hat{i}+2\hat{j}+4\hat{k}$

$(\lambda+2)\hat{i}+7\hat{j}-3\hat{k}=\bar{a}_1-\bar{a}_2$

$\vec{p}\times\vec{q}=-6\hat{i}-15\hat{j}+3\hat{k}$

$\frac{44}{\sqrt{30}}=\frac{|-6\lambda-12-105-9|}{\sqrt{(-6)^2+(-15)^2+3^2}}$

$\frac{44}{\sqrt{30}}=\frac{|6\lambda+126|}{3\sqrt{30}}$

$132=|6\lambda+126|$

$\lambda=1,\lambda=-43$

$|\lambda|=43$

Question 283

If the line, $2x-y+3=0$ is at a distance $\frac{1}{\sqrt{5}}$ and $\frac{2}{\sqrt{5}}$ from the lines $4x-2y+\alpha=0$ and $6x-3y+\beta=0$, respectively, then the sum of all possible values of α and β is

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 30

Source solution

30

$L_1: 2x-y+3=0$

$L_2: 4x-2y+\alpha=0$

$L_3: 6x-3y+\beta=0$

$$\frac{\left|\frac{\alpha}{2}-3\right|}{\sqrt{5}}=\frac{1}{\sqrt{5}} \quad \Rightarrow \frac{\alpha}{2}-3=1,-1$$
$$\Rightarrow \alpha=8,4$$

$$\frac{\left|\frac{\beta}{3}-3\right|}{\sqrt{5}}=\frac{2}{\sqrt{5}} \quad \Rightarrow \frac{\beta}{3}-3=2,-2$$
$$\Rightarrow \beta=15,3$$

Question 284

If the shortest distance between the lines

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{\lambda}$$

and

$$\frac{x-2}{1}=\frac{y-4}{4}=\frac{z-5}{5}$$

is $\frac{1}{\sqrt{3}}$, then the sum of all possible values of λ is :

A. 16

B. 6

C. 12

D. 10

Correct answer: A

Source solution

$$\text{S.D.}=\frac{|5-2\lambda|}{\sqrt{(15-4\lambda)^2+(\lambda-10)^2+25}}=\frac{1}{\sqrt{3}}$$

$$\Rightarrow 3(5-2\lambda)^2=(15-4\lambda)^2+(\lambda-10)^2+25$$

$$\Rightarrow \lambda^2-4\lambda-12=0$$

$$(\lambda-6)(\lambda+2)=0$$

$$\lambda=6 \text{ or } \lambda=-2$$

$$\text{Sum of all possible values of } \lambda=6+(-2)=4$$

Question 285

The shortest distance between the lines $\frac{x+2}{1}=\frac{y}{-2}=\frac{z-5}{2}$ and $\frac{x-4}{1}=\frac{y-1}{2}=\frac{z+3}{0}$ is

A. 6

B. 9

C. 7

D. 8

Correct answer: B

Source solution

Sol. Given lines

$\frac{x+2}{1}=\frac{y}{-2}=\frac{z-5}{2}\&\frac{x-4}{1}=\frac{y-1}{2}=\frac{z+3}{0}$

Formula for shortest distance

S.D. $=\frac{\left|\begin{matrix}x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2\end{matrix}\right|}{\left|\begin{matrix}\hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2\end{matrix}\right|}$

$=\frac{\left|\begin{matrix}6 & 1 & -8 \\ 1 & -2 & 2 \\ 2 & 1 & 0\end{matrix}\right|}{\left|\begin{matrix}\hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 2 & 1 & 0\end{matrix}\right|}=\frac{54}{6}=9$

Question 286

Let the line passing through the points $(-1, 2, 1)$ and parallel to the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{4}$ intersect the line $\frac{x+2}{3} = \frac{y-3}{2} = \frac{z-4}{1}$ at the point P. Then the distance of P from the point $Q(4, -5, 1)$ is :

A. 5

B. 4

C. (2) 10

D. (3) $5\sqrt{6}$

Correct answer: D

Source solution

$$\text{Given line } L_1: \quad \frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{4}.$$

Its direction vector is

$$\vec{d}_1 = (2, 3, 4).$$

Therefore, the line passing through $(-1, 2, 1)$ and parallel to L_1 is

$$\frac{x+1}{2} = \frac{y-2}{3} = \frac{z-1}{4} = \lambda.$$

Hence,

$$x = -1 + 2\lambda, \qquad y = 2 + 3\lambda, \qquad z = 1 + 4\lambda.$$

The second line is

$$\frac{x+2}{3} = \frac{y-3}{2} = \frac{z-4}{1} = \mu.$$

Thus,

$$x = -2 + 3\mu, \qquad y = 3 + 2\mu, \qquad z = 4 + \mu.$$

Since the two lines intersect at P,

$$-1 + 2\lambda = -2 + 3\mu, \tag{1}$$

$$2 + 3\lambda = 3 + 2\mu, \tag{2}$$

$$1 + 4\lambda = 4 + \mu. \tag{3}$$

From (3),

$$\mu = 4\lambda - 3.$$

Substituting into (1),

$$-1 + 2\lambda = -2 + 3(4\lambda - 3),$$

$$-1 + 2\lambda = 12\lambda - 11,$$

$$10 = 10\lambda,$$

$$\lambda = 1.$$

Hence,

$$\mu = 4(1) - 3 = 1.$$

Therefore,

$$P = (-1 + 2,\ 2 + 3,\ 1 + 4) = (1, 5, 5).$$

Given

$$Q = (4, -5, 1).$$

Hence,

$$PQ = \sqrt{(4-1)^2 + (-5-5)^2 + (1-5)^2}.$$

$$= \sqrt{3^2 + (-10)^2 + (-4)^2}.$$

$$= \sqrt{9 + 100 + 16}.$$

$$= \sqrt{125} = 5\sqrt{5}.$$

$$\boxed{5\sqrt{6}}$$

Therefore, the distance of P from Q is

$\boxed{5\sqrt{6}}$.

Question 287

Let two vertices of triangle ABC be $(2,4,6)$ and $(0,-2,-5)$, and its centroid be $(2,1,-1)$. If the image of third vertex in the plane $x+2y+4z=11$ is (α,β,γ), then $\alpha\beta+\beta\gamma+\gamma\alpha$ is equal to

A. 72

B. 74

C. 76

D. 70

Correct answer: B

Source solution

Given points $A(2,4,6)$, $B(0,-2,-5)$, and $G(2,1,-1)$, we need to find the coordinates of vertex $C(x,y,z)$.

1. **Finding x**:

$$\frac{2+0+x}{3}=2 \Rightarrow 2+0+x=6 \Rightarrow x=4$$

2. **Finding y**:

$$\frac{4-2+y}{3}=1 \Rightarrow 4-2+y=3 \Rightarrow y=1$$

3. **Finding z**:

$$\frac{6-5+z}{3}=-1 \Rightarrow 6-5+z=-3 \Rightarrow z=-4$$

Thus, the coordinates of vertex C are $C(4,1,-4)$.

Next, we find the image of vertex C in the plane, denoted as (α,β,γ):

$$\frac{\alpha-4}{1}=\frac{\beta-1}{2}=\frac{\gamma+4}{4}=\frac{-2(4+2-16-11)}{21}$$

Calculating the right-hand side:

$$4+2-16-11=-21 \Rightarrow -2(-21)=42 \Rightarrow \frac{42}{21}=2$$

Now, we have:

$$\frac{\alpha-4}{1}=2 \Rightarrow \alpha=6$$

$$\frac{\beta-1}{2}=2 \Rightarrow \beta-1=4 \Rightarrow \beta=5$$

$$\frac{\gamma+4}{4}=2 \Rightarrow \gamma+4=8 \Rightarrow \gamma=4$$

Thus, the image coordinates are $(\alpha,\beta,\gamma)=(6,5,4)$.

Finally, we calculate:

$$\alpha\beta+\beta\gamma+\gamma\alpha=6\cdot5+5\cdot4+4\cdot6=30+20+24=74$$

The final answer is:

$$\alpha\beta + \beta\gamma + \gamma\alpha = 74$$

Question 288

If the square of the shortest distance between the lines

$$\frac{x-2}{1}=\frac{y-1}{2}=\frac{z+3}{-3}$$

and

$$\frac{x+1}{2}=\frac{y+3}{4}=\frac{z+5}{-5}$$

is $\frac{\mathbf{m}}{\mathbf{n}}$, where $\mathbf{m}, \mathbf{n}$ are coprime numbers, then $\mathbf{m}+\mathbf{n}$ is equal to:

A. 6

B. 9

C. 21

D. 14

Correct answer: B

Source solution

Sol.

Given vectors:

$$\vec{a}=(2,1,-3), \quad \vec{b}=(-1,-3,-5)$$

Calculate the cross product $\vec{p} \times \vec{q}$:

$$\vec{p} \times \vec{q}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix}$$

Calculating the determinant:

$$=2\hat{i}-\hat{j}$$

Next, find $\vec{b}-\vec{a}$:

$$\vec{b}-\vec{a}=(-1-2,-3-1,-5+3)=(-3,-4,-2)=-3\hat{i}-4\hat{j}-2\hat{k}$$

Now, calculate the scalar distance S_d:

$$S_d=\frac{|(\vec{b}-\vec{a}) \cdot (\vec{p} \times \vec{q})|}{|\vec{p} \times \vec{q}|}$$

First, compute the dot product:

$$(\vec{b}-\vec{a}) \cdot (\vec{p} \times \vec{q})=(-3,-4,-2) \cdot (2,-1,0)=-6+4+0=-2$$

Next, find the magnitude of $\vec{p} \times \vec{q}$:

$$|\vec{p} \times \vec{q}|=\sqrt{(2)^2+(-1)^2+(0)^2}=\sqrt{4+1}=\sqrt{5}$$

Substituting back into the equation for S_d:

$$S_d = \frac{|-2|}{\sqrt{5}} = \frac{2}{\sqrt{5}}$$

Now, square S_d:

$$(S_d)^2 = \left(\frac{2}{\sqrt{5}}\right)^2 = \frac{4}{5}$$

Let $m = 4$ and $n = 5$:

$$m + n = 9$$

Question 289

If the foot of the perpendicular drawn from $(1, 9, 7)$ to the line passing through the point $(3, 2, 1)$ and parallel to the planes $x + 2y + z = 0$ and $3y - z = 3$ is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to

A. 3

B. 1

C. -1

D. 5

Correct answer: D

Source solution

Sol. 4

$\overline{n}_1 = (1, 2, 1) \overline{n}_2 = (0, 3, -1)$

$\overline{P} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} 1 & 2 & 10 & 3 & -1 \end{vmatrix} = \hat{i}(-5) - \hat{j}(-1) + \hat{k}(3)$

$= (-5, 1, 3)$

$\therefore$ line : $\frac{x-3}{-5} = \frac{y-2}{1} = \frac{z-1}{3} = \lambda$

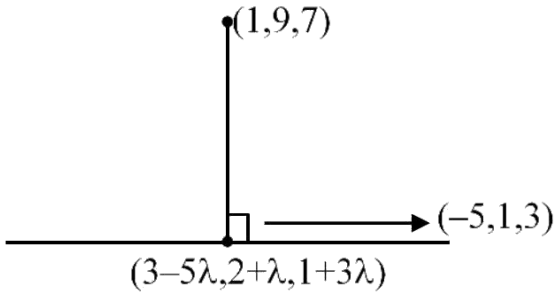
$(2 - 5\lambda) \cdot -5 + (\lambda - 7) \cdot 1 + (3\lambda - 6) \cdot 3 = 0$

$25\lambda - 10 + \lambda - 7 + 9\lambda - 18 = 0$

$35\lambda = 35$

$\lambda = 1$

$\therefore$ Point is $(-2, 3, 4) \quad \alpha + \beta + \gamma = 5$



Question 290

Let the line L pass through the point $(0,1,2)$, intersect the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and be parallel to the plane $2x+y-3z=4$. Then the distance of the point $P(1,-9,2)$ from the line L is

A. 9

B. $\sqrt{54}$

C. $\sqrt{69}$

D. $\sqrt{74}$

Correct answer: D

Source solution

$$\text{Let } \overrightarrow{AB}\cdot\vec{n}=0$$

$$\overrightarrow{AB}=(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1+4\lambda)\hat{k}$$

$$\vec{n}=2\hat{i}+\hat{j}-3\hat{k}$$

Calculating the dot product:

$$(1+2\lambda)\cdot 2+(1+3\lambda)\cdot 1+(1+4\lambda)\cdot(-3)=0$$

Expanding this gives:

$$2+4\lambda+1+3\lambda-3-12\lambda=0$$

Combining like terms:

$$5\lambda=0\Rightarrow\lambda=0$$

Now, the line $\overrightarrow{AB}$ can be expressed as:

$$\vec{r}=\hat{j}+2\hat{k}+\mu(\hat{i}+\hat{j}+\hat{k})$$

The general form of point Q is:

$$Q(\mu,1+\mu,2+\mu)$$

Since $\overrightarrow{PQ}\cdot\overrightarrow{AB}=0$:

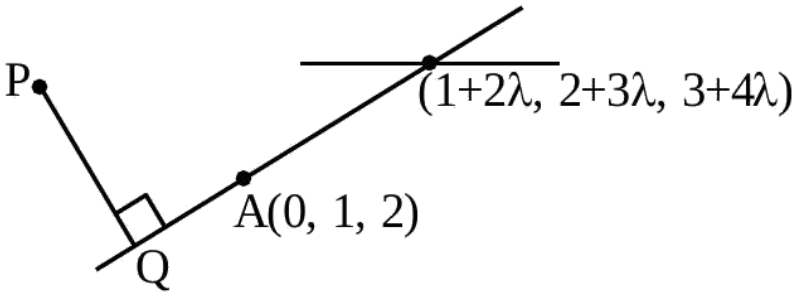
$$(\mu-1)+(10+\mu)+\mu=0$$

Simplifying this:

$$3\mu - 1 + 10 = 0 \Rightarrow 3\mu = -9 \Rightarrow \mu = -3$$

Finally, the distance is calculated as:

$$\text{Distance} = \sqrt{16 + 49 + 9} = \sqrt{74}$$



Question 291

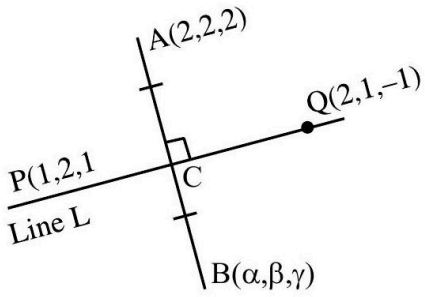
Consider a line L passing through the points $\mathbf{P(1,2,1)}$ and $\mathbf{Q(2,1,-1)}$. If the mirror image of the point $\mathbf{A(2,2,2)}$ in the line L is (α,β,γ), then $\alpha+\beta+6\gamma$ is equal to .....

Numerical answer: _____

Enter the requested value (an integer; 0 decimal places). Use the units and scaling stated in the question.

Correct answer: 6

Source solution



DR's of Line $\mathbf{L\equiv -1:1:2}$
DR's of $\mathbf{AB\equiv \alpha-2:\beta-2:\gamma-2}$
$\mathbf{AB\perp_{ar} L\Rightarrow 2-\alpha+\beta-2+2\gamma-4=0}$

$$\mathbf{2\gamma+\beta-\alpha=4} \qquad \mathbf{(1)}$$

Let C is mid-point of AB
$\mathbf{C\left(\frac{\alpha+2}{2},\frac{\beta+2}{2},\frac{\gamma+2}{2}\right)}$
DR's of $\mathbf{PC=\frac{\alpha}{2}:\frac{\beta-2}{2}:\frac{\gamma}{2}}$
line $\mathbf{L\|PC\Rightarrow \frac{-\alpha}{2}=\frac{\beta-2}{2}=\frac{\gamma}{4}=K(}$ let)
$\mathbf{\alpha=-2K}$
$\mathbf{\beta=2K+2}$
$\mathbf{\gamma=4K}$
use in $\mathbf{(1)\Rightarrow K=\frac{1}{6}}$
value of $\mathbf{\alpha+\beta+6\gamma=24K+2=6}$

Question 292

Let the line $\frac{x-3}{7}=\frac{y-2}{-1}=\frac{z-3}{-4}$ intersect the plane containing the lines $\frac{x-4}{1}=\frac{y+1}{-2}=\frac{z}{1}$ and $4ax-y+5z-7a=0=2x-5y-z-3, a \in \mathbb{R}$ at the point $P(\alpha, \beta, \gamma)$. Then the value of $\alpha+\beta+\gamma$ equals .

Numerical answer: _____

Enter the requested value or coefficient rounded to the nearest integer (0 decimal places).

Correct answer: 12

Source solution

Use the plane family $4ax-y+5z-7a+\lambda(2x-5y-z-3)=0$. Containing the point (4,-1,0) gives $9a+1+10\lambda=0$; containing the direction (1,-2,1) gives $4a+7+11\lambda=0$. Hence a=1, lambda=-1 and the plane is $x+2y+3z-2=0$. Substitution of (7t+3,2-t,3-4t) gives t=2, so P=(17,0,-5), whose coordinate sum is 12.

Question 293

Let in a $\triangle ABC$, the length of the side AC be 6, the vertex B be $(1, 2, 3)$ and the vertices A, C lie on the line

$$\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}.$$

Then the area (in sq. units) of $\triangle ABC$ is

A. 42

B. 21

C. 56

D. 17

Correct answer: B

Source solution

Let $\mathbf{M(3\lambda + 6, 2\lambda + 7, -2\lambda + 7)}$.

First, we find the vector $\overrightarrow{\mathbf{BM}}$:

$$\overrightarrow{\mathbf{BM}} = (3\lambda + 5)\hat{\mathbf{i}} + (2\lambda + 5)\hat{\mathbf{j}} + (-2\lambda + 4)\hat{\mathbf{k}}$$

Next, we compute the dot product $\overrightarrow{\mathbf{AC}} \cdot \overrightarrow{\mathbf{BM}} = \mathbf{0}$:

$$0 = 3(3\lambda + 5) + 2(2\lambda + 5) - 2(-2\lambda + 4)$$

We also have:

$$\overrightarrow{\mathbf{BM}} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$$

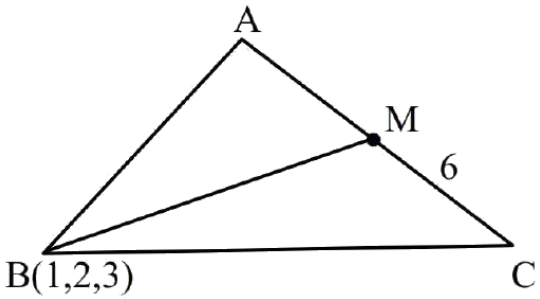
Now, we calculate the magnitude of $\overrightarrow{\mathbf{BM}}$:

$$|\overrightarrow{\mathbf{BM}}| = 7$$

Finally, we find the area:

$$\text{Area} = \frac{1}{2} \times 6 \times 7 = 21$$

Thus, the final answer is option (2).


A
M
6
B(1,2,3)
C

Mathematical Reasoning

Questions 294-300 / 7 selected / 2.3% practice weight / Historical appendix

Question 294

The compound statement $(\sim (P \wedge Q)) \vee ((\sim P) \wedge Q) \Rightarrow ((\sim P) \wedge (\sim Q))$ is equivalent to

A. $(\sim Q) \vee P$

B. $((\sim P) \vee Q) \wedge (\sim Q)$

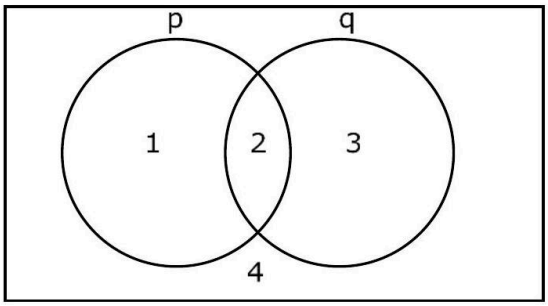
C. $(\sim P) \vee Q$

D. $((\sim P) \vee Q) \wedge ((\sim Q) \vee P)$

Correct answer: D

Source solution

$$(\sim (p \wedge q) \vee (\sim p \wedge q)) \rightarrow \sim p \wedge \sim q$$

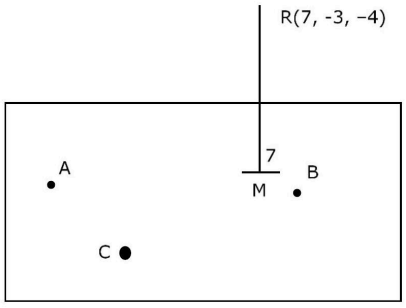


$$(1+2+4)+2 \Rightarrow 4$$
$$(1+2+4) \Rightarrow 4 \quad = \sim (1+2+4)+4$$
$$= 3+4$$

Ans. 4

8o. The distance of the point $(7, -3, -4)$ from the plane passing through the points $(2, -3, 1), (-1, 1, -2)$ and $(3, -4, 2)$ is :
(1) 5
(2) 4
(3) $5\sqrt{2}$
(4) $4\sqrt{2}$

Sol. (3)



$$\overline{n}_p = \overline{AB} \times \overline{DC} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 3 & -4 & 3 \\ -1 & 1 & -1 \end{vmatrix} = \langle 1, 0, -1 \rangle$$

Equation of plane

$$P: \quad 1(x-2) + 0(y+3) - (z-1) = 0$$
$$P: \quad x - z - 1 = 0$$

$$d(RM) = \left| \frac{7+4-1}{\sqrt{2}} \right| = 5\sqrt{2}$$

Question 295

The statement $(p \wedge (\sim q)) \Rightarrow (p \Rightarrow (\sim q))$ is

A. a tautology

B. a contradiction

C. equivalent to $p \vee q$

D. equivalent to $(\sim p) \vee (\sim q)$

Correct answer: A

Source solution

$(p \wedge (\sim q)) \Rightarrow (p \Rightarrow (\sim q))$

P	q	$\sim q$	$p \wedge \sim q$	$p \Rightarrow \sim q$	$(p \wedge \sim q) \Rightarrow (p \Rightarrow \sim q)$
T	T	F	F	F	T
T	F	T	T	T	T
F	T	F	F	T	T
F	F	T	F	T	T

Tautology

Question 296

(S1) $(p \Rightarrow q) \vee (p \wedge (\sim q))$ is a tautology
(S2) $((\sim p) \Rightarrow (\sim q)) \wedge ((\sim p) \vee q)$ is a contradiction.
Then

A. both (S1) and (S2) are correct

B. only (S1) is correct

C. only (S2) is correct

D. both (S1) and (S2) are wrong

Correct answer: B

Source solution

$S_1 : (P \Rightarrow q) V (P \wedge (\sim q))$

P	q	$P \Rightarrow q$	$\sim q$	$P \wedge \sim q$	$(P \Rightarrow q) V (P \wedge \sim q)$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	T	F	T

S_1 is a tautology
$S_2 : ((\sim P) \Rightarrow (\sim q)) \wedge ((\sim P) \vee q)$

$\sim P$	$\sim q$	$\sim P \Rightarrow \sim q$	$\sim P v q$	$((\sim P) \Rightarrow (\sim q)) \Lambda (\sim P) v q)$
F	F	T	T	T
F	T	T	F	F
T	F	F	T	F
T	T	T	T	T

S_2 is not a contradiction

Question 297

The negation of the expression $q \vee ((\sim q) \wedge p)$ is equivalent to

A. $(\sim p) \vee (\sim q)$

B. $p \wedge (\sim q)$

C. $(\sim p) \vee q$

D. $(\sim p) \wedge (\sim q)$

Correct answer: D

Source solution

$$\begin{aligned}&\sim (q \vee (\sim q) \wedge p)\\ &=\sim q \wedge (q \vee \sim p)\\ &=(\sim q \wedge q) \vee (\sim q \wedge \sim p)\\ &=\mathbf{F} \vee (\sim q \wedge \sim p)=(\sim q) \wedge (\sim p)\end{aligned}$$

Question 298

The statement $B \Rightarrow ((\sim A) \vee B)$ is equivalent to :

A. $A \Rightarrow (A \Leftrightarrow B)$

B. $A \Rightarrow ((\sim A) \Rightarrow B)$

C. $A \wedge B$

D. $A \vee B$

The original exam question has multiple correct choices and a conflicting printed ALLEN answer label. Independent truth-table verification establishes that original options 2, 3 and 4 are equivalent. In this corrected single-answer practice version, original options 1 and 2 are preserved; options 3 and 4 are replaced by A AND B and A OR B. Only option 2 is correct. The original defective choices remain in the review evidence.

Correct answer: B

Source solution

The target is $\neg B \vee \neg A \vee B$, a tautology. Option 2 is $\neg A \vee A \vee B$, also a tautology. Option 1 reduces to $\neg A \vee B$ and is false at A=true, B=false. Option 3 is false at A=false, B=false, and option 4 is also false there. Therefore only option 2 is equivalent for every truth assignment.

Question 299

Which of the following statements is a tautology ?

A. $p \vee (p \wedge q)$

B. $(\mathrm{p} \wedge (\mathrm{p} \rightarrow \mathrm{q})) \rightarrow \sim \mathrm{q}$

C. $(\mathrm{p} \wedge \mathrm{q}) \rightarrow (\sim (\mathrm{p}) \rightarrow \mathrm{q})$

D. $\mathrm{p} \rightarrow (\mathrm{p} \wedge (\mathrm{p} \rightarrow \mathrm{q}))$

Correct answer: C

Source solution

(i) $\mathrm{p} \rightarrow (\mathrm{p} \wedge (\mathrm{p} \rightarrow \mathrm{q}))$
$(\sim p) \vee (p \wedge (\sim p \vee q))$
$(\sim p) \vee (f \vee (p \wedge q))$
$\sim p \vee (p \wedge q) = (\sim p \vee p) \wedge (\sim p \vee q)$
$= \sim p \vee q$
(ii) $(p \wedge q) \rightarrow (\sim p \rightarrow q)$
$\sim (p \wedge q) \vee (p \vee q) = t$
$\{a, b, d\} \vee \{a, b, c\} = V$
Tautology
(iii) $(p \wedge (p \rightarrow q)) \rightarrow \sim q$
$\sim (p \wedge (\sim p \vee q)) \vee \sim q = \sim (p \wedge q) \vee \sim q = \sim p \vee \sim q$
Not tautology
(iv) $pV(p \wedge q) = p$

Not tautology.
So, option (2) is correct.

Question 300

Consider the following statements:

P : I have fever
Q: I will not take medicine
R : I will take rest.
The statement "If I have fever, then I will take medicine and I will take rest" is equivalent to:

A. $((\sim \mathrm{P}) \vee \sim \mathrm{Q}) \wedge ((\sim \mathrm{P}) \vee \mathrm{R})$

B. $(\mathrm{P} \vee \mathrm{Q}) \wedge ((\sim \mathrm{P}) \vee \mathrm{R})$

C. $((\sim \mathrm{P}) \vee \sim \mathrm{Q}) \wedge ((\sim \mathrm{P}) \vee \sim \mathrm{R})$

D. $(\mathrm{P} \vee \sim \mathrm{Q}) \wedge (\mathrm{P} \vee \sim \mathrm{R})$

Correct answer: A

Source solution

$$\mathrm{P} \rightarrow (\sim \mathrm{Q} \wedge \mathrm{R})$$
$$\sim \mathrm{P} \vee (\sim \mathrm{Q} \wedge \mathrm{R})$$
$$\Rightarrow ((\sim \mathrm{P}) \vee (\sim \mathrm{Q})) \wedge ((\sim \mathrm{P}) \vee \mathrm{R})$$
